1-5. A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.003 Wb. With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.

\[ R_1 = \frac{l}{\mu A} \left( \frac{\mu_r \mu_0 A}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} \right) = 58.36 \text{kA} \cdot \text{t/Wb} \]

\[ R_2 = \frac{l}{\mu A} \left( \frac{\mu_r \mu_0 A}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} \right) = 47.75 \text{kA} \cdot \text{t/Wb} \]

\[ R_3 = \frac{l}{\mu A} \left( \frac{\mu_r \mu_0 A}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} \right) = 95.49 \text{kA} \cdot \text{t/Wb} \]

The total reluctance is thus

\[ R_{TOT} = R_1 + R_2 + R_3 = 58.36 + 47.75 + 95.49 = 201.6 \text{kA} \cdot \text{t/Wb} \]

and the magnetomotive force required to produce a flux of 0.003 Wb is

\[ \mathcal{F} = \phi R = (0.003 \text{ Wb})(95.45 \text{kA} \cdot \text{t/Wb}) = 286.4 \text{ A} \cdot \text{t} \]

and the required current is

\[ i = \frac{\mathcal{F}}{N} = \frac{286.4 \text{ A} \cdot \text{t}}{500 \text{ t}} = 0.573 \text{ A} \]

The flux density on the top of the core is

\[ B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.4 \text{ T} \]

The flux density on the right side of the core is

\[ B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 1.2 \text{ T} \]
A 20-kVA 8000/277-V distribution transformer has the following resistances and reactances:

\[
R_p = 32 \, \Omega \quad R_s = 0.05 \, \Omega \\
X_p = 45 \, \Omega \quad R_s = 0.06 \, \Omega \\
R_e = 250 \, k\Omega \quad X_M = 30 \, k\Omega
\]

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the high-voltage side.

(b) Find the per-unit equivalent circuit of this transformer.

(c) Assume that this transformer is supplying rated load at 277 V and 0.8 PF lagging. What is this transformer’s input voltage? What is its voltage regulation?

(d) What is the transformer’s efficiency under the conditions of part (c)?

**Solution**

(a) The turns ratio of this transformer is \( a = 8000/277 = 28.89 \). Therefore, the secondary impedances referred to the primary side are

\[
R_s' = a^2 R_s = (28.89)^2 (0.05 \, \Omega) = 41.7 \, \Omega \\
X_s' = a^2 X_s = (28.89)^2 (0.06 \, \Omega) = 50.1 \, \Omega
\]

The resulting equivalent circuit is

(b) The rated kVA of the transformer is 20 kVA, and the rated voltage on the primary side is 8000 V, so the rated current in the primary side is 20 kVA/8000 V = 2.5 A. Therefore, the base impedance on the primary side is

\[
Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{8000 \, V}{2.5 \, A} = 3200 \, \Omega
\]

Since \( Z_{\text{pu}} = Z_{\text{actual}} / Z_{\text{base}} \), the resulting per-unit equivalent circuit is as shown below:
(c) To simplify the calculations, use the simplified equivalent circuit referred to the primary side of the transformer:

\[
\begin{align*}
I_p & \quad 32 \Omega \quad j45 \Omega \quad 41.7 \Omega \quad j50.1 \Omega \quad I_a \\
\left( \begin{array}{c}
V_p \\
\end{array} \right) & \quad 250 \text{ k}\Omega \quad j30 \text{ k}\Omega \\
\end{align*}
\]

The secondary current in this transformer is

\[
I_s = \frac{20 \text{kVA}}{277 \text{ V}} \angle -36.87^\circ \text{ A} = 72.2 \angle -36.87^\circ \text{ A}
\]

The secondary current referred to the primary side is

\[
I_s' = I_s a = \frac{72.2 \angle -36.87^\circ \text{ A}}{28.89} = 2.50 \angle -36.87^\circ \text{ A}
\]

Therefore, the primary voltage on the transformer is

\[
V_p = V_s' + \left( R_{EQ} + jX_{EQ} \right) I_s'
\]

\[V_p = 8000 \angle 0^\circ \text{ V} + (73.7 + j95.1)(2.50 \angle -36.87^\circ \text{ A}) = 8290 \angle 0.55^\circ \text{ V}\]

The voltage regulation of the transformer under these conditions is

\[
VR = \frac{8290 - 8000}{8000} \times 100\% = 3.63\%
\]

(d) Under the conditions of part (c), the transformer’s output power copper losses and core losses are:

\[
P_{OUT} = S \cos \theta = (20 \text{kVA})(0.8) = 16 \text{kW}
\]

\[
P_{CU} = \left( I_s' \right)^2 R_{EQ} = (2.5)^2(73.7) = 461 \text{ W}
\]

\[
P_{core} = \frac{V_s'^2}{R_c} = \frac{8290^2}{250,000} = 275 \text{ W}
\]

The efficiency of this transformer is

\[
\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{16,000}{16,000 + 461 + 275} \times 100\% = 95.6\%
\]
Matlab Code to Solve Problem 2-2

% hw #3, problem 2-2

RP = 32;
XP = 45;
RS = 0.05;
XS = 0.06;
RC = 250e3;
XM = 30e3;
VP = 8000;
VS = 277;
a = VP / VS;
S = 20e3;

% (a) eq ckt referred to HV side
RSp = a^2 * RS;
XSp = a^2 * XS;

fprintf('(a):
');
fprintf('RSp = %6.2f 
XSp = %6.2f

', RSp, XSp);

% (b) per-unit eq ckt
Sb = S;
Vb = VP;
Zb = Vb^2 / Sb;

RPpu = RP / Zb;
XPpu = XP / Zb;
RSpu = RSp / Zb;
XSpu = XSp / Zb;
RCpu = RC / Zb;
XMpu = XM / Zb;

fprintf('(b):
');
fprintf('RPpu = %8.4f 
XPpu = %8.4f 
', RPpu, XPpu);
fprintf('RSpu = %8.4f 
XSpu = %8.4f 
', RSpu, XSpu);
fprintf('RCpu = %8.4f 
XMpu = %8.4f

', RCpu, XMpu);

% (c) input voltage and voltage regulation

pf = .8;
phi = acos(pf);
SS = S * exp(j*phi);  % e^(j*phi) = cos(phi) + j*sin(phi)
IS = conj(SS / VS);
ISp = IS / a;
VSp = VS * a;

ZP = RP + j*XP;
ZSp = RSp + j*XSp;
ZC = RC + j*XM;
% Zeq = ZP + ZC/ZSp;
Zeq = ZP + ZSp*ZC/(ZSp+ZC);
Vin = VSp + ISp * Zeq;
VR = (abs(Vin) - VP) / VP * 100;  % voltage regulation

fprintf('(c):
');
fprintf('Vin = %10.2f /_ %6.2f 
', abs(Vin), angle(Vin)*180/pi);
fprintf('VR = %8.4f%% 

', VR);

% (d) input voltage and voltage regulation
Pout = S * pf;
If = (VSp + lSp * ZSp) / ZC;    % excitation current
IP = If + lSp;
Sin = Vin * conj(IP);
Pin = real(Sin);
eita = Pout / Pin * 100;

% Or use the simplified approximation method
% Pcu = (abs(lSp))^2 * real(Zeq);
% Pcore = (abs(VSp))^2 / RC;
% Pin = Pout + Pcu + Pcore;
% eita = Pout / Pin * 100;

fprintf('(d):
');
fprintf('Pout = %10.2f 
Pin = %10.2f
', Pout, Pin);
fprintf('eita = %8.4f%% 

', eita);

% Results
% (a):
% RSp =  41.71
% XSp =  50.05
%
% (b):
% RPpu =   0.0100
% XPpu =   0.0141
% RSpu =   0.0130
% XSpu =   0.0156
% RCpu =  78.1250
% XMpu =   9.3750
%
% (c):
% Vin =    8290.34 /_   0.55
% VR =   3.6292%
%
% (d):
% Pout =   16000.00
% Pin =   16727.21
% eita =  95.6525%
Special Problem No. 1

\[
\begin{align*}
\frac{I_P}{V_P} & \quad R = R_p + R'_s \\
I_S/a & \quad X = X_p + X'_s
\end{align*}
\]

Ignore the excitation branch.

At high voltage side:

\[
P_{sc} = \text{Re}\{V_{sc} \times I_{sc}^*\} = \text{Re}\{I_{sc} \times (R + jX) \times I_{sc}^*\}
\]

\[
= \text{Re}\{I_{sc}^2 \times (R + jX)\} = I_{sc}^2 \times R
\]

so, \( R = P_{sc} / I_{sc}^2 = 36 / 60^2 = 0.01 \Omega \)

and, \( V_{sc} = I_{sc} \times (R + jX) \Rightarrow |R + jX| = \frac{|V_{sc}|}{|I_{sc}|} \)

or, \( \sqrt{R^2 + X^2} = \frac{|V_{sc}|}{|I_{sc}|} = 20 / 60 = .3333 \)

so, \( X = \sqrt{.3333^2 - R^2} = \sqrt{.3333^2 - .01^2} = .3332 \Omega \)

(a) Referred to low voltage side:

\[
R' = R / a^2 = .01 / (1200 / 120)^2 = .0001 \Omega \\
X' = X / a^2 = .3332 / (1200 / 120)^2 = .0033 \Omega
\]

(b) per unit

\[
S_b = 72000 \text{VA} \quad V_b = 1200 \text{V} \\
Z_b = V_b^2 / S_b = 1200^2 / 72000 = 20 \Omega \\
R_{pu} = R / Z_b = .01 / 200 = .0005 \\
X_{pu} = X / Z_b = .3332 / 200 = .0166
\]