Chapter 1

SEARCH, CLASSIFICATION AND ATTACK DECISIONS FOR COOPERATIVE WIDE AREA SEARCH MUNITIONS *

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Abstract There are currently several wide area search munitions in the research and development phase within the Department of Defense. While the work on the airframes, sensors, target recognition algorithms and navigation schemes is promising, there are insufficient analytical tools for evaluating the effectiveness of these concept munitions. Simulation can be used effectively for this purpose, but analytical results are necessary for validating the simulations and facilitating the design trades early in the development process. Recent research into cooperative behavior for autonomous munitions has further highlighted the importance of fundamental analysis to steer the direction of this new research venture. This paper presents extensions to some classic work in the area of search and detection. The unique aspect of the munition problem is that a search agent is lost whenever an attack is executed. This significantly impacts the overall effectiveness in a multi-target/false target environment. While the analytic development here will concentrate on the single munition case, extensions to the multi-munition will be discussed to include the potential benefit from cooperative classification and engagement.

*The views expressed in this article are those of the authors and do not reflect the official policy of the U.S. Air Force, Department of Defense, or the U.S. Government.
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1. Introduction

Several types of wide area search munitions are currently being investigated within the U.S. Department of Defense research labs. These munitions are being designed to autonomously search, detect, recognize and attack mobile and relocatable targets. Additional work at the basic research level is investigating the possibility of having these autonomous munitions share information and act in a cooperative fashion[1][2][3]. While some of the research is promising, most of it is relying heavily on simulation to evaluate the performance of the multi-munition system. Analysis appears to be lacking with regards to the fundamental nature of the wide area search munition problem, to include identification of the critical munition and target environment parameters that must be adequately modeled for a valid simulation. Some classic work has been done in the area of optimal search [4][5][6], but this work does not address address the multi-target/false target scenario where an engagement comes at the expense of a search agent. Further, this work needs to be extended for application in cooperative behavior algorithms. This paper presents extensions to some of this classic work in the area of search and detection. Section 2 will present the basic method of analysis for the single munition/single target case. A uniform target distribution in a Poisson field of false targets will be considered. Section 3 will essentially repeat this analysis for the multiple target case, where a Poisson distribution will be assumed for both real and false targets. Section 4 will provide some analytic extensions for the multiple munition case and establish a basis for comparison with cooperative behavior approaches. Section 5 will use the analytical approaches to suggest methods and performance limitations for both cooperative engagement and classification. While the scenarios being considered are somewhat simplistic, the goal is to obtain closed form analytic results that can provide insight as to the fundamental nature of the wide area search munition problem.

2. The Single Munition/Single Target Case

A formula describing the probability of mission success for the single munition/single target scenario is as follows:

$$P_{MS} = P_K \cdot P_{TR} \cdot P_{LOS} \cdot P_E \tag{1}$$

where

P_K	$\equiv$	probability of target kill given Target Report (TR)
P_{TR}	$\equiv$	prob. of Target Report given clear Line of Sight to target
P_{LOS}	$\equiv$	prob. of clear LOS given target in Field of Regard (FOR)
P_E	$\equiv$	prob. the target will appear in the FOR

The expression in (1) is not the most general, but is easily shown to be equivalent to the more general equations. For example, P_K represents the product of guidance, hit, and kill probabilities. P_{TR} represents the product of detection and confirmation probabilities, where confirmation could be either classification or identification depending upon the level of discrimination being employed by the munition being considered. P_{LOS} could also be included in P_{TR}, and that is the convention that will be followed for the remainder of the development.

With the exception of P_E, the other probabilities are expressed as either single numerical values, or, in the case of P_{TR}, a table of values sometimes referred to as a confusion matrix. The term confusion matrix stems from the fact that it represents the probability of both correct *and* incorrect target reports. P_E is a function of the area to be searched, the density function describing the probable target location, and the ordering of the search process. Consider an autonomous munition looking for a single target (see Figure 1.1). For now we shall assume a single target is uniformly distributed amongst a Poisson field of false targets in the area A_S. A false target is considered to be something that has the potential for fooling the autonomous target recognition (ATR) algorithm (e.g., similar size, shape). Because we are considering single shot munitions, the probability of successfully engaging a target in the incremental area ΔA is conditioned on not engaging a false target prior to arriving at ΔA. The incremental probability of encountering a target in ΔA can be expressed as:

$$\Delta P_E \;=\; P_{\overline{FTA}}(A) \cdot \frac{\Delta A}{A_S} \tag{2}$$

where $P_{\overline{FTA}}(A)$ is the probability of having no false target attacks while searching the area A leading up to ΔA. A closed form expression $P_E(A_S)$ can be obtained as follows. Let

η	$\equiv$	false target probability density
$P_{FTA\mid FT}$	$\equiv$	probability of false target attack given encounter
α	$\equiv$	False Target Attack Rate (FTAR), $\alpha = \eta \; P_{FTA\mid FT}$
$P_{FT_{j,A}}$	$\equiv$	false target attack probability distribution

$P_{FT_{j,A}}$ represents the distribution of j, the expected number of false target attacks which would be reported by the seeker in a non-commit

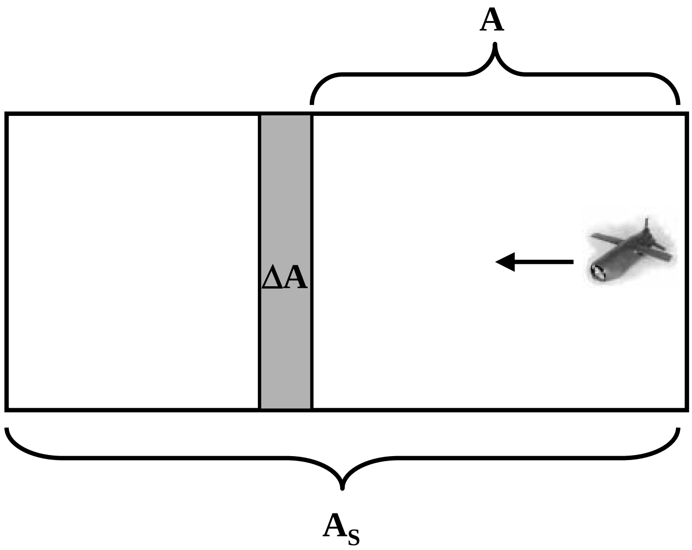

Figure 1.1. Single Target Search

mode, as a function of the area searched, A. It is a Poisson distribution with parameter $\lambda_{false} = \alpha A$.

$$P_{FT_{j,A}} = \frac{(\alpha A)^j e^{-\alpha A}}{j!} \tag{3}$$

The probability of searching A without executing a false target attack is

$$P_{\overline{FTA}}(A) = P_{FT_{0,A}} = e^{-\alpha A} \tag{4}$$

We can now formulate and solve an expression for the probability of encountering a target within A_S.

$$P_E(A_S) = \int_0^{A_S} \frac{e^{-\alpha A}}{A_S} dA = \frac{1 - e^{-\alpha A_S}}{\alpha A_S} \tag{5}$$

Note that the expression above assumes that the target is contained within A_S with probability one. For the case of uniform target/Poisson false target distribution, $P_E(A_S)$ can simply be multiplied by the probability that the target is contained within A_S. For cases of non-uniform target distributions a simple multiplication factor is no longer sufficient because the order of the search affects the probability of encountering the target.

Figure 1.2 shows the sensitivity of mission success to the FTAR (α) and probability of correct target report (P_{TR}) for $P_k = 0.8$ and $A_S =$

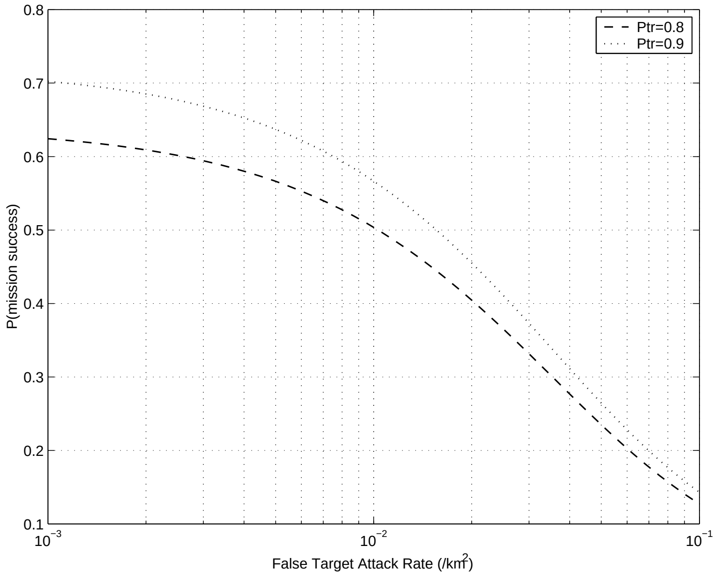

Figure 1.2. Single Target - Uniform Distribution

50 km^2. As shown, the probability of success begins to drop off rapidly for $\alpha > .01/km^2$. The problem is more sensitive to P_{TR} for low values of α than it is for higher values. While the probability of success may seem low, it can be improved by assigning multiple munitions to the same search area as will be discussed later.

3. The Single Munition/Multiple Target Case

There are several ways of looking at the multiple target case. If the objective is to find a specific target within a field of other targets, this could be treated in the same manner as the single target case; the other targets merely serve to increase the density of false targets. If any of the targets is considered valid then we need to be able to evaluate the probability of a successful encounter with any one of the targets. The single target case allowed us to determine the probability of finding and recognizing a target within a searchable area as

$$P_{RT}(A_S) = P_{TR} \, P_E(A_S) \tag{6}$$

For that case P_{TR} did not appear in the formulation for $P_E(A_S)$. For the multiple target case we will formulate it in a slightly different fashion. Referring back to Figure 1.1, the ability to find and recognize a target in the element of area ΔA is now conditioned on no false target attacks *and* no real target declarations/attacks prior to getting to ΔA. Assuming a Poisson distribution for both real and false targets (with $\lambda_{real} \neq \lambda_{false}$), our new formulation for the elemental probability of recognizing the target is

$$\Delta P_{RT}(A) = P_{TR}\ P_{\overline{FTA}}(A)\ P_{\overline{RT}}(A)\ \eta_T\ \Delta A \tag{7}$$

where η_T is the uniform target probability density. Implicit in this formulation is the assumption that $\eta_T \Delta A$, loosely interpreted as the probability of finding a target in the elemental area ΔA, is sufficiently less than one. This assumption is typically met for munitions with relatively small instantaneous sensor footprints relative to the average target density in the area. $P_{\overline{RT}}(A)$, the probability of not having recognized a real target after searching A, is obtained in the same manner as $P_{\overline{FTA}}(A)$. Specifically, $P_{RT_{k,A}}$ represents the distribution of k, the number of target recognitions that would be reported by the seeker in a non-commit mode, as a function of the area searched, A. It is a Poisson distribution with parameter $\lambda_{real} = P_{TR}\ \eta_T\ A$.

$$P_{RT_{k,A}} = \frac{(P_{TR}\ \eta_T\ P_{LOS}\ A)^k e^{-P_{TR}\ \eta_T\ A}}{k!} \tag{8}$$

The probability of searching A without executing a real *or* false target attack is

$$P_{\overline{RT,FTA}}(A) = P_{RT_{0,A}} \cdot P_{FT_{0,A}} = e^{-(P_{TR}\ \eta_T\ +\ \alpha)\ A} \tag{9}$$

We can now formulate and solve an expression for the probability of recognizing a target within A_S.

$$\begin{aligned} P_{RT_m}(A_S) &= \int_0^{A_S} P_{TR}\ \eta_T\ e^{-(P_{TR}\ \eta_T\ +\ \alpha)\ A}\ dA \\ &= \frac{P_{TR}\ \eta_T}{(P_{TR}\ \eta_T\ +\ \alpha)}(1 - e^{-(P_{TR}\ \eta_T\ +\ \alpha)\ A_S}) \end{aligned} \tag{10}$$

Figure 1.3 shows P_{MS} vs. α for the Poisson distributed multi-target case, with $\eta_T = .1/km^2$, $P_{TR} = 0.8$ and $P_k = 0.8$. As one would anticipate, it is far less sensitive to α than the single target case. Of greater interest is that the sensitivity to P_{TR} is greater for low values of α than it is for higher values; the opposite of the trend for the single target case. The reason for this is that a missed target is no longer a failed mission because there are other targets to be found. Further, the probability that these other targets will be encountered is high if the FTAR is sufficiently low.

4. Analytic Multi-Munition Extensions

The single target scenario can be extended to the multi-munition case in several ways. The easiest way is to divide the total search area

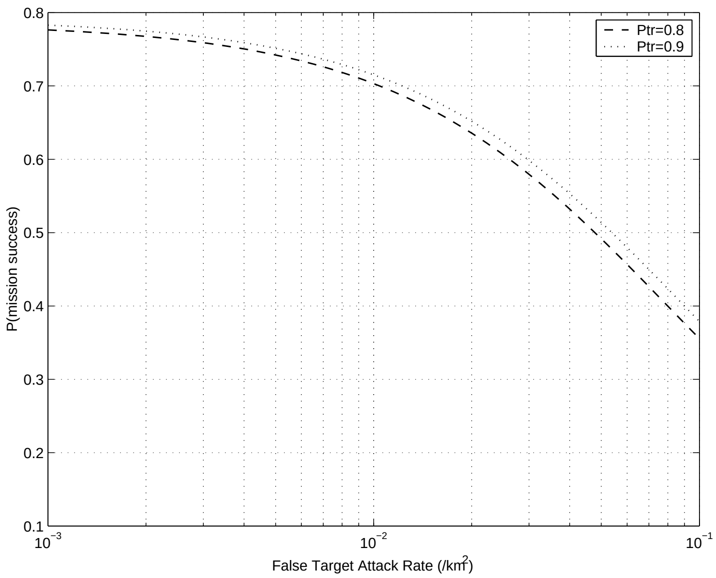

Figure 1.3. Multi Target - Uniform Distribution

by the number of munitions, and determine the P_{MS} for the munition searching the subarea that the target appears in. All other munitions find nothing for the single target case. P_{MS} will increase because A_S will decrease for all munitions, including the munition searching the subarea where the target happens to be. However, because this method assumes zero overlap in the subareas being searched, the probability of mission success is ultimately limited by the P_K for the single munition. If the warhead is not lethal enough to provide the desired probability of mission success from a single munition-target engagement, then overlapping search areas and multi-munition engagements must be considered. It should also be noted that unnecessarily limiting the area over which the munitions can search is not an efficient use of valuable search assets.

Extending expressions (1) and (5) above for multiple munitions becomes quickly complicated by path considerations and the degree of correlation assumed for the behavior of the munitions as they encounter either real or false targets. Considering only the terminal engagement for the time being, we can assume independent events for each warhead shot yielding an expression for $P_K^{[N]}$, the probability of kill for the case of N munitions engaging the target.

$$P_K^{[N]} = 1 - (1 - P_K)^N \tag{11}$$

One could (incorrectly) assume a similar roll-up of P_{MS} for the case of N munitions all searching the same area for a single target

$$P_{MS}^{[N]} = P_C(1 - (1 - \frac{P_{MS}}{P_C})^N) \quad (12)$$

where P_C refers to the probability that the target is located within the area being searched. The reason this formulation is incorrect is because it assumes the placement of clutter, non-targets and the real target is re-randomized for each munition. This can never be true for the case of several munitions looking for the same target in the same location. To address the problem correctly requires some consideration of the search path followed by the individual munitions.

The analysis in this section will be limited to the single target/multi-munition scenario. It is based on some unpublished results from Henderson[7] and some previous analysis by this author[8]. For this analysis, we not only need the probability of kill given access by N munitions, but we also need the probability that n munitions will encounter the target, $P_E^{[n]}$. For N munitions searching for the target, we can set up an expression for the probability of killing the target contained within the area A_S.

$$P_{MS}^{[N]}(A_S) \;=\; \sum_{n=1}^{N} P_K^{[n]} \cdot P_E^{[n]}(A_S) \quad (13)$$

While this expression appears simple, the complication arises when we attempt to define $P_E^{[n]}(A_S)$. For this, we will consider the cases of munitions searching over the same path and over opposing paths. For both cases we will limit the analysis to uncorrelated behavior on the part of the munitions. Certainly the assumption of uncorrelated behavior of homogeneous munitions searching over the same path is not valid, but it simplifies the development. For munitions searching over opposing paths there should be much less correlation because the munitions are seeing the targets, and false targets, at very different aspect angles.

4.1. Multi-Munition, Same Path Formulation

Consider the case of two munitions searching identical paths for a uniformly distributed single target, with a Poisson distribution of false targets in the area A_S. Equation 5 provides the expression for the probability that any given munition will have access to the target, but we need the probability that any combination of the munitions will have access to the target. The probability that one of the two munitions will have access is

$$P_E^{[1]}(A) \;=\; 2e^{-\alpha A}(1 - e^{-\alpha A}) \quad (14)$$

and the probability that both will have access is

$$P_E^{[2]}(A) \;=\; e^{-2\alpha A}. \tag{15}$$

The probability of kill given access by n weapons is $(1-(1-P_K P_{TR})^n)$. The two-weapon formula can now be expressed as

$$\begin{aligned} P_{MS_S}^{[2]}(A_S) \;&=\; P_C \int_0^{A_S} \Big(P_K P_{TR} 2e^{-\alpha A}(1-e^{-\alpha A}) \\ &\qquad + (1-(1-P_K P_{TR})^2)\; e^{-2\alpha A}\Big) \;\frac{dA}{A_S} & (16) \\ &=\; P_C P_K P_{TR} \left(\frac{2}{\alpha A_S}(1-e^{-\alpha A_S}) - \frac{P_K P_{TR}(1-e^{-2\alpha A_S})}{2\alpha A_S} \right). & (17) \end{aligned}$$

The more general expression for N munitions traversing the same search path can be expressed as

$$\begin{aligned} P_{MS_S}^{[N]}(A_S) \;&=\; \frac{1}{A_S} \sum_{n=1}^{N} \Bigg[(1-(1-P_K P_{TR})^n)\; \frac{N!}{n!(N-n)!} \\ &\qquad \cdot \int_0^{A_S} (e^{-\alpha A})^n \;(1-e^{-\alpha A})^{N-n} dA \Bigg]. \end{aligned} \tag{18}$$

4.2. Multi-Munition, Opposing Path Formulation

Now consider the case of two munitions searching opposing paths for a uniformly distributed single target, again with a Poisson distribution of false targets in the area A_S. The probability that the munition conducting the forward search will have access to the target is

$$P_{E_f}(A) \;=\; e^{-\alpha A} \tag{19}$$

and the probability that the munition conducting the reverse search will have access to the target is

$$P_{E_r}(A) \;=\; e^{-\alpha(A_s - A)}. \tag{20}$$

The probability that both munitions will have access to the target is

$$P_E^{[2]}(A) \;=\; e^{-\alpha A}\; e^{-\alpha(As-A)} \;=\; e^{-\alpha A_s} \tag{21}$$

and we note that it is constant! With these expressions, we can now lay out the expression for two munitions searching opposing paths.

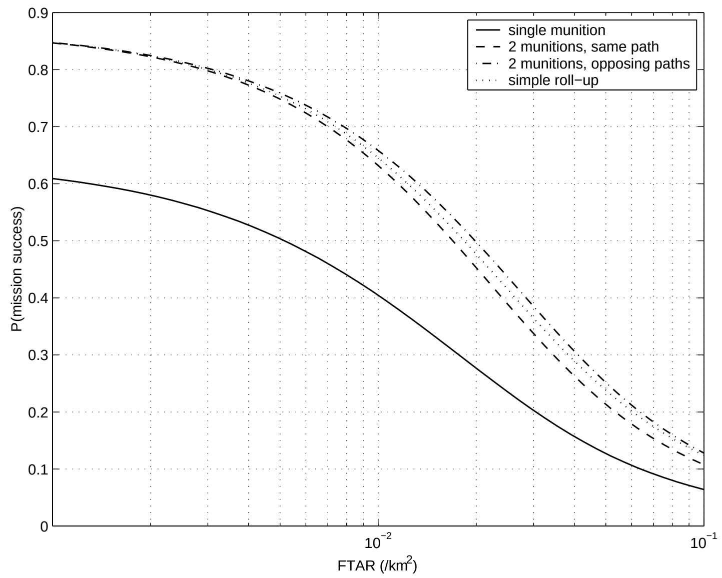

Figure 1.4. Path Considerations for Multi-Munition Case

$$
\begin{aligned}
P^{[2]}_{MS_O} &= P_C \int_0^{A_S} P_K P_{TR} \Big(e^{-\alpha A} + e^{-\alpha(A_S - A)} \\
&\qquad - 2e^{-\alpha A_S} + (2 - P_K P_{TR}) e^{-\alpha A_S} \Big) \frac{dA}{A_S} \\
&= P_C P_K P_{TR} \left(\frac{2}{\alpha A_S}(1 - e^{-\alpha A_S}) - P_K P_{TR}\, e^{-\alpha A_S} \right) \qquad (22)
\end{aligned}
$$

Similar expressions have been derived for four munitions (two searching each direction) and six munitions (three searching each direction) but a general formula for N munitions has yet to be defined.

For the same total number of munitions, the opposing path case will produce the highest mission success value, the same path case will produce the lowest mission success value, and the simple multi-munition roll-up will produce a value in between the other two. The graph shown in Figure 1.4 is for two munitions, but it should be noted that the differences between the curves increases with the number of munitions used in the analysis.

It is worth repeating that the assumption of uncorrelated behavior (at either a real or false target) is not strictly valid, and we should expect a high degree of correlation for the case where the munitions are traversing the same path in the same direction. For scenarios where the potential false targets greatly outnumber the real targets, correlated behavior will degrade the overall mission success rate. For this reason, search patterns should be planned which decrease the degree of correlated behavior at false targets. This can be done through the use of lateral offsets between munitions and/or different approach vectors. While this does not make the assumption of uncorrelated behavior valid, it can reduce the degree of correlation at both targets and false targets. Analytically it becomes intractable to define an expression for arbitrary numbers of munitions executing arbitrarily specified search patterns and degrees of correlation. However, for any realistic effectiveness analysis these are the cases we are most interested in. A numerical simulation with Monte-Carlo runs is the only practical way of performing this more general analysis, and work is currently being done in this area.

5. Implications for Cooperative Behavior

5.1. Cooperative Engagement of Targets

The analysis above indicates that munition success is quite sensitive to FTAR and the target location probability distribution. Cooperative behavior and control has recently become an active area of research, and one of the objectives of the research is to reduce the sensitivity to FTAR and target location error. While there are many aspects of cooperative behavior and control, the two most applicable to the wide area search munition problem are cooperative engagement and cooperative classification. Cooperative engagement is defined as a munition initiating an attack on a target that a second munition has declared. Cooperative classification involves using multiple looks from one or more munitions in order to improve the probability of making a correct target declaration.

Cooperative engagement has potential benefits in several areas. For a target that has been correctly identified, it increases the probability of kill for that target by virtue of multiple warhead events. This increase was described earlier in equation (11). If munitions are chosen for cooperative engagement that are unlikely to find additional targets through continued search, the probability of kill for found targets could be increased without significantly degrading the probability of finding additional targets. Complications arise due to the possibility that declared targets are not real targets (incorrect classification), thus diverting valuable resources for no real gain. Ultimately what is required is a way to compare the probability of success from continued search with the probability of success from attacking a found target. Figure 1.5 depicts all the possible events from searching a Poisson field of targets and non-targets. Starting from the top, the munition can either encounter a target, incorrectly declare (and attack) a false target, or run out of gas prior to the occurrence of either a target encounter or false target attack. For a given target encounter the munition may recognize it as such or incorrectly bypass it. A recognized target will be attacked with an uncertain outcome of the warhead event. A incorrectly bypassed target essentially means that the munition is still in a search mode, but the time remaining for search will be decreased by (on average) the expected time to next target encounter, $E[t_E] = (\eta_T V W)^{-1}$. The basic tree structure repeats at each occurrence of the search state. If we define success as a lethal warhead event on any one of the real targets appearing within the search area, the entire tree can be collapsed to a single level, with the probability of real target attack, false target attack, and running out of gas given as a function of search time remaining,

$$P_{RT}(t_r) = \frac{P_{TR}\eta_T}{\alpha + P_{TR}\eta_T}(1 - e^{-(\alpha+P_{TR}\eta_T)Vt_rW}) \quad (23)$$

$$P_{FTA}(t_r) = \frac{\alpha}{\alpha + P_{TR}\eta_T}(1 - e^{-(\alpha+P_{TR}\eta_T)Vt_rW}) \quad (24)$$

$$P_{OOG}(t_r) = e^{-(\alpha+P_{TR}\eta_T)Vt_rW} \quad (25)$$

respectively. The term Vt_rW represents the area that can be searched in the time remaining t_r for a given munition velocity V and search width W. The probability for success in search is simply

$$P_{SS}(t_r) = P_K \cdot P_{RT}(t_r) \quad (26)$$

Figure 1.6 shows a similar tree structure for the event of a declared target to be attacked. Any declared target may or may not be a real target, and any real declared target may or may not be recognized by a second munition being sent to engage it. A correctly found real target again results in a warhead event with uncertain outcome. If the declared target is actually a false target, a second munition may or may not make the same mistake as the first munition making the initial declaration. If it correctly identifies it as a false target it resumes search with the time remaining decreased by the time to arrive at the initially declared target. The analysis in this paper assumes independent events for target/false target declarations, so the probabilities for all individual munitions are the same without regard to order of occurrence. The probability of a real target given that a target declaration has been made can be expressed as

$$P_{RT|TR} = \frac{P_{TR}\eta_T}{P_{TR}\eta_T + P_{FTA|FT}\eta} \quad (27)$$

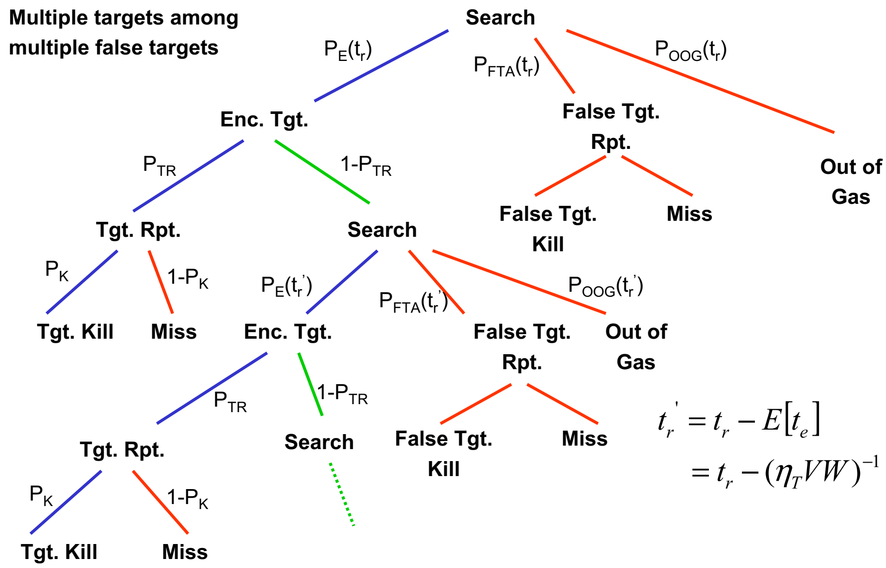

Figure 1.5. Possible Search Outcomes

With this, we can now define the probability of success from engaging a declared target.

$$\begin{aligned} P_{SA} &= P_K P_{TR} P_{RT|TR} + P_{SS}(t_r - t_{ETA}) \cdot (1 - P_{TR}) \cdot P_{RT|TR} \\ &\quad + P_{SS}(t_r - t_{ETA}) \cdot (1 - P_{FTA|FT}) \cdot (1 - P_{RT|TR}) \end{aligned} \tag{28}$$

Note that this expression includes success increments from continued search after either missing a real target or bypassing false targets initially declared as real. It should also be noted that once a munition has declared a target, the probability of success in attacking it is

$$P_{SA|TR} = P_K \, P_{RT|TR}. \tag{29}$$

Further note that $P_{SA|TR}$ will always be greater than $P_{SS}(t_r)$, regardless of the amount of search time remaining.

Equations (28) and (29) apply to the case where the target has not yet been attacked by a previous munition. For the case where one or more attacks on the target have previously taken place, the value of attacking the target again should be decreased due to the chance of the target being previously killed. With the assumption of independent warhead events, the probability of a live target $(1 - P_K)^N$ given N previous attacks can be used as a multiplying factor with equation (29) or the

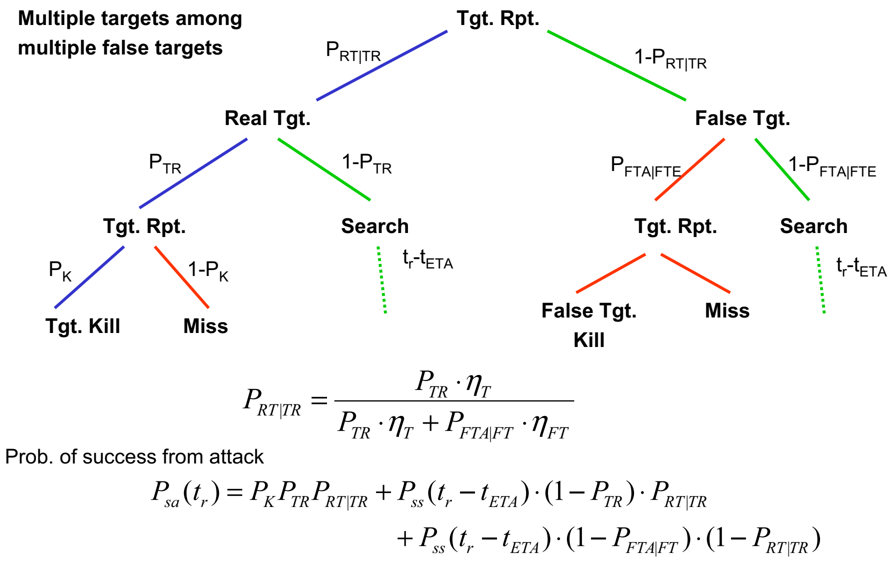

Figure 1.6. Possible Engagement Outcomes

first term in (28). Equations (28), (29) with the appropriate multiplying factor for multiple attacks, and equation (26) are quantitative measures of the value associated with either attacking a target or continuing to search, respectively. Although the situation gets quickly complicated when one considers multiple target types of differing priorities, these basic measures can serve as a basis for making decisions on cooperative engagement. Work is progressing to incorporate these quantitative measures into overall schemes for cooperative engagement, and simulation based analysis is being used to evaluate the schemes under more general multi-target scenarios.

5.2. Cooperative Classification of Targets

Cooperative engagement has proven effective in increasing the probability of success for cases where FTAR is low, but it provides no benefit, and is possibly detrimental, for cases where FTAR is higher[2]. The reason for this is that cooperative engagement increases the chance that additional munitions will be lost due to false target attacks. Cooperative classification may provide some help for this problem because it can effectively reduce the false target attack rate; however, cooperative classification can potentially increase the chance of missing real targets.

For the case of independent classification events, a simple analysis can provide some insight into the potential strengths and weaknesses of cooperative classification. While independent events may not be a realistic assumption, it can be used to provide a first-cut sensitivity analysis.

Figure 1.7 shows a surface plot of P_{MS} vs. FTAR and P_{TR} for two munitions conducting a non-cooperative, opposing path search for a single target in a 100 km^2 area. The extreme sensitivity to FTAR is clearly evident (note the log scale for FTAR), as well as the general insensitivity to P_{TR}. The most basic implementation of cooperative classification would require two subsequent looks with the same classification prior to declaring a real target. For this simple two look scenario the effective probability of false target attack given false target encounter is now the square of the value for non-cooperative classification. This yields an effective FTAR of $\alpha = (P_{FTA|FT})^2\eta$ which is obviously reduced from the non-cooperative case. The downside of this approach is that the effective probability of correct target report is also the square of the value for the non-cooperative case, thus reducing a value that you would like to keep as close to unity as possible. Figure 1.8 shows a similar surface plot for two munitions conducting a cooperative search along parallel paths. Each munition is responsible for searching the total area along the same path, and each can cooperatively classify and attack targets without delay. The sensitivity to FTAR is reduced using cooperative classification, but there is obviously an increase in the sensitivity to P_{TR}. Of great interest is the difference between the two surfaces, thus indicating where cooperative classification improves or degrades the overall probability of success. Figures 1.9 and 1.10 show the difference between the two surface plots, and clearly there are regions where cooperative classification can help ($\Delta P_{MS} > 0$) and hurt ($\Delta P_{MS} < 0$). For low single munition FTAR, the decrease in ${P_{TR}}^2$ clearly outweighs any benefit from further decreasing FTAR through cooperative behavior.

A better implementation of cooperative behavior might be to attempt an opposing path formulation. In order to keep the assumption of zero delay for cooperative classification and engagement, we need to assume that each munition is on a parallel track covering half the total area, and at the end of the track they switch lanes and reverse direction. We can add the probability of success for this second pass conditioned on not having engaged the target *or* any false target on the first pass. Assuming independent passes (not completely unreasonable because of the reversed direction), we can assume that the conditional probability of success for the second pass is the same as the unconditional probability of success for the first pass. The probability of not having engaged the target or a

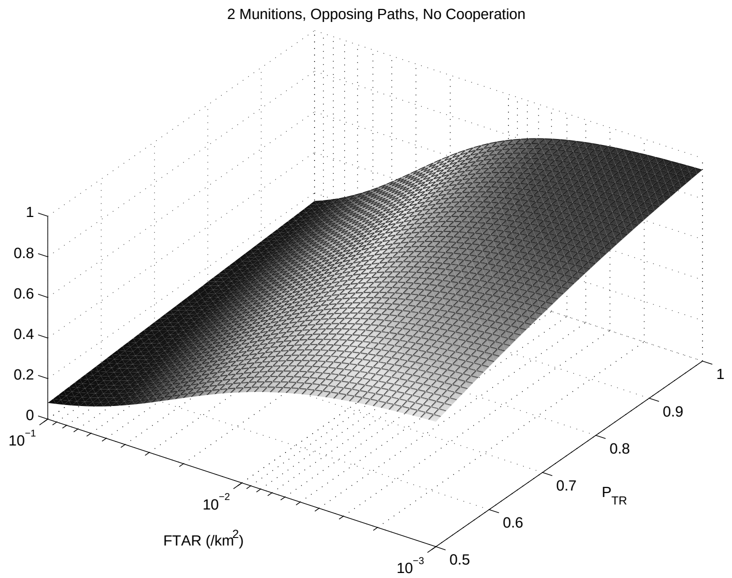

Figure 1.7. Complementary Search w/ 2 Munitions

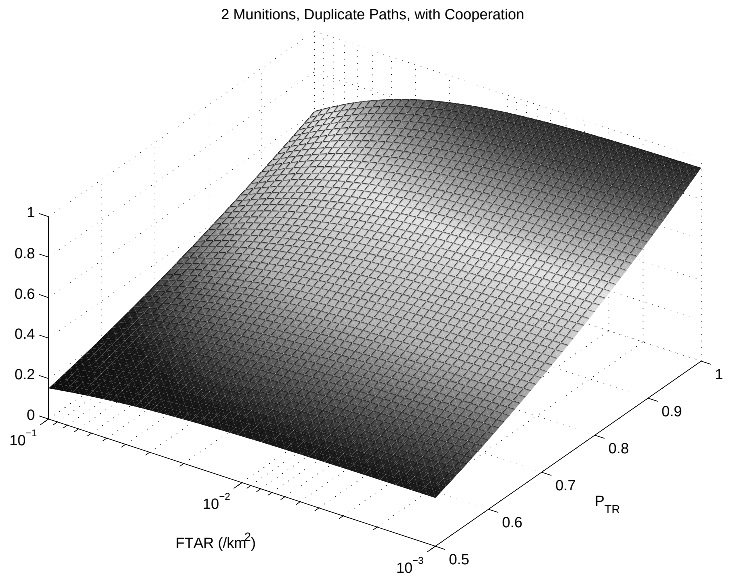

Figure 1.8. Cooperative Classification/Engagement w/ 2 Munitions

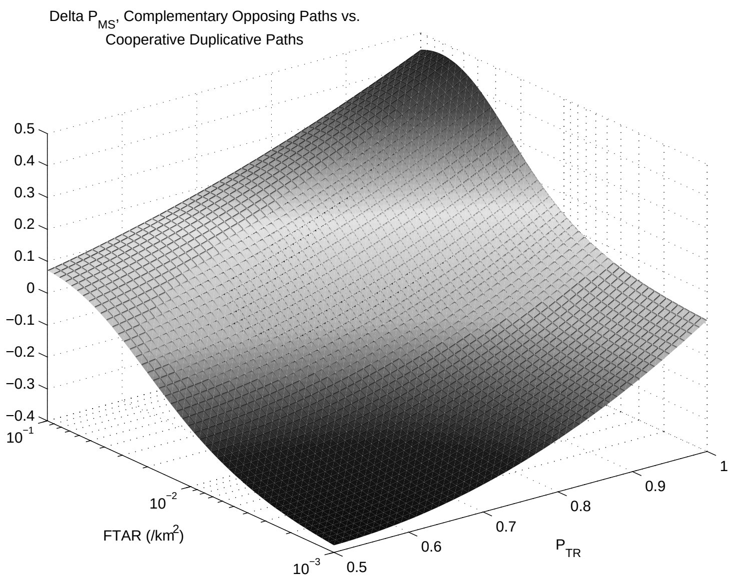

Figure 1.9. Difference of Cooperative and Complementary Behavior

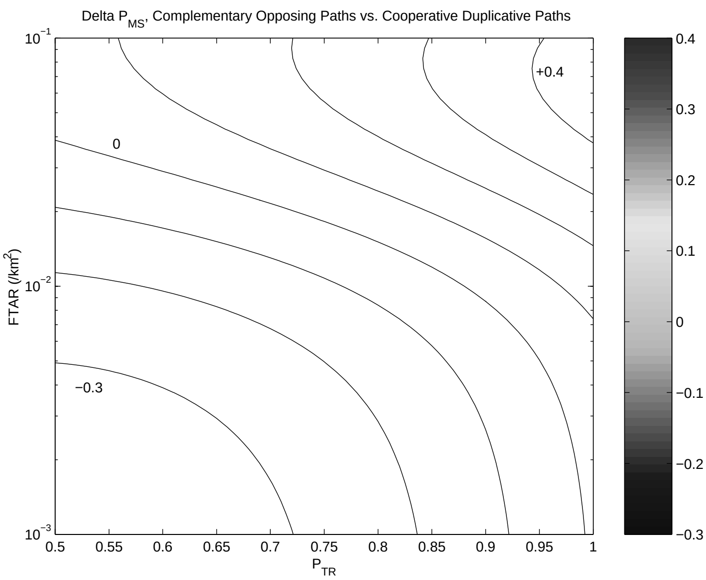

Figure 1.10. Difference Contour of Cooperative and Complementary Behavior

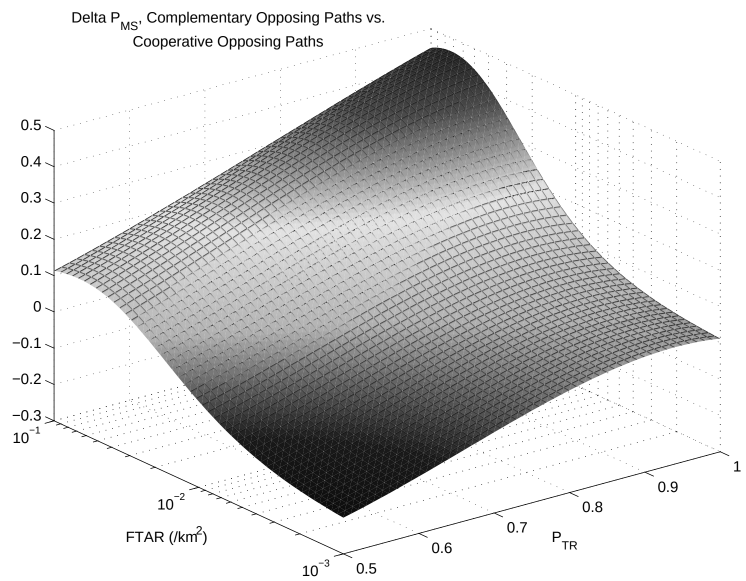

Figure 1.11. Difference of Cooperative and Complementary Behavior

false target on the first pass is

$$P_{\bar{1}} = e^{-P_{FTA|FT}{}^{2}\eta A_S} \left(1 - P_{TR}{}^{2}\right) \tag{30}$$

Figures 1.11 and 1.12 show the ΔP_{MS} for the complementary (non-cooperating) opposing path vs. the cooperative opposing path scenarios. Once again, a positive ΔP_{MS} indicates the cooperative scheme outperforms the non-cooperating scheme. While the situation is improved over the cooperative, same path scheme, there is still a significant operating region for low FTAR's where it is no longer beneficial to employ cooperation. It should be noted that these results are for a very limited single target, two munition scenario, but they highlight some important considerations for the more complex multi-target/multi-munition scenarios we would like to address. Mobile targets will make cooperative classification even more difficult because the target may not be in the same location when the second munition arrives for its confirming look. Ultimately the benefit of analysis such as this may be to provide desired operating regions for the operating characteristic of the ATR algorithm. P_{TR} and $P_{FTA|FT}$ are competing objectives, so system trades will need to be made.

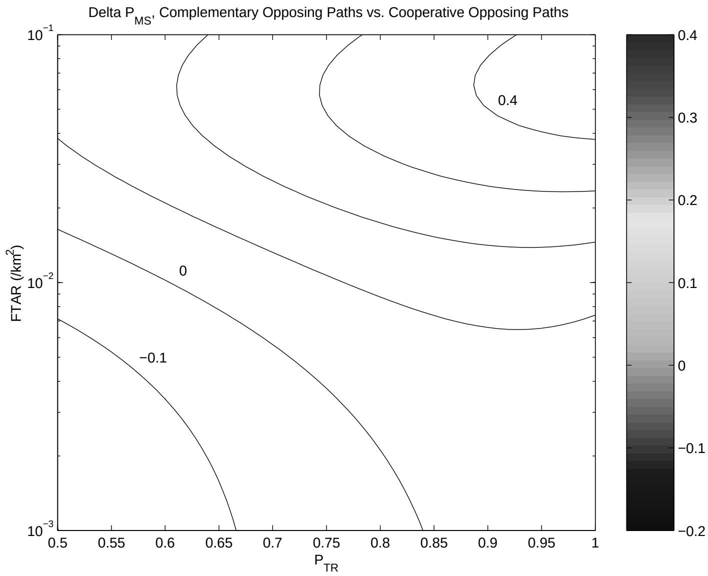

Figure 1.12. Difference Contour of Cooperative and Complementary Behavior

6. Conclusions

This paper has presented some fundamental analysis of the wide area search munition problem. False target attack rate and the distribution of targets have been identified as critical factors in this problem. Extensions to existing search theory have been presented, specifically in the area of multiple target/false target scenarios. Finally, the implication of this analysis for cooperative behavior has been discussed. Decision factors for cooperative engagement were developed, and the strengths and limitations of cooperative classification were highlighted. Overall, cooperative behavior holds promise for the autonomous wide area search munition problem, but analysis such as has been presented here is required in order to develop behavior algorithms that degrade gracefully in the presence of uncertain target location and/or false targets.

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