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RaÚl OrdÓÑez; Kevin M. Passino

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Indirect adaptive control for a class of non-linear systems with a time-varying structure

RAÚL ORDÓÑEZ†* and KEVIN M. PASSINO‡

In this paper we present an indirect adaptive control method for a class of uncertain non-linear systems with a time-varying structure. We view the non-linear systems as composed of a finite number of 'pieces', which are interpolated by functions that depend on a possibly exogenous scheduling variable. We assume that each piece is in strict feedback form, and show that the indirect adaptive method yields stability of all signals in the closed-loop, as well as convergence of the state vector to a residual set around the equilibrium, whose size can be set by the choice of several design constants. The class of systems considered here is a generalization of the class of strict feedback systems traditionally considered in the backstepping literature. Finally, we apply the indirect adaptive method to the problem of regulation of aircraft wing rock with a time-varying angle of attack.

1. Introduction

The field of non-linear adaptive control has had a rapid development in the last decade. The papers by Narendra and Parthasarathy (1990), Polycarpou and Ioannou (1991) and Sanner and Slotine (1992) gave birth to an important branch of adaptive control theory, the non-linear on-line function approximation based control (which includes neural and fuzzy approaches). The papers by Narendra and Parthasarathy (1990), Polycarpou and Ionnou (1991), Sanner and Slotine (1992), Yabuta and Yamada (1992, Liu and Chen (1993), Sadegh (1993), Liu and Chen (1993), Rovithakis and Christodoulou (1994), Yeșildirek and Lewis (1995), Farrell (1996), Polycarpou (1996) and Polycarpou and Mears (1998) make use of neural networks as approximators of non-linear functions, whereas Su and Stepanenko (1994), Wang (1994), Hsu and Fu (1995), Chen *et al.* (1996) and Lee and Wang (1996) use fuzzy systems for the same purpose, and Narendra and Parthasarathy (1990) and Rovithakis and Christodoulou (1994) use dynamical neural networks. The neural and fuzzy approaches are most of the time equivalent, differing between each other only for the structure of the approximator chosen (Spooner and Passino 1996). Among those works in which tunable parameterized functions are used, a major difference can be devised in the choice of the parameterization: linear in Polycarpou and Ioannou (1991), Sanner and Slotine (1992), Sadegh (1993), Su and Stepanenko (1994), Carelli *et al.* (1995), Hsu and Fu (1995), Yin and Lee (1995), Chen *et al.* (1996), Farrell (1996), Fabri and Kadirkamanathan (1996), Polycarpou (1996) and Spooner and Passino (1996) and non-linear in Narendra and Parthasarathy (1990), Yabuta and Yamada (1992), Liu and Chen (1993), Chen and Liu (1994), Yeșildirek and Lewis (1995), Lewis *et al.* (1996) and Polycarpou and Mears (1998). Indirect adaptive control seems to be the most commonly explored strategy to approach the adaptive control problem, as is done in the present paper. The direct adaptive control approach, where the control law is generated without need for approximation of the plant's dynamics, is taken more infrequently (e.g., in Rovithakis and Christodoulou 1995, Spooner and Passino 1996, Ordóñez and Passino 2001).

Some researchers have attempted to examine classes of systems other than that of feedback linearizable plants. In particular, plants whose dynamics can be expressed in the so-called 'strict feedback form' have been considered, and techniques like backstepping and adaptive backstepping (Kanellakpoulos *et al.* 1991, Krstić *et al.* 1995) have emerged for their control. Backstepping currently appears to be the most systematic method for non-linear control design through step-by-step construction of quadratic Lyapunov functions. The stability analysis is constructive, and it generates stabilizing control laws. In addition, backstepping provides some clear guidelines on the effects of design parameters on transient performance of the closed loop system. Adaptive backstepping (in particular through the tuning functions approach (Kristic *et al.* 1992)) builds on these results, and, through the element of adaptation, provides a systematic method to stabilize 'parametric strict feedback systems', a class of non-linear systems that have a linear dependence on unknown parameters. The papers by Polycarpou (1996) and Polycarpou and Mears (1998) present an extension of the tuning functions approach in which the non-linearities of the strict feedback system are not assumed to be parametric uncertainties, but rather completely unknown non-linearities to be approximated

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* Author for correspondence. e-mail: ordonez@rowan.edu
† Department of Electrical and Computer Engineering, Rowan University, 201 Mullica Hill Road, Glassboro, NJ 08028, USA.
‡ Department of Electrical Engineering, The Ohio State University, 2015 Neil Avenue, Columbus, OH 43210, USA.

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on-line with non-linearly parameterized function approximators. Both the adaptive methods in Krstić *et al.* (1995) and in Polycarpou (1996) and Polycarpou and Mears (1998) attempt to approximate the dynamics of the plant on-line, so they may be classified as indirect adaptive schemes. Recently, the paper by Zhang *et al.* (1999) studies the problem of indirect adaptive control for a class of strict feedback systems more general than in Polycarpou and Mears (1998). However, the authors use integral Lyapunov functions, which causes the control law to include an integral term that needs to be evaluated on-line. The present paper does not have this limitation, and it achieves indirect adaptive control for a class of systems still more general than the ones considered in the aforementioned works.

In this paper, we have combined an extension of the class of strict feedback systems considered in Polycarpou (1996), Polycarpou and Mears (1998) and Zhang *et al.* (1999) with the concept of a dynamic structure that depends on time through a scheduling variable. In this manner we propose a class of uncertain non-linear systems with a time-varying structure for which we develop an indirect adaptive control approach. This class of systems is a generalization of the class of strict feedback systems traditionally considered in the literature. Additionally, the indirect approach proposed here, presents the possibility of exploiting the time-varying structure of the plant to yield a 'localized' controller with scheduled gains.

This paper is organized as follows. In § 2 we present the indirect adaptive control method and provide a closed loop stability proof in the Appendix. In § 3 we illustrate the practical significance of the method by proposing a solution to the problem of aircraft wing rock regulation when the angle of attack is not held constant, but allowed to vary within a range of interest. Section 4 concludes the paper.

2. Indirect adaptive control

Consider the class of continuous time non-linear systems given by

$$\left.\begin{aligned} \dot{x}_i &= \sum_{j=1}^{R} \rho_j(v)(\phi_i^j(X_i) + \psi_i^j(X_i)x_{i+1}) \\ \dot{x}_n &= \sum_{j=1}^{R} \rho_j(v)(\phi_n^j(X_n) + \psi_n^j(X_n)u) \end{aligned}\right\} \quad (1)$$

where $i = 1, 2, \ldots, n-1$, $X_i = [x_1, \ldots, x_i]^\top$, and $X_n \in \mathrm{R}^n$ is the state vector, which we assume measurable, and $u \in \mathrm{R}$ is the control input. The variable $v \in \mathrm{R}^q$ may be an additional input or a possibly exogenous 'scheduling variable.' We assume that v and its derivatives up to and including the $(n-1)$th one are bounded and available for measurement, which may imply that v is given by an external dynamical system. The functions ρ_j, $j = 1, \ldots, R$ may be considered to be 'interpolating functions' that introduce the varying structural nature of system (1), since they combine R systems in strict feedback form (given by the ϕ_i^j and ψ_i^j functions, $i = 1, \ldots, n$, $j = 1 \ldots, R$) and the combination depends on the variable v. Here, we assume that the functions ρ_j are n times continuously differentiable, and that they satisfy, for all $v \in \mathrm{R}^q$

$$0 < \sum_{j=1}^{R} \rho_j(v) < \infty \quad (2)$$

Denote for convenience

$$\left.\begin{aligned} \phi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v)\phi_i^j(X_i) \\ \psi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v)\psi_i^j(X_i) \end{aligned}\right\} \quad (3)$$

We will assume that ϕ_i^c and ψ_i^c are sufficiently smooth in their arguments, and that they satisfy, for all $X_i \in \mathrm{R}^i$ and $v \in \mathrm{R}^q$, $i = 1, \ldots, n$

$$\left.\begin{aligned} \phi_i^c(0, v) &= 0 \\ \psi_i^c(X_i, v) &\neq 0 \end{aligned}\right\} \quad (4)$$

Here, we will develop an indirect adaptive control method for the class of systems (1). We assume that the interpolation functions ρ_j are known, but the functions ϕ_i^j and ψ_i^j (which constitute the underlying dynamics of the system) are unknown. Note that this assumption is made for technical reasons, and it constitutes a limitation of the method proposed here. Removing this assumption would make (1) a true representation of a time-varying non-linear system, instead of a system with a time-varying structure. Following the indirect adaptive control methodology, we will attempt to identify the unknown functions and then construct a stabilizing control law based on the approximations to the plant dynamics. This approximation will be performed within a compact set $\mathcal{S}_n \subset \mathrm{R}^n$ of arbitrary size but known and *a priori* fixed which contains the origin. In this manner, the results obtained are semi-global, in the sense that they are valid as long as the state remains within $\mathcal{S}_n$, but this set can be made as large as desired by the designer. In particular, with enough plant information it can be made large enough that the state never exits it.

For each vector X_i we will assume the existence of a compact set $\mathcal{S}_i \subset \mathrm{R}^i$ specified *a priori* by the designer. We will consider trajectories within the compact sets $\mathcal{S}_i, i = 1, \ldots, n$, where the sets are constructed such that $\mathcal{S}_i \subset \mathcal{S}_{i+1}$, for $i = 1, \ldots, n-1$. We will also assume

$$0 < \underline{\psi}_i(X_i, v) \leq \psi_i^c(X_i, v), \qquad i = 1, \ldots, n \quad (5)$$

with the lower bounds $\underline{\psi}_i$ known (e.g., $\underline{\psi}_i$ may be constant, or given by $\underline{\psi}_i = \sum_{j=1}^{R} \rho_j \underline{\psi}_i^j$, with $\underline{\psi}_i^j > 0$).

The class of plants (1) is, to our knowledge, the most general class of systems considered so far within the context of adaptive control based on backstepping. In particular, in Krstić *et al.* (1995), Polycarpou (1996) and Polycarpou and Mears (1998), which are indirect adaptive approaches, the input functions ψ_i^j are assumed to be constant for $i = 1, \dots, n$. This assumption allows the authors of those works to perform a simpler stability analysis, which becomes more complex in the general case treated here. Also, the addition of the interpolation functions ρ_j, $j = 1, \dots, R$, extends the class of strict feedback systems to one including some cases of gain-scheduling, and it can be related to the multiple-model (Narendra and Balakrishnan 1997) and model-switching (Morse 1996) approaches (see Remark 6 for more discussion on this topic). Note that if we let $R = 1$ and $\rho_1(v) = 1$ for all v, together with $\psi_i^c = 1$, $i = 1, \dots, n$, we have the particular case considered in Polycarpou (1996) and Polycarpou and Mears (1998).

For the ith state, consider the local representation of the system dynamics within a compact set $\mathcal{S}_i \subset \mathrm{R}^i$

$$\left.\begin{aligned} \phi_i^j(X_i) &= \theta_{\phi_i^j}^{*\top} \zeta_{\phi_i^j}(X_i) + \delta_{\phi_i^j}(X_i) \qquad \text{for all } X_i \in \mathcal{S}_i \\ \psi_i^j(X_i) &= \theta_{\psi_i^j}^{*\top} \zeta_{\psi_i^j}(X_i) + \delta_{\psi_i^j}(X_i) \qquad \text{for all } X_i \in \mathcal{S}_i \end{aligned}\right\} \tag{6}$$

where the parameter vectors $\theta_{\phi_i^j}^*$ and $\theta_{\psi_i^j}^*$ are optimum in the sense that they minimize the representation errors $\delta_{\phi_i^j}$ and $\delta_{\psi_i^j}, j = 1, \dots, R$, respectively, within some suitable compact sets. More specifically, let Ω_{ϕ_i} and Ω_{ψ_i} be some compact parameter sets, within which we let

$$\left.\begin{aligned} \theta_{\phi_i^j}^* &= \arg \min_{\theta_{\phi_i^j} \in \Omega_{\phi_i}} \left[\sup_{X_i \in \mathcal{S}_i} \left| \phi_i^j - \theta_{\phi_i^j}^{\top} \zeta_{\phi_i^j}(x_i) \right| \right] \\ \theta_{\psi_i^j}^* &= \arg \min_{\theta_{\psi_i^j} \in \Omega_{\psi} i} \left[\sup_{X_i \in \mathcal{S}_i} \left| \psi_i^j - \theta_{\psi_i^j}^{\top} \zeta_{\psi_i^j}(x_i) \right| \right] \end{aligned}\right\} \tag{7}$$

Therefore, there exist constants $d_{\phi_i^j}$ and $d_{\psi_i^j}$ such that, for all $X_i \in \mathcal{S}_i$

$$\left.\begin{aligned} |\delta_{\phi_i^j}(X_i)| &\le d_{\phi_i^j} \\ |\delta_{\psi_i^j}(X_i)| &\le d_{\psi_i^j} \end{aligned}\right\} \tag{8}$$

Let $\delta_{\phi_i}(X_i, v) = \sum_{j=1}^{R} \rho_j(v) \delta_{\phi_i^j}(X_i)$ and $\delta_{\psi_i}(X_i, v) = \sum_{j=1}^{R} \rho_j(v) \delta_{\psi_i^j}(X_i)$. Then,

$$\left.\begin{aligned} \phi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \theta_{\phi_i^j}^{*\top} \zeta_{\phi_i^j}(X_i) + \delta_{\phi_i}(X_i, v) \\ \psi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \theta_{\psi_i^j}^{*\top} \zeta_{\psi_i^j}(X_i) + \delta_{\psi_i}(X_i, v) \end{aligned}\right\} \tag{9}$$

Also, for $i = 1, \dots, n$, assume known bounds d_{ϕ_i} and d_{ψ_i} are known such that

$$\left.\begin{aligned} d_{\phi_i} &= \sup_{v \in \mathrm{R}^q, X_i \in \mathcal{S}_i} |\delta_{\phi_i}(X_i, v)| \\ d_{\psi_i} &= \sup_{v \in \mathrm{R}^q, X_i \in \mathcal{S}_i} |\delta_{\psi_i}(X_i, v)| \end{aligned}\right\} \tag{10}$$

We choose the function approximators as

$$\left.\begin{aligned} \hat{\phi}_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \hat{\theta}_{\phi_i^j}^{\top} \zeta_{\phi_i^j}(X_i) \\ \hat{\psi}_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \hat{\theta}_{\psi_i^j}^{\top} \zeta_{\psi_i^j}(X_i) \end{aligned}\right\} \tag{11}$$

Let the parameter errors for the jth approximator in the ith state be $\Phi_{\phi_i^j} = \theta_{\phi_i^j}^* - \hat{\theta}_{\phi_i^j}$ and $\Phi_{\psi_i^j} = \theta_{\psi_i^j}^* - \hat{\theta}_{\psi_i^j}$, $i = 1, \dots, n, j = 1, \dots, R$. We assume $\zeta_{\phi_i^j}$ and $\zeta_{\psi_i^j}$ to be at least $n - i$ times continuously differentiable, and to satisfy, for $i = 1, \dots, n, j = 1, \dots, R$

$$\left| \frac{\partial^{n-i} \zeta_{\phi_i^j}}{\partial X_i^{n-i}} \right| < \infty, \qquad \left| \frac{\partial^{n-i} \zeta_{\psi_i^j}}{\partial X_i^{n-i}} \right| < \infty \tag{12}$$

2.1. *Indirect adaptive control theorem*

Here, we state the main control result for the indirect adaptive case. For convenience, we use the notation $\nu_i = [v, \dot{v}, \dots, v^{(i-1)}] \in \mathrm{R}^{q \times i}, i = 1, \dots, n$. We will generally omit arguments of the functions, except when the dependencies need to be emphasized.

Theorem 1: *Consider system* (1) *with the state vector* X_n *available for measurement and the scheduling vector* ν_{n-1} *bounded and available for measurement, together with assumptions* (2), (4) *and* (5). *Assume also that* $X_i(0)$ *lies within a sufficiently small subset*† *of* $\mathcal{S}_i \subset \mathrm{R}^i, i = 1, \dots, n$, *where* $\mathcal{S}_i$ *are compact sets specified* a priori, *and large enough that* X_i *does not exit them. Consider the diffeomorphism*

$$\left.\begin{aligned} z_1 &= x_1 \\ z_i &= x_i - \alpha_i - 1 - \alpha_{i-1}^s, \qquad i = 2, \dots, n \end{aligned}\right\} \tag{13}$$

where $\alpha_m = \alpha_m(X_i, \nu_i, \theta_{\phi_i^j}, \theta_{\psi_i^j}; i = 1, \dots, m; j = 1, \dots, R)$, $\alpha_m^s = \alpha_m^s(X_i, \nu_i, \theta_{\phi_i^j}, \theta_{\psi_i^j}; i = 1, \dots, m; j = 1, \dots, R)$, *and they are given by*

$$\alpha_1 = \frac{1}{\hat{\psi}_1^c}(-\hat{\phi}_1^c - c_1 z_1) \tag{14}$$

$$\alpha_1^s = -\frac{z_1}{\underline{\psi}_1}(k_{11} + k_{12}\alpha_1^2) \tag{15}$$

† See Remark 8 for elaboration.

and, for $m = 2, \ldots, n$

$$\alpha_m = \frac{1}{\hat{\psi}_m^c}\Bigg(-\hat{\phi}_m^c - c_m z_m - \hat{\psi}_{m-1}^c z_{m-1} + \sum_{i=1}^{m-1}\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)(\hat{\phi}_i^c + \hat{\psi}_i^c x_{i+1}) + \sum_{i=1}^{m-1}\sum_{j=1}^{R}\Bigg[\left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\phi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\phi_i^j}}\right)^\top \gamma_{\phi_i^j}\tau_{\phi^j_{i(m+1-i)}} + \left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\psi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\psi_i^j}}\right)^\top \gamma_{\psi_i^j}\tau_{\psi^j_{i(m+1-i)}}\Bigg] + \left(\frac{\partial \alpha_{m-1}}{\partial \nu_{m-1}} + \frac{\partial \alpha_{m-1}^s}{\partial \nu_{m-1}}\right)\dot{\nu}_{m-1} + \kappa_m\Bigg) \tag{16}$$

$$\alpha_m^s = -\frac{z_m}{\underline{\psi}_m}\Bigg(k_{m1} + k_{m2}\alpha_m^2 + \sum_{i=1}^{m-1} k_{i1}\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)^2 + \sum_{i=1}^{m-2} k_{i2}x_{i+1}^2\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)^2 + 2k_{(m-1)2}x_m^2\left(\frac{\partial \alpha_{m-1}}{\partial x_{m-1}} + \frac{\partial \alpha_{m-1}^s}{\partial x_{m-1}}\right)^2 + 2k_{(m-1)2}z_{m-1}^2\Bigg) \tag{17}$$

where $c_i > 0, k_{i1} > 0$ *and* $k_{i2} > 0, i = 1, \ldots, n$ *are design constants,* $\gamma_{\phi_i^j} > 0, \gamma_{\psi_i^j} > 0, i = 1, \ldots, n, j = 1, \ldots, R$ *are adaptation constants, and we define auxiliary functions* $(m = 2, \ldots, n)$

$$\tau_{\phi^j_{i1}} = \rho_j\zeta_{\phi_i^j} z_i - \frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\hat{\theta}_{\phi_i^j}, \qquad i = 1, \ldots, n \tag{18}$$

$$\tau_{\psi^j_{i1}} = \rho_j\zeta_{\psi_i^j} z_i\alpha_i - \frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\hat{\theta}_{\psi_i^j}, \qquad i = 1, \ldots, n \tag{19}$$

$$\tau_{\psi^j_{(m-1)2}} = \tau_{\psi^j_{(m-1)1}} - \rho_j\zeta_{\psi^j_{(m-1)1}} \times \left(\left(\frac{\partial \alpha_{m-1}}{\partial x_{m-1}} + \frac{\partial \alpha_{m-1}^s}{\partial x_{m-1}}\right)z_m x_m - z_{m-1}z_m\right) \tag{20}$$

$$\tau_{\phi^j_{i(m+1-i)}} = \tau_{\phi^j_{i(m-i)}} - \rho_j\zeta_{\phi_i^j}\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)z_m, \qquad i = 1, \ldots, m-1 \tag{21}$$

$$\tau_{\psi^j_{i(m+1-i)}} = \tau_{\psi^j_{i(m-i)}} - \rho_j\zeta_{\psi_i^j}\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)z_m x_{i+1}, \qquad i = 1, \ldots, m-2 \tag{22}$$

with $\sigma_{\phi_i^j} > 0, \sigma_{\psi_i^j} > 0, i = 1, \ldots, n, j = 1, \ldots, R$ *'leakage' constants, and, for* $m = 3, \ldots, n$ *(letting* $\kappa_2 = 0$*)*

$$\kappa_m = -\sum_{i=1}^{m-2}\sum_{j=1}^{R}\Bigg[\left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial \alpha_{l-1}}{\partial \hat{\theta}_{\phi_i^j}} + \frac{\partial \alpha_{l-1}^s}{\partial \hat{\theta}_{\phi_i^j}}\right)^\top\right)\gamma_{\phi_i^j}\zeta_{\phi_i^j} \times \left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right) + \left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial \alpha_{l-1}}{\partial \hat{\theta}_{\psi_i^j}} + \frac{\partial \alpha_{l-1}^s}{\partial \hat{\theta}_{\psi_i^j}}\right)^\top\right)\gamma_{\psi_i^j}\zeta_{\phi_i^j} \times \left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)x_{i+1}\Bigg] \tag{23}$$

Assume the functions $\zeta_{\phi_i^j}(X_i)$ *and* $\zeta_{\phi_i^j}(X_i)$ *to satisfy assumption* (12)*. Consider the adaptation laws for the parameter vectors*

$$\hat{\theta}_{\phi_i^j} \in \mathrm{R}^{N_{\phi_i^j}} \qquad \text{and} \qquad \hat{\theta}_{\psi_i^j} \in \mathrm{R}^{N_{\psi_i^j}}$$

$N_{\phi_i^j} \in \mathrm{N}, N_{\psi_i^j} \in \mathrm{N}, i = 1, \ldots, n, j = 1, \ldots, R,$ *and letting* $z_{n+1} = 0, x_{n+1} = 0$

$$\left.\begin{aligned} \dot{\hat{\theta}}_{\phi_i^j} &= \gamma_{\phi_i^j}\tau_{\phi^j_{i(n+1-i)}} \\ &= \gamma_{\phi_i^j}\rho_j\zeta_{\phi_i^j}\left(z_i - \sum_{l=1}^{n-1}\left(\frac{\partial \alpha_l}{\partial x_i} + \frac{\partial \alpha_l^s}{\partial x_i}\right)z_{l+1}\right) - \sigma_{\phi_i^j}\hat{\theta}_{\phi_i^j} \\ \dot{\hat{\theta}}_{\psi_i^j} &= \gamma_{\psi_i^j}\tau_{\psi^j_{i(n+1-i)}} \\ &= \gamma_{\psi_i^j}\rho_j\zeta_{\psi_i^j}\left(z_i\alpha_i - \sum_{l=1}^{n-1}\left(\frac{\partial \alpha_l}{\partial x_i} + \frac{\partial \alpha_l^s}{\partial x_i}\right)z_{l+1}x_{i+1} + z_i z_{i+1}\right) - \sigma_{\psi_i^j}\hat{\theta}_{\psi_i^j} \end{aligned}\right\} \tag{24}$$

Then, the control law

$$u = \alpha_n + \alpha_n^s \tag{25}$$

guarantees boundedness of all signals and convergence of the states to the residual set

$$\mathcal{D}_i = \left\{X_n \in \mathrm{R}^n : \sum_{i=1}^{n} z_i^2 \leq \frac{2W_i}{\beta_i}\right\} \tag{26}$$

where β_i *is a design constant, and* W_i *measures approximation errors and ideal parameter sizes, and its magnitude can be reduced through the choice of the design constants* $k_{i1}, k_{i2}, \gamma_{\phi_i^j}, \gamma_{\psi_i^j}, \sigma_{\phi_i^j}$ *and* $\sigma_{\psi_i^j}$.

Proof: See Appendix. □

Remark 1: Note that the assumption that $\psi_i^c > 0$, $i = 1, \ldots, n$ is only to simplify the analysis and implies no loss of generality, in the sense that the underlying fundamental requirement is that ψ_i^c be bounded away from zero by a constant of known sign. The stability proof can easily accommodate negative cases in the usual fashion.

Remark 2: The representation error bounds and the size of the ideal parameter vectors do not need to be known, but they affect the size of the residual set to which the states converge. The size of this set is specified by appropriately setting the design constants, but meeting performance specifications for the size of the residual set can only be done *a posteriori* if knowledge of the magnitudes of errors and parameters is lacking. In general, however, it is difficult for a designer to have access to these magnitudes. In spite of this fact, it is possible to perform a performance analysis of the transient bounds in a manner similar to Krstić *et al.* (1995) and Ordóñez and Passino (2001).

Remark 3: Notice that the indirect adaptive approach presented here relies on linearly parameterized function approximators. Non-linearly parameterized approximators can be integrated into the analysis in an analogous way to Polycarpou and Mears (1998), where the mean value theorem is used. Nevertheless, even though the analysis can be performed in this manner, it may not be clear how to keep the approximators for the functions ψ_i^j bounded away from zero if they are non-linearly parameterized (e.g. if we use feedforward neural networks). For the linearly parameterized approximators considered here, simple projection algorithms can be employed. For this reason, and to simplify the analysis, we concentrate on linearly parameterized approximators.

Remark 4: It may be the case that part of the system's dynamics are known, so that we may rewrite (1) as

$$\left.\begin{aligned} \dot{x}_i &= \sum_{j=1}^{R} \rho_j(v)((\phi_i^j(X_i) + \phi_{k_i}^j(X_i)) \\ &\quad + (\psi_i^j(X_i) + \psi_{k_i}^j(X_i))x_{i+1}) \\ \dot{x}_n &= \sum_{j=1}^{R} \rho_j(v)((\phi_n^j(X_n) + \phi_{k_n}^j(X_n)) \\ &\quad + (\psi_n^j(X_n) + \psi_{k_n}^j(X_n))u) \end{aligned}\right\} \tag{27}$$

where the subscript k is not an index, and it denotes the *known* part of the plant dynamics. We may then define the known functions

$$\left.\begin{aligned} \phi_i^k(X_i, v) &= \sum_{j=1}^{R} \rho_j(v)\phi_{k_i}^j(X_i) \\ \psi_i^k(X_i, v) &= \sum_{j=1}^{R} \rho_j(v)\psi_{k_i}^j(X_i) \end{aligned}\right\} \tag{28}$$

where the only restriction made is that ϕ_i^k and ψ_i^k are smooth, and that $\psi_i^k \geq \underline{\psi}_i^k > 0, i = 1, \ldots, n$, for some constants $\underline{\psi}_i^k$. The stability proof of Theorem 1 can be carried out with these additions, provided we replace $\hat{\phi}_i^c$ for $\hat{\phi}_i^c + \phi_i^k$ and $\hat{\psi}_i^c$ for $\hat{\psi}_i^c + \psi_i^k, i = 1, \ldots, n$ in (14) and (16). Also, for simplicity, we may replace the lower bounds $\underline{\psi}_i$ in (15) and (17) for $\underline{\psi}_i + \underline{\psi}_i^k, i = 1, \ldots, n$.

Remark 5: Instead of using constants, one may let for $i = 1, \ldots, n$,

$$c_i(v) = \sum_{j=1}^{R} \rho_j(v)c_i^j, \qquad \text{with constants } c_i^j > 0 \tag{29}$$

i.e. use 'scheduled' gains. In this way, the control law becomes more 'localized,' in the sense that the designer may choose distinct gains for each 'piece' in (1). Thus, the indirect adaptive result in Theorem 1 allows for a more detailed fine-tuning of the closed-loop performance. Note that with this choice of $c_i(v)$, the signals $\alpha_i, i = 2, \ldots, n$ have to compensate for the derivatives of the time-varying gains.

Remark 6: The presence of the functions $\rho_j(v)$ in the definition of the plant (1) gives rise to the possibility of having a very general representation of non-linear systems. Here, we have emphasized the idea of having the $\rho_j(v)$ act as interpolating functions that give the plant a dynamic structure that changes according to a scheduling variable. However, these functions may also be thought of within the context of gain-scheduling, where each of the R pieces in (1) is a non-linear representation of a system at an operating point given by the scheduling variable. Moreover, note that system (1) can be related to the idea of multiple models (Narendra and Balakrishnan 1997), where each subsystem is chosen according to the value of v. In our case, the switching between models occurs smoothly, rather than discontinuously as in Narendra and Balakrishnan (1997), and v would act as the switching variable. Even more generally, the plant (1) is an interpolation performed on the space of v of 'local' representations, or 'pieces', each in strict feedback form, where the jth piece is given by the functions ϕ_i^j and ψ_i^j, $i = 1, \ldots, n$. This important point can be made more clear if we consider the particular case where the functions $\rho_j(v)$ form a convex combination, i.e. they satisfy

$$\left.\begin{aligned} 0 \leq \rho_j(v) \leq 1 \\ \sum_{j=1}^{R} \rho_j(v) = 1 \end{aligned}\right\} \tag{30}$$

a subcase of (2). Then, the locality is readily reflected in the structure of the adaptation laws: observe in (24) that, due to the presence of ρ_j, only those parameter vectors corresponding to an active 'subsystem' are updated (disregarding the leakage terms, which are used to provide parameter stability). That is, the method in Theorem 1 is able to *localize* the adaptation (within the scheduling space of v).

Remark 7: As stated, the vector v corresponds in general to some possibly exogenous variable. However, v may also contain the state x_1, since all we require from it is that its derivatives up to the $(n-1)$th are available for measurement. Moreover, if we allow each state equation $\dot{x}_i$ to have its own set of interpolation functions, i.e.

$$\left.\begin{aligned} \dot{x}_i &= \sum_{j=1}^{R_i} \rho_i^j(v_i)(\phi_i^j(X_i) + \psi_i^j(X_i)x_{i+1}, \\ &\qquad\qquad i = 1, \dots, n-1 \\ \dot{x}_n &= \sum_{j=1}^{R_n} \rho_n^j(v_n)(\phi_n^j(X_n) + \psi_n^j(X_n)u \end{aligned}\right\} \tag{31}$$

where $R_i \in \mathrm{N}, i = 1, \dots, n$, we may let v_i contain X_i (i.e. the elements of X_i are also elements of v_i, in addition to other exogenous variables). Letting v_i contain states not in X_i leads to technical problems where the control has to be augmented with integrators, in spite of which it is generally not well defined. For this reason, we avoid such case here.

Remark 8: Note that the stability result of Theorem 1 is semi-global, in the sense that it is valid within the compact sets $\mathcal{S}_i, i = 1, \dots, n$, which can be made arbitrarily large (but of bounded size). The initial value of the state has to lie within a sufficiently small subset of $\mathcal{S}_i, i = 1, \dots, n$, so that the initial derivatives do not force the state to leave the compact sets before the controller is able to stop the state from leaving the allowable region of approximation. The stability result may be made global by adding a high gain bounding control term to the control law. Such a term may be particularly useful when, due to a complete lack of *a priori* knowledge, the control designer is unable to guarantee that the compact sets $\mathcal{S}_i, i = 1, \dots, n$, are large enough so that the state will not exit them before the controller has time to bring the state inside $\mathcal{D}_i$; moreover, it may also happen that due to a poor design and poor system knowledge, $\mathcal{D}_i$ is not contained in $\mathcal{S}_n$. In this case, too, bounding control terms may be helpful until the design is refined and improved. However, using bounding control requires the additional knowledge of functional upper bounds of $|\psi_i^c(X_i, v)|$. Bounding terms may be added to the diffeomorphism (13), but we do not present the analysis since it is similar to the one in Theorem 1 and it is algebraically tedious; we simply note, though, that the bounding terms have to be smooth (because they need to be differentiable), so they need to be defined in terms of smooth approximations to the sign, saturation and absolute value functions that are typically used in this approach.

Remark 9: The result in Theorem 1 is for regulation of system (1) to zero, but it may be easily modified to allow for tracking of a reference model. Consider a bounded reference input $r(t) \in \mathrm{R}$ and a reference model in controllable canonical form

$$\left.\begin{aligned} \dot{x}_{r_i} &= x_{r_{i+1}}, \qquad i = 1, 2, \dots, n-1 \\ \dot{x}_{r_n} &= f_r(X_{r_n}, r) \end{aligned}\right\} \tag{32}$$

with the origin globally asymptotically stable, and $X_{r_n} = [x_{r_1}, \dots, x_{r_n}]^\top \in \mathrm{R}^n$ is the reference state vector, which is available for measurement. The objective is to have x_1 track the reference model state x_{r_1}. To this end, we replace the change of coordinates (13) with

$$\left.\begin{aligned} z_1 &= x_1 - x_{r_1} \\ z_i &= x_i - \alpha_{i-1} - \alpha_{i-1}^s, \qquad i = 2, \dots, n \end{aligned}\right\} \tag{33}$$

and, instead of (14) and (16), we let

$$\alpha_1 = \frac{1}{\hat{\psi}_1^c}(-\hat{\phi}_1^c - c_1 z_1 + x_{r_2}) \tag{34}$$

$$\begin{aligned} \alpha_m = \frac{1}{\hat{\psi}_m^c}\Bigg(&- \hat{\phi}_m^c - c_m z_m - \hat{\psi}_{m-1}^c z_{m-1} \\ &+ \sum_{i=1}^{m-1}\left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i}\right)(\hat{\phi}_i^c + \hat{\psi}_i^c x_{i+1}) \\ &+ \sum_{i=1}^{m-1}\sum_{j=1}^{\mathrm{R}}\Bigg[\left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\phi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\phi_i^j}}\right)\gamma_{\phi_i^j}\tau_{\phi^j_{i(m+1-i)}} \\ &+ \left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\psi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\psi_i^j}}\right)\gamma_{\psi_i^j}\tau_{\psi^j_{i(m+1-i)}}\Bigg] \\ &+ \left(\frac{\partial \alpha_{m-1}}{\partial \nu_{m-1}} + \frac{\partial \alpha_{m-1}^s}{\partial \nu_{m-1}}\right)\dot{\nu}_{m-1} \\ &+ \left(\frac{\partial \alpha_{m-1}}{\partial x_{r_m}} + \frac{\partial \alpha_{m-1}^s}{\partial x_{r_m}}\right)\dot{x}_{r_m} + \kappa_m\Bigg) \end{aligned} \tag{35}$$

The proof can then be carried as shown in Theorem 1, provided the terms arising from partial derivatives with respect to the model states are taken into account, and from (71) we conclude that the tracking error converges to a neighbourhood of the origin of size $\sqrt{(2W_i/\beta_i)}$.

3. Aircraft wing rock regulation with varying angle of attack

Subsonic wing rock is a non-linear phenomenon experienced by aircraft with slender delta wings, in which limit cycle roll and roll rate oscillations or unstable behaviour are experienced by aircraft with pointed forebodies at high angles of attack. Wing rock may diminish flight effectiveness or even present serious danger due to potential instability of the aircraft. Here, we will apply the indirect adaptive control method in Theorem 1 to the problem of wing rock regulation.

Other approaches to this problem can be found in Luo and Lan (1993), Krstić *et al.* (1995), Singh *et al.* (1995) and Joshi *et al.* (1998), among others. In Singh *et al.* (1995) the authors present conventional adaptive and neural adaptive control methods for wing rock control. In Luo and Lan (1993), an optimal feedback control using Beecham-Titchener's averaging technique is applied. The paper by Joshi *et al.* (1998) presents a single-neuron controller trained with backpropagation to regulate wing rock, and this controller is tested in a wind tunnel. In Krstić *et al.* (1995) the authors use the tuning functions method of adaptive backstepping to develop a wing rock regulator.

It is interesting to note that all these methods are developed at a *fixed* angle of attack, and then in some cases tested at another angle close to the design point, which serves to help the researchers claim robustness of the designs. Here, the problem is considered in a more general setting, where the angle of attack is allowed to vary with time according to the evolution of an external dynamical system (which may represent the commands of the pilot together with the aircraft dynamics). As will be noted below, the dynamics of the wing rock phenomenon change non-linearly with the angle of attack, which makes the problem of developing controllers that are robust against angle of attack a challenging one. However, this problem fits the class of time-varying systems (1) considered in this paper, so development of a controller which can operate at all angles of attack is greatly simplified by following Theorem 1.

There exist several analytical non-linear models that characterize the phenomenon of wing rock (Hsu and Lan 1985, Elzebda *et al.* 1989, Nayfeh *et al.* 1989). The model we use here is the one presented in Elzebda (1989) and Nayfeh *et al.* (1989), which has the advantage over the model in Hsu and Lan (1985) of being differentiable and, according to the authors, slightly more accurate. This model is given by

$$\ddot{\phi} = -w_j^2\phi + \mu_1^j\dot{\phi} + b_1^j\dot{\phi}^3 + \mu_2^j\phi^2\dot{\phi} + b_2^j\phi\dot{\phi}^2 + g\delta_a \quad (36)$$

where ϕ is the roll angle, δ_a is the output of an actuator with first order dynamics, $g = 1.5$ is an input gain, and

$$\left.\begin{aligned} w_j^2 &= -c_1 a_1^j \\ \mu_1^j &= c_1 a_2^j - c_2 \\ b_1^j &= c_1 a_3^j \\ \mu_2^j &= c_1 a_4^j \\ b_2^j &= c_1 a_5^j \end{aligned}\right\} \quad (37)$$

are system coefficients that depend on the parameters a_i^j, which in turn are functions of the angle of attack, denoted here by v (aircraft notation conventions dictate the use of α as the angle of attack; however, to avoid confusion with the notation here, we will use v instead). From Nayfeh *et al.* (1989) we let $c_1 = 0.354$ and $c_2 = 0.001$, constants given by the physical parameters of a delta wing used in wind tunnel experiments in Levin and Katz (1984) to develop the analytical model (36). In Nayfeh *et al.* (1989), four angles of attack are considered, at which the coefficients a_i^j are given. We added three points to the table in Nayfeh *et al.* (1989) by assuming that the functions passing through the points a_i^j are approximately piecewise linear (a reasonable assumption, considering the plots presented in Nayfeh *et al.* (1989)). Thus, the points used are given in table 1, where the points at $v = 17$, 19 and 23.75 have been added to the table in Nayfeh *et al.* (1989).

In order to build a smooth, time-varying model of the wing rock that depends on the angle of attack v, we will consider the interpolation functions

$$\rho_j(v) = \frac{\exp\left[-\left(\dfrac{v - v_j}{s_j}\right)^2\right]}{\sum_{l=1}^{7} \exp\left[-\left(\dfrac{v - v_l}{s_l}\right)^2\right]} \quad (38)$$

where the centres v_j and spreads $s_j, j = 1, \ldots, 7$, are given in table 2. Notice that the interpolation functions (38) satisfy assumption (2).

v	a_1^j	a_2^j	a_3^j	a_4^j	a_5^j
15	−0.010 26	−0.021 17	−0.141 81	0.997 35	−0.834 78
17	−0.020 07	−0.010 2	−0.083 7	0.633 33	−0.503 4
19	−0.029 8	0.000 818	−0.025 5	0.269 2	−0.171 9
21.5	−0.042 07	0.014 56	0.047 14	−0.185 83	0.242 34
22.5	−0.046 81	0.019 66	0.056 71	−0.226 91	0.590 65
23.75	−0.051 8	0.026 1	0.065	−0.293 3	1.029 4
25	−0.056 86	0.032 54	0.073 34	−0.359 7	1.468 1

Table 1. Parameters for the coefficients in the wing rock model.

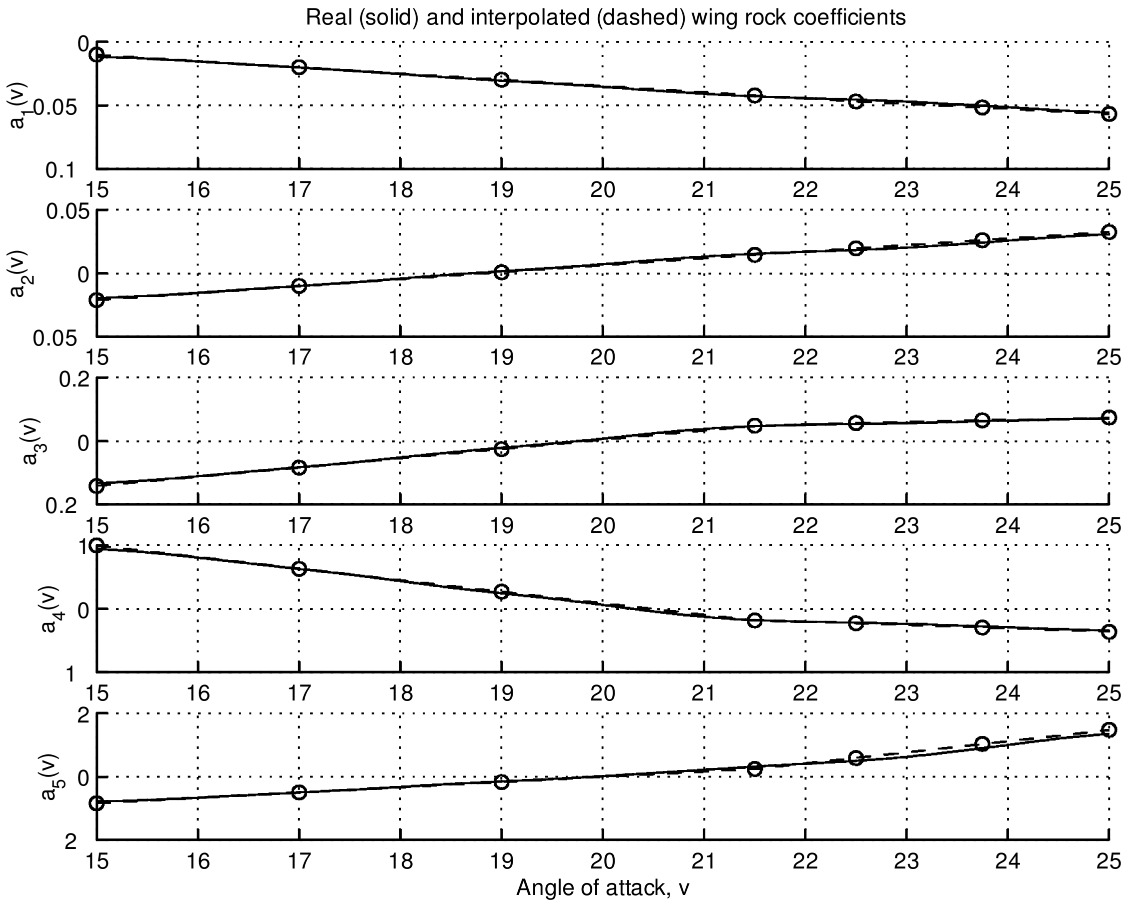

Figure 1. Interpolated coefficients for the time-varying wing rock model.

j	1	2	3	4	5	6	7
v_j	15	17	19	21.5	22.5	23.75	25
s_j	1.5	1.5	1.5	2.0	1	1	1

Table 2. Centres and spreads for wing rock interpolation functions.

In order to test the accuracy of the interpolations, let

$$a_i(v) = \sum_{j=1}^{7} \rho_j(v) a_i^j \qquad (39)$$

for $i = 1, \ldots, 5$. Figure 1 contains the plots of the interpolated coefficients $a_i(v)$ (solid lines), as well as the data points in table 1, marked by circles. We see that the interpolations are generally close to the data points, so we may consider the resulting time-varying model accurate enough.

We will assume the control input u affects the wing through an actuator with linear, first order dynamics. In order to express the model in the form (1), we let $x_1 = \phi, x_2 = \dot{\phi}$ and $x_3 = \delta_a$. Then, the time-varying wing rock model is given by

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \sum_{j=1}^{7} \rho_j(v)(-w_j^2\phi + \mu_1^j\dot{\phi} + b_1^j\dot{\phi}^3 + \mu_2^j\phi^2\dot{\phi} + b_2^j\phi\dot{\phi}^2) \\ &\quad + g x_3 \\ \dot{x}_3 &= -\frac{1}{\tau}x_3 + \frac{1}{\tau}u \end{aligned}\right\} \qquad (40)$$

where the actuator time constant is $\tau = \frac{1}{15}$. We will assume that the angle of attack v varies according to an exogenous dynamical system,

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 25 \\ -25 & -10 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 500 \end{bmatrix} + \begin{bmatrix} 0 \\ 62.5 \end{bmatrix} r \qquad (41)$$

where $v_1 = v, v_2 = \dot{v}$, and r is a command input that can take values between minus one and one. System (41) has its poles at $-5 \pm 24.5i$ ($i = \sqrt{-1}$), and its equilibrium is at $v_1 = 20, v_2 = 0$.

According to the analysis performed in Nayfeh *et al.* (1989), the wing rock system has a stable focus at the origin for angles of attack v less than approximately 19.5 degrees. For higher angles, the origin becomes an

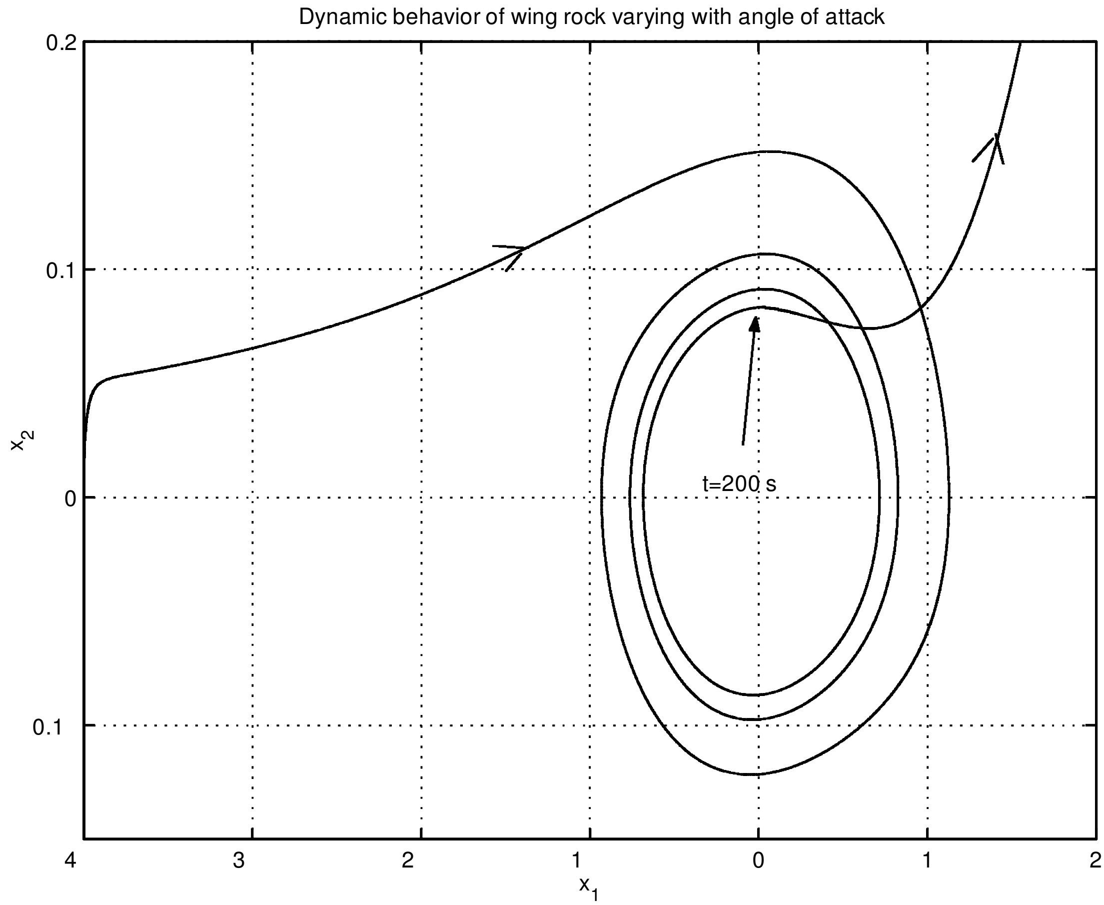

Figure 2. Qualitative change in wing rock dynamics with varying angle of attack.

unstable equilibrium, and a limit cycle appears around it. In both cases, however, the system is unstable and may diverge to infinity if the initial conditions are large enough (since we are dealing with angles, such a divergence means that the wings rotate faster and faster). The problem we consider here has the angle of attack varying within the range between 15 and 25 degrees, so the qualitative behaviour of (40) changes periodically, as v becomes respectively smaller or larger than 19.5. To gain a better insight into how the dynamic behaviour of the wing rock phenomenon changes qualitatively with v, consider figure 2, where we let the system start at the initial condition $X_3(0) = [-4,\ 0,\ 0]$, $\nu_2(0) = [20,\ 0]$. Initially, we set $r = 1$, so the angle of attack stabilizes at 22.5, and we let the system run in open loop for 200 s. We observe that x_1 and x_2 approach a limit cycle, which would be reached if the system were allowed to run for a longer time; however, at $t = 200$ we let $r = -1$ (this is marked by an arrow in figure 2), so the angle of attack changes and after a short transient stabilizes at 17.5. Not being close enough to the origin to be attracted by the local stable focus, the system starts to diverge.

We will consider the regulation problem, where the indirect adaptive controller tries to bring the states of the system to zero, or a neighborhood of zero. We use radial basis function neural networks (Moody and Darken 1989) as the function approximators for the plant dynamics. No approximation is required for the first state equation in (40), which simplifies the subsequent steps of the design. For $\zeta_{\phi_2^j}$ we choose, for $j = 1, \ldots, 7$

$$\zeta_{\phi_2^j} = \left[1, \exp\left(\frac{(x_1 - c_1^l)^2}{s_1^2}\right)\exp\left(\frac{(x_2 - c_2^m)^2}{s_2^2}\right)\right]^\top \qquad l = 1, \ldots, 5, \qquad m = 1, \ldots, 5 \quad (42)$$

with the centres c_1^l and c_2^m both evenly spaced along the interval $[-10,\ 10]$, and the spreads $s_1 = s_2 = 5$ (i.e. $\mathcal{S}_{x_1} = \{x_1 \in \mathrm{R} : -10 \le x_1 \le 10\}$, and $\mathcal{S}_{x_2} = \{x_2 \in \mathrm{R} : -10 \le x_2 \le 10\}$. Since $\phi_2^j = g$ for $j = 1, \ldots, 7$, we set $\zeta_{\psi_2^j} = 1$ and only update a single coefficient to get $\hat{\psi}_2^c$. We do the same for $\hat{\phi}_3^c$ and $\hat{\psi}_3^c$. The coefficient vectors for $\hat{\phi}_2^c$ are initialized with zeros, $\hat{\phi}_3^c$ is initially equal to one, and the coefficients for $\hat{\psi}_2^c$ and $\hat{\psi}_3^c$ are initialized

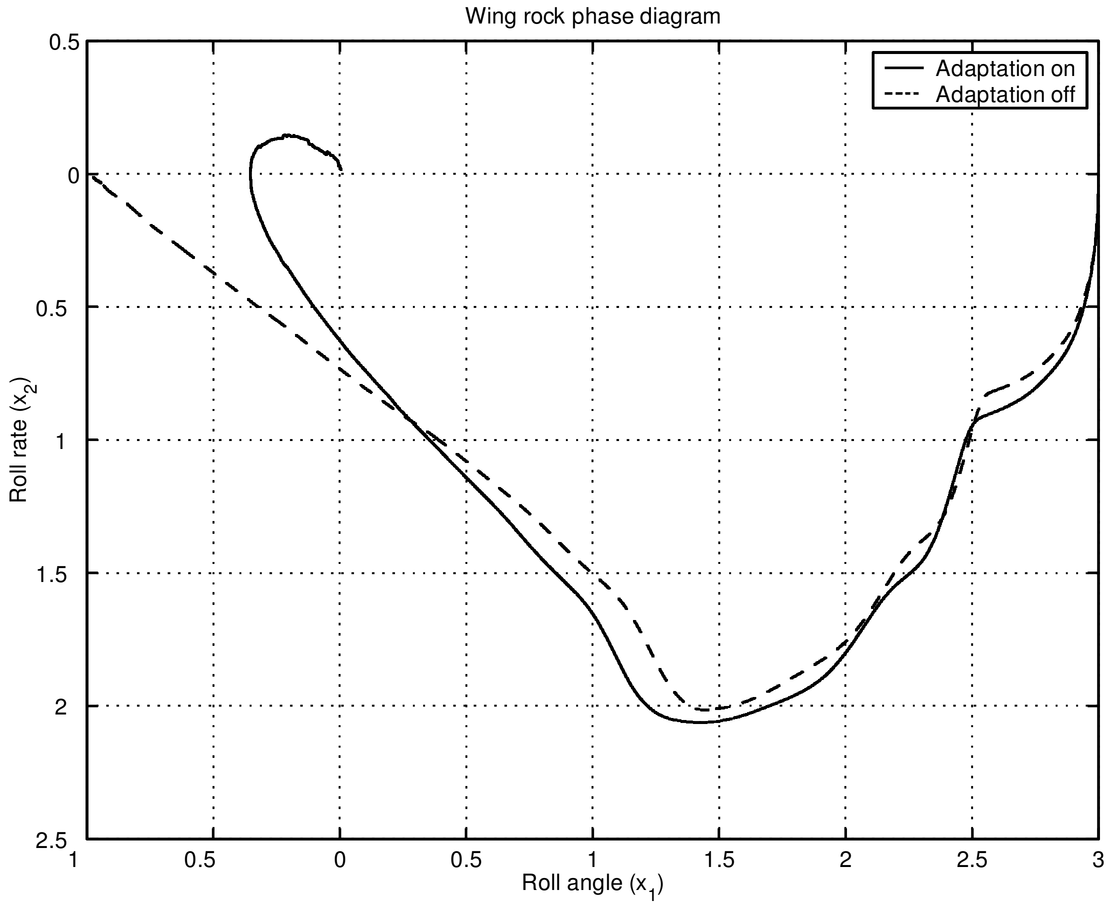

Figure 3. Indirect adaptive wing rock regulation: roll and roll rate.

with ones; we use projection to keep these coefficients bounded away from zero.

We let the reference to the angle of attack system alternate between -1 and 1 every 0.75 s, and we choose $\nu_2(0) = [20, \ 0]$ as the initial condition for the angle of attack system. For our design, we set $\underline{\psi}_2 = 1$ and $\underline{\psi}_3 = 10$. Moreover, we let $c_1 = 0.2$, $c_2 = 0.3$, $c_3 = 0.6$, and $k_{21} = k_{22} = k_{31} = k_{32} = 0.1$. For the adaptation laws, we pick $\gamma_{\phi_2^j} = 0.01$, $\sigma_{\phi_2^j} = 0.03$, $j = 1, \ldots, 7$, and $\gamma_{\psi_2} = \gamma_{\psi_3} = 0.05$, $\sigma_{\psi_2} = \sigma_{\psi_3} = 0.08$, $\gamma_{\phi_3} = 0.5$, $\sigma_{\phi_3} = 0.8$. These gains have been selected to illustrate the effect of operating with and without adaptation turned on. In this indirect adaptive approach (similar in this regard to most indirect adaptive methods in the literature) there is no guarantee of parameter convergence for the function approximators. Thus, although stability is guaranteed, the plant dynamics may or may not be properly identified during closed-loop operation. Many times the question arises about what, if anything at all, is the contribution of the adaptive mechanisms to closed-loop performance. In this application we want to show an example of a case where having adaptation adds significantly to the closed-loop performance versus using only the high gain stabilizing terms with no adaptation. Clearly, no conclusive answer is given to the question posed above, but still this example is of interest because it illustrates one particular case where significant improvement can be achieved by using adaptation.

Figure 3 shows the regulation results, where we run the simulation for 8 s. The solid line corresponds to running the plant with adaptation turned on, and the dashed line corresponds to turning adaptation off. Note that by turning off adaptation we no longer have a guarantee of closed loop stability; however, with the gains chosen and for this particular plant stable behaviour seems to be maintained using only the stabilizing terms. It is interesting to note that, when adaptation is turned off, the controller is unable to regulate the states to zero. The indirect adaptive controller, on the other hand, manages to do so as expected (convergence is achieved to a small neighbourhood of the origin).

In figure 4 we have in the top plot the changing value of the angle of attack, ν. As noted, the wing rock dynamics change as this angle takes on different values.

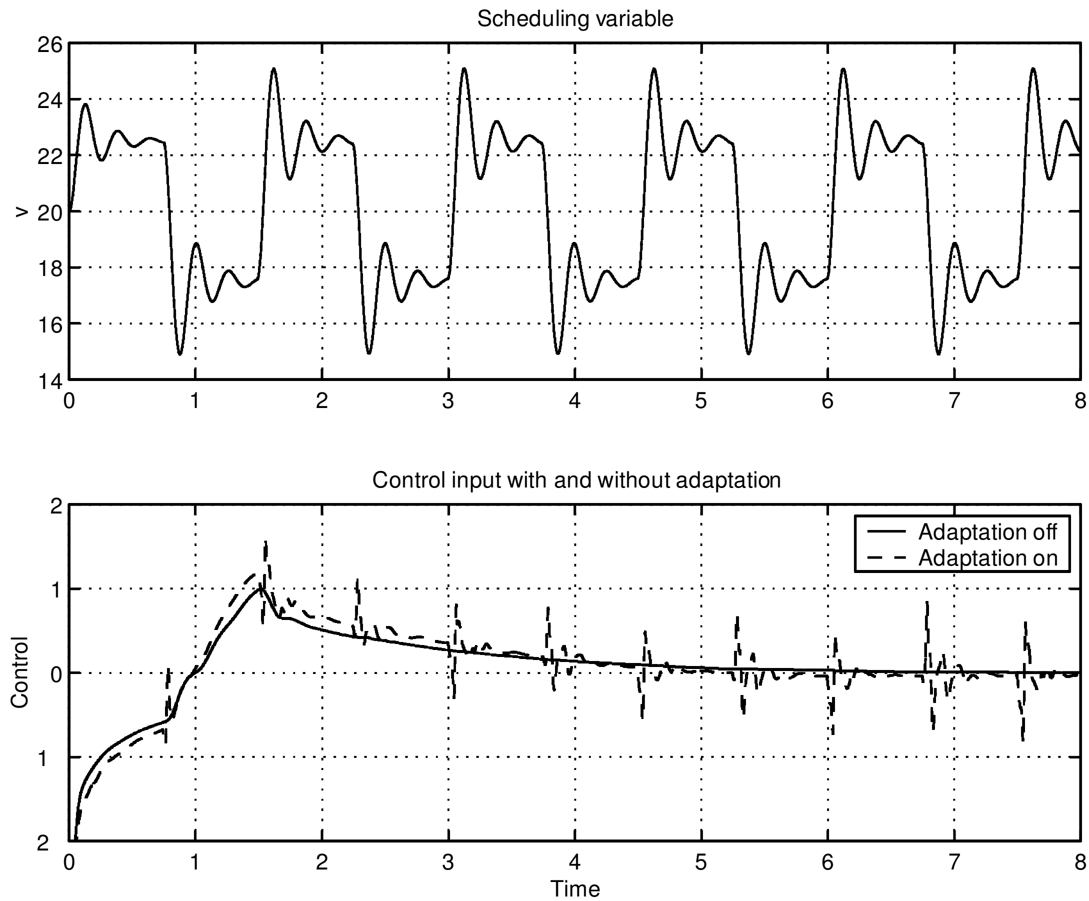

Figure 4. Indirect adaptive wing rock regulation: scheduling variable and control input.

In the bottom plot we observe the control inputs generated with and without adaptation in the controller. Notice that the control generated by the non-adaptive controller appears to be in a sense an 'average' of the adaptive control input. However, this non-adaptive control is unable to perform adequate regulation.

4. Conclusions

In this paper we have developed an indirect adaptive control method for a class of uncertain non-linear systems with a time-varying structure using a Lyapunov approach to construct the stability proof. The non-linear systems we consider are composed of a finite number of 'pieces,' or dynamic subsystems, which are interpolated by functions that depend on a possibly exogenous scheduling variable. We assume that each piece is in strict feedback form, and show that the method yields stability of all signals in the closed-loop, as well as convergence of the state vector to a residual set around the equilibrium, whose size can be set by the choice of several design constants. We argue that the indirect adaptive method has the advantage of providing a large design flexibility by allowing a 'localized' fine-tuning of the controller. Finally, we apply the indirect adaptive method to regulation of aircraft wing rock when the angle of attack is allowed to change with time.

Appendix. Stability proof

The proof is in n steps and is performed by induction. Let $z_1 = x_1$ and $z_2 = x_2 - \alpha_1 - \alpha_1^s$. Consider the expression for α_1 given in (14). Then

$$\begin{aligned}\dot{z}_1 &= \phi_1^c + \psi_1^c(z_2 + \alpha_1 + \alpha_1^s) + (-\hat{\phi}_1^c - c_1 z_1) \\ &\quad - (-\hat{\phi}_1^c - c_1 z_1) \\ &= -c_1 z_1 + \psi_1^c z_2 + (\phi_1^c - \hat{\phi}_1^c) \\ &\quad + \left(\frac{\psi_1^c}{\hat{\psi}_1^c} - 1\right)(-\hat{\phi}_1^c - c_1 z_1) + \psi_1^c \alpha_1^s \end{aligned} \tag{43}$$

Note that

$$\left(\frac{\psi_1^c}{\hat{\psi}_1^c} - 1\right)(-\hat{\phi}_1^c - c_1 z_1) = (\psi_1^c - \hat{\psi}_1^c)\alpha_1$$

so that

$$\dot{z}_1 = -c_1 z_1 + \psi_1^c z_2 + \sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_1^j}^\top \zeta_{\phi_1^j} + \Phi_{\psi_1^j}^\top \zeta_{\psi_1^j} \alpha_1 \right) + (\delta_{\phi_1} + \delta_{\psi_1}\alpha_1) + \psi_1^c \alpha_1^s \quad (44)$$

Let

$$V_1 = \tfrac{1}{2} z_1^2 + \tfrac{1}{2} \sum_{j=1}^{R} \left(\frac{1}{\gamma_{\phi_1^j}} \Phi_{\phi_1^j}^\top \Phi_{\phi_1^j} + \frac{1}{\gamma_{\psi_1^j}} \Phi_{\psi_1^j}^\top \Phi_{\psi_1^j} \right)$$

and consider its derivative

$$\begin{aligned} \dot{V}_1 = & -c_1 z_1^2 + \psi_1^c z_1 z_2 \\ & + z_1 \sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_1^j}^\top \zeta_{\phi_1^j} + \Phi_{\psi_1^j}^\top \zeta_{\psi_1^j} \alpha_1 \right) \\ & + z_1(\delta_{\phi_1} + \delta_{\psi_1}\alpha_1) + z_1 \psi_1^c \alpha_1^s \\ & - \sum_{j=1}^{R} \left(\frac{1}{\gamma_{\phi_1^j}} \Phi_{\phi_1^j}^\top \dot{\hat{\theta}}_{\phi_1^j} + \frac{1}{\gamma_{\psi_1^j}} \Phi_{\psi_1^j}^\top \dot{\hat{\theta}}_{\psi_1^j} \right) \\ \leq & -c_1 z_1^2 + \psi_1^c z_1 z_2 \\ & + \sum_{j=1}^{R} \left[\Phi_{\phi_1^j}^\top \left(\rho_j \zeta_{\phi_1^j} z_1 - \frac{1}{\gamma_{\phi_1^j}} \dot{\hat{\theta}}_{\phi_1^j} \right) \right. \\ & \left. + \Phi_{\psi_1^j}^\top \left(\rho_j \zeta_{\psi_1^j} z_1 \alpha_1 - \frac{1}{\gamma_{\psi_1^j}} \dot{\hat{\theta}}_{\psi_1^j} \right) \right] \\ & + \left(\frac{d_{\phi_1^2}}{4k_{11}} + \frac{d_{\psi_1^2}}{4k_{12}} \right) \end{aligned} \quad (45)$$

where the inequality comes from the term (15), since we can establish

$$z_1 \delta_{\phi_1} + z_1 \alpha_1 \delta_{\psi_1} \leq k_{11} z_1^2 + \frac{d_{\phi_i}^2}{4k_{11}} + k_{12} z_1^2 \alpha_1^2 + \frac{d_{\psi_i}^2}{4k_{12}} \quad (46)$$

for any $k_{11} > 0$, $k_{12} > 0$, and we have used the facts that $\dot{\hat{\theta}}_{\phi_i^j} = -\dot{\Phi}_{\phi_i^j}$ and $\dot{\hat{\theta}}_{\psi_i^j} = -\dot{\Phi}_{\psi_i^j}$. Letting $m = 1$ in (18) and (19) we obtain

$$\begin{aligned} \dot{V}_1 \leq & -c_1 z_1^2 + \sum_{j=1}^{R} \left(\frac{\sigma_{\phi_1^j}}{\gamma_{\phi_1^j}} \Phi_{\phi_1^j}^\top \hat{\theta}_{\phi_1^j} + \frac{\sigma_{\psi_1^j}}{\gamma_{\psi_1^j}} \Phi_{\psi_1^j}^\top \hat{\theta}_{\psi_1^j} \right) \\ & + \sum_{j=1}^{R} \left[\Phi_{\phi_1^j}^\top \left(\tau_{\phi_{11}^j} - \frac{1}{\gamma_{\phi_1^j}} \dot{\hat{\theta}}_{\phi_1^j} \right) + \Phi_{\psi_1^j}^\top \left(\tau_{\psi_{11}^j} - \frac{1}{\gamma_{\psi_1^j}} \dot{\hat{\theta}}_{\psi_1^j} \right) \right] \\ & + \left(\frac{d_{\phi_1}^2}{4k_{11}} + \frac{d_{\psi_1}^2}{4k_{12}} \right) \end{aligned} \quad (47)$$

Notice that we will not pick the adaptation laws for $\dot{\hat{\theta}}_{\phi_1^j}$ and $\dot{\hat{\theta}}_{\psi_1^j}$ yet, but we will rather wait until the nth step to do so. The first step of the proof is therefore completed.

In the second step, we let $z_3 = x_3 - \alpha_2 - \alpha_2^s$. We examine the dynamics of z_2

$$\begin{aligned} z_2 = & \phi_2^c + \psi_2^c (z_3 + \alpha_2 + \alpha_2^s) \\ & - \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right) (\phi_1^c + \psi_1^c x_2) - \left(\frac{\partial \alpha_1}{\partial v} + \frac{\partial \alpha_1^s}{\partial v} \right) \dot{v} \\ & - \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\phi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\phi_1^j}} \right)^\top \dot{\hat{\theta}}_{\phi_1^j} + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\psi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\psi_1^j}} \right)^\top \dot{\hat{\theta}}_{\psi_1^j} \right] \end{aligned} \quad (48)$$

With α_2 as given in (16) with $m = 2$ one can show that

$$\begin{aligned} \dot{z}_2 = & -c_2 z_2 - \hat{\psi}_1^c z_1 + \psi_2^c z_3 + \sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_2^j}^\top \zeta_{\phi_2^j} + \Phi_{\psi_2^j}^\top \zeta_{\psi_2^j} \alpha_2 \right) \\ & + (\delta_{\phi_2} + \delta_{\psi_2} \alpha_2) + \psi_2^c \alpha_2^s \\ & - \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right) \\ & \times \left[\sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_1^j}^\top \zeta_{\phi_1^j} + \Phi_{\psi_1^j}^\top \zeta_{\psi_1^j} x_2 \right) + (\delta_{\phi_1} + \delta_{\psi_1} x_2) \right] \\ & + \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\phi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\phi_1^j}} \right)^\top \left(\gamma_{\phi_1^j} \tau_{\phi_{12}^j} - \dot{\hat{\theta}}_{\phi_1^j} \right) \right. \\ & \left. + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\psi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\psi_1^j}} \right)^\top \left(\gamma_{\psi_1^j} \tau_{\psi_{12}^j} - \dot{\hat{\theta}}_{\psi_1^j} \right) \right] \end{aligned} \quad (49)$$

Let

$$V_2 = V_1 + \tfrac{1}{2} z_2^2 + \tfrac{1}{2} \sum_{j=1}^{R} \left(\frac{1}{\gamma_{\phi_2^j}} \Phi_{\phi_2^j}^\top \Phi_{\phi_2^j} + \frac{1}{\gamma_{\psi_2^j}} \Phi_{\psi_2^j}^\top \Phi_{\psi_2^j} \right)$$

so that

$$\begin{aligned} \dot{V}_2 = & \dot{V}_1 + z_2 \dot{z}_2 - \sum_{j=1}^{R} \left(\frac{1}{\gamma_{\phi_2^j}} \Phi_{\phi_2^j}^\top \dot{\hat{\theta}}_{\phi_2^j} + \frac{1}{\gamma_{\psi_2^j}} \Phi_{\psi_2^j}^\top \dot{\hat{\theta}}_{\psi_2^j} \right) \\ \leq & -c_1 z_1^2 - c_2 z_2^2 + z_1 z_2 (\psi_1^c - \hat{\psi}_1^c) + \psi_2^c z_2 z_3 \\ & + \sum_{j=1}^{R} \left(\frac{\sigma_{\phi_1^j}}{\gamma_{\phi_1^j}} \Phi_{\phi_1^j}^\top \hat{\theta}_{\phi_1^j} + \frac{\sigma_{\psi_1^j}}{\gamma_{\psi_1^j}} \Phi_{\psi_1^j}^\top \hat{\theta}_{\psi_1^j} \right) \\ & + \sum_{j=1}^{R} \left[\Phi_{\phi_1^j}^\top \left(\tau_{\phi_{11}^j} - \frac{1}{\gamma_{\phi_1^j}} \dot{\hat{\theta}}_{\phi_1^j} \right) + \Phi_{\psi_1^j}^\top \left(\tau_{\psi_{11}^j} - \frac{1}{\gamma_{\psi_1^j}} \dot{\hat{\theta}}_{\psi_1^j} \right) \right] \\ & + \sum_{j=1}^{R} \left[\Phi_{\phi_2^j}^\top \left(\rho_j \zeta_{\phi_2^j} z_2 - \frac{1}{\gamma_{\phi_2^j}} \dot{\hat{\theta}}_{\phi_2^j} \right) \right. \\ & \left. + \Phi_{\psi_2^j}^\top \left(\rho_j \zeta_{\psi_2^j} z_2 \alpha_2 - \frac{1}{\gamma_{\psi_2^j}} \dot{\hat{\theta}}_{\psi_2^j} \right) \right] \\ & - z_2 \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right) \sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_1^j}^\top \zeta_{\phi_1^j} + \Phi_{\psi_1^j}^\top \zeta_{\psi_1^j} x_2 \right) \end{aligned}$$

$$+ z_2 \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\phi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\phi_1^j}} \right)^{\top} \left(\gamma_{\phi_1^j} \tau_{\phi_{12}^j} - \dot{\hat{\theta}}_{\phi_1^j} \right) + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\psi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\psi_1^j}} \right)^{\top} \left(\gamma_{\psi_1^j} \tau_{\psi_{12}^j} - \dot{\hat{\theta}}_{\psi_1^j} \right) \right]$$
$$+ z_2 \left((\delta_{\phi_2} + \delta_{\psi_2} \alpha_2) - \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right) (\delta_{\phi_1} + \delta_{\psi_1} x_2) \right)$$
$$+ \left(\frac{d_{\phi_1}^2}{4k_{11}} + \frac{d_{\psi_1}^2}{4k_{12}} \right) + z_2 \psi_2^c \alpha_2^s \tag{50}$$

Observe that

$$z_1 z_2 (\psi_1^c - \hat{\psi}_1^c) = z_1 z_2 \left(\sum_{j=1}^{R} \rho_j \Phi_{\psi_1^j}^{\top} \zeta_{\psi_1^j} + \delta_{\psi_1} \right) \tag{51}$$

Also, we have the inequality

$$z_2 \left((\delta_{\phi_2} + \delta_{\psi_2} \alpha_2) - \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right) (\delta_{\phi_1} + \delta_{\psi_1} x_2) \right) + z_1 z_2 \delta_{\psi_1}$$
$$\leq k_{21} z_2^2 + \frac{d_{\phi_2}^2}{4k_{21}} + k_{22} z_2^2 \alpha_2^2 + \frac{d_{\psi_2}^2}{4k_{22}} + k_{11} z_2^2 \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right)^2$$
$$+ \frac{d_{\phi_1}^2}{4k_{11}} + 2k_{12} z_2^2 x_2^2 \left(\frac{\partial \alpha_1}{\partial x_1} + \frac{\partial \alpha_1^s}{\partial x_1} \right)^2$$
$$+ \frac{d_{\psi_1}^2}{8k_{12}} + 2k_{12} z_1^2 z_2^2 + \frac{d_{\psi_1^2}}{8k_{12}} \tag{52}$$

Then, setting $m = 2$ in the terms (17) and (20), and $m = 2$ and $i = 1$ in (21) yields

$$\dot{V}_2 \leq -\sum_{i=1}^{2} c_i z_i^2 + \psi_2^c z_2 z_3$$
$$+ \sum_{i=1}^{2} \sum_{j=1}^{R} \left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}} \Phi_{\phi_i^j}^{\top} \hat{\theta}_{\phi_i^j} + \frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}} \Phi_{\psi_i^j}^{\top} \hat{\theta}_{\psi_i^j} \right)$$
$$+ \sum_{j=1}^{R} \left[\Phi_{\phi_1^j}^{\top} \left(\tau_{\phi_{12}^j} - \frac{1}{\gamma_{\phi_1^j}} \dot{\hat{\theta}}_{\phi_1^j} \right) + \Phi_{\psi_1^j}^{\top} \left(\tau_{\psi_{12}^j} - \frac{1}{\gamma_{\psi_1^j}} \dot{\hat{\theta}}_{\psi_1^j} \right) \right]$$
$$+ \sum_{j=1}^{R} \left[\Phi_{\phi_2^j}^{\top} \left(\tau_{\phi_{21}^j} - \frac{1}{\gamma_{\phi_2^j}} \dot{\hat{\theta}}_{\phi_2^j} \right) + \Phi_{\psi_2^j}^{\top} \left(\tau_{\psi_{21}^j} - \frac{1}{\gamma_{\psi_2^j}} \dot{\hat{\theta}}_{\psi_2^j} \right) \right]$$
$$+ z_2 \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\phi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\phi_1^j}} \right)^{\top} \left(\gamma_{\phi_1^j} \tau_{\phi_{12}^j} - \dot{\hat{\theta}}_{\phi_1^j} \right) + \left(\frac{\partial \alpha_1}{\partial \hat{\theta}_{\psi_1^j}} + \frac{\partial \alpha_1^s}{\partial \hat{\theta}_{\psi_1^j}} \right)^{\top} \left(\gamma_{\psi_1^j} \tau_{\psi_{21}^j} - \dot{\hat{\theta}}_{\psi_1^j} \right) \right]$$
$$+ 2 \left(\frac{d_{\phi_1}^2}{4k_{11}} + \frac{d_{\psi_1}^2}{4k_{12}} \right) + \left(\frac{d_{\phi_2}^2}{4k_{21}} + \frac{d_{\psi_2}^2}{4k_{22}} \right) \tag{53}$$

This completes the second step.

We may now perform the mth step of the proof, for $m = 3, \ldots, n-1$, where we select $z_{m+1} = x_{m+1} - \alpha_m - \alpha_m^s$, and we have

$$\dot{z}_m = \phi_m^c + \psi_m^c (z_{m+1} + \alpha_m + \alpha_m^s)$$
$$- \sum_{i=1}^{m-1} \left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i} \right) (\phi_i^c + \psi_i^c x_{i+1})$$
$$- \left(\frac{\partial \alpha_{m-1}}{\partial \nu_{m-1}} + \frac{\partial \alpha_{m-1}^s}{\partial \nu_{m-1}} \right) \dot{\nu}_{m-1}$$
$$- \sum_{i=1}^{m-1} \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\phi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\phi_i^j}} \right)^{\top} \dot{\hat{\theta}}_{\phi_i^j} + \left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\psi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\psi_i^j}} \right)^{\top} \dot{\hat{\theta}}_{\psi_i^j} \right] \tag{54}$$

With the term (16) we obtain

$$\dot{z}_m = -c_m z_m - \hat{\psi}_{m-1}^c z_{m-1} + \psi_m^c z_{m+1}$$
$$+ \sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_m^j}^{\top} \zeta_{\phi_m^j} + \Phi_{\psi_m^j}^{\top} \zeta_{\psi_m^j} \alpha_m \right) + (\delta_{\phi_m} + \delta_{\psi_m} \alpha_m)$$
$$- \sum_{i=1}^{m-1} \left(\frac{\partial \alpha_{m-1}}{\partial x_i} + \frac{\partial \alpha_{m-1}^s}{\partial x_i} \right) \times \left[\sum_{j=1}^{R} \rho_j \left(\Phi_{\phi_i^j}^{\top} \zeta_{\phi_i^j} + \Phi_{\psi_i^j}^{\top} \zeta_{\psi_i^j} x_{i+1} \right) + (\delta_{\phi_i} + \delta_{\psi_i} x_{i+1}) \right]$$
$$+ \sum_{i=1}^{m-1} \sum_{j=1}^{R} \left[\left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\phi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\phi_i^j}} \right)^{\top} \times \left(\gamma_{\phi_i^j} \tau_{\phi_{i(m+1-i)}^j} - \dot{\hat{\theta}}_{\phi_i^j} \right) + \left(\frac{\partial \alpha_{m-1}}{\partial \hat{\theta}_{\psi_i^j}} + \frac{\partial \alpha_{m-1}^s}{\partial \hat{\theta}_{\psi_i^j}} \right)^{\top} \left(\gamma_{\psi_i^j} \tau_{\psi_{i(m+1-i)}^j} - \dot{\hat{\theta}}_{\psi_i^j} \right) \right]$$
$$+ \kappa_m + \psi_m^c \alpha_m^s \tag{55}$$

We let

$$V_m = V_{m-1} + \tfrac{1}{2} z_m^2 + \tfrac{1}{2} \sum_{j=1}^{R} \left(\frac{1}{\gamma_{\phi_m^j}} \Phi_{\phi_m^j}^{\top} \Phi_{\phi_m^j} + \frac{1}{\gamma_{\psi_m^j}} \Phi_{\psi_m^j}^{\top} \Phi_{\psi_m^j} \right)$$

so that

$$
\begin{aligned}
\dot V_m &= \dot V_{m-1} + z_m \dot z_m - \sum_{j=1}^{R}\left(\frac{1}{\gamma_{\phi_m^j}}\Phi_{\phi_m^j}^\top \dot{\hat\theta}_{\phi_m^j} + \frac{1}{\gamma_{\psi_m^j}}\Phi_{\psi_m^j}^\top \dot{\hat\theta}_{\psi_m^j}\right)\\
&\le -\sum_{i=1}^{m} c_i z_i^2 + z_{m-1} z_m(\psi_{m-1}^c - \hat\psi_{m-1}^c)\\
&\quad + \psi_m^c z_m z_{m+1} + z_m \kappa_m + z_m \alpha_m^s\\
&\quad + \sum_{i=1}^{m-1}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\Phi_{\phi_i^j}^\top \hat\theta_{\phi_i^j} + \frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\Phi_{\psi_i^j}^\top \hat\theta_{\psi_i^j}\right)\\
&\quad + \sum_{i=1}^{m-1}\sum_{j=1}^{R}\left[\Phi_{\phi_i^j}^\top\left(\tau_{\phi_{i(m-i)}^j} - \frac{1}{\gamma_{\phi_i^j}}\dot{\hat\theta}_{\phi_i^j}\right)\right.\\
&\quad \left. + \Phi_{\psi_i^j}^\top\left(\tau_{\psi_{i(m-i)}^j} - \frac{1}{\gamma_{\psi_i^j}}\dot{\hat\theta}_{\psi_i^j}\right)\right]\\
&\quad + \sum_{j=1}^{R}\left[\Phi_{\phi_m^j}^\top\left(\rho_j\zeta_{\phi_m^j} z_m - \frac{1}{\gamma_{\phi_m^j}}\dot{\hat\theta}_{\phi_m^j}\right)\right.\\
&\quad \left. + \Phi_{\psi_m^j}^\top\left(\rho_j\zeta_{\psi_m^j} z_m \alpha_m - \frac{1}{\gamma_{\psi_m^j}}\dot{\hat\theta}_{\psi_m^j}\right)\right]\\
&\quad - z_m \sum_{i=1}^{m-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right)\\
&\quad \times \sum_{j=1}^{R}\rho_j\left(\Phi_{\phi_i^j}^\top\zeta_{\phi_i^j} + \Phi_{\psi_i^j}^\top\zeta_{\psi_i^j} x_{i+1}\right)\\
&\quad + \sum_{i=1}^{m-2}\sum_{j=1}^{R}\left[\left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\phi_i^j}} + \frac{\partial\alpha_{l-1}^s}{\partial\hat\theta_{\phi_i^j}}\right)^\top\right)\right.\\
&\quad \times\left(\gamma_{\phi_i^j}\tau_{\phi_{i(m-i)}^j} - \dot{\hat\theta}_{\phi_i^j}\right)\\
&\quad \left. + \left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\psi_i^j}} + \frac{\partial\alpha_{l-1}^s}{\partial\hat\theta_{\psi_i^j}}\right)^\top\right)\left(\gamma_{\psi_i^j}\tau_{\psi_{i(m-i)}^j} - \dot{\hat\theta}_{\psi_i^j}\right)\right]\\
&\quad + z_m\sum_{i=1}^{m-1}\sum_{j=1}^{R}\left[\left(\frac{\partial\alpha_{m-1}}{\partial\hat\theta_{\phi_i^j}} + \frac{\partial\alpha_{m-1}^s}{\partial\hat\theta_{\phi_i^j}}\right)^\top\left(\gamma_{\phi_i^j}\tau_{\phi_{i(m+1-i)}^j} - \dot{\hat\theta}_{\phi_i^j}\right)\right.\\
&\quad \left. + \left(\frac{\partial\alpha_{m-1}}{\partial\hat\theta_{\psi_i^j}} + \frac{\partial\alpha_{m-1}^s}{\partial\hat\theta_{\psi_i^j}}\right)^\top\left(\gamma_{\psi_i^j}\tau_{\psi_{i(m+1-i)}^j} - \dot{\hat\theta}_{\psi_i^j}\right)\right]\\
&\quad + z_m\left((\delta_{\phi_m} + \delta_{\psi_m}\alpha_m) - \sum_{i=1}^{m-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right)\right.\\
&\quad \left.\times(\delta_{\phi_i} + \delta_{\psi_i}x_{i+1})\right)\\
&\quad + \sum_{i=1}^{m-1}(m+1-i)\left(\frac{d_{\phi_i}^2}{4k_{i1}} + \frac{d_{\psi_i}^2}{4k_{i2}}\right)
\end{aligned}
\tag{56}
$$

Notice that, by definition, for $i = 1, \ldots, m-2$, it holds that

$$
\left.
\begin{aligned}
\gamma_{\phi_i^j}\tau_{\phi_{i(m-i)}^j} - \dot{\hat\theta}_{\phi_i^j} &= \gamma_{\phi_i^j}\tau_{\phi_{i(m+1-i)}^j}\\
&\quad + \rho_j\zeta_{\phi_i^j}\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right) z_m - \dot{\hat\theta}_{\phi_i^j}\\
\gamma_{\psi_i^j}\tau_{\psi_{i(m-i)}^j} - \dot{\hat\theta}_{\psi_i^j} &= \gamma_{\psi_i^j}\tau_{\psi_{i(m+1-i)}^j}\\
&\quad + \rho_j\zeta_{\psi_i^j}\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right) z_m x_{i+1} - \dot{\hat\theta}_{\psi_i^j}
\end{aligned}
\right\}
\tag{57}
$$

Moreover

$$
\begin{aligned}
&z_m\left((\delta_{\phi_m} + \delta_{\psi_m}\alpha_m) - \sum_{i=1}^{m-1}\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right)\right.\\
&\qquad\left.\times(\delta_{\phi_i} + \delta_{\psi_i}x_{i+1}) + z_{m-1}\delta_{\psi_{m-1}}\right)\\
&\le k_{m1}z_m^2 + \frac{d_{\phi_m}^2}{4k_{m1}} + k_{m2}z_m^2\alpha_m^2 + \frac{d_{\psi_m}^2}{4k_{m2}}\\
&\quad + \sum_{i=1}^{m-1}\left(k_{i1}z_m^2\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right)^2 + \frac{d_{\phi_i}^2}{4k_{11}}\right)\\
&\quad + \sum_{i=1}^{m-2}\left(k_{i2}z_m^2x_{i+1}^2\left(\frac{\partial\alpha_{m-1}}{\partial x_i} + \frac{\partial\alpha_{m-1}^s}{\partial x_i}\right)^2 + \frac{d_{\psi_i}^2}{4k_{i2}}\right)\\
&\quad + 2k_{(m-1)2}z_m^2x_m^2\left(\frac{\partial\alpha_{m-1}}{\partial x_{m-1}} + \frac{\partial\alpha_{m-1}^s}{\partial x_{m-1}}\right)^2\\
&\quad + \frac{d_{\psi_{m-1}}^2}{8k_{(m-1)2}} + 2k_{(m-1)2}z_{m-1}^2z_m^2 + \frac{d_{\psi_{m-1}}^2}{8k_{(m-1)2}}
\end{aligned}
\tag{58}
$$

so that with the term (17) we obtain

$$
\begin{aligned}
\dot V_m &\le -\sum_{i=1}^{m} c_i z_i^2 + \psi_m^c z_m z_{m+1}\\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\Phi_{\phi_i^j}^\top\hat\theta_{\phi_i^j} + \frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\Phi_{\psi_i^j}^\top\hat\theta_{\psi_i^j}\right)\\
&\quad + \sum_{i=1}^{m}(m+1-i)\left(\frac{d_{\phi_i}^2}{4k_{i1}} + \frac{d_{\psi_i}^2}{4k_{i2}}\right)\\
&\quad + \sum_{i=1}^{m}\sum_{j=1}^{R}\left[\Phi_{\phi_i^j}^\top\left(\tau_{\phi_{i(m+1-i)}^j} - \frac{1}{\gamma_{\phi_i^j}}\dot{\hat\theta}_{\phi_i^j}\right)\right.\\
&\quad \left. + \Phi_{\psi_i^j}^\top\left(\tau_{\psi_{i(m+1-i)}^j} - \frac{1}{\gamma_{\psi_i^j}}\dot{\hat\theta}_{\psi_i^j}\right)\right]
\end{aligned}
$$

$$+\sum_{i=1}^{m-1}\sum_{j=1}^{R}\left[\left(\sum_{l=2}^{m} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\phi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\phi_i^j}}\right)^{\top}\right)\right.$$
$$\times\left(\gamma_{\phi_i^j}\tau_{\phi^j_{i(m+1-i)}}-\dot{\hat\theta}_{\phi_i^j}\right)$$
$$\left.+\left(\sum_{l=2}^{m} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\psi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\psi_i^j}}\right)^{\top}\right)\left(\gamma_{\psi_i^j}\tau_{\psi^j_{i(m+1-i)}}-\dot{\hat\theta}_{\psi_i^j}\right)\right]$$
$$+z_m\left(\kappa_m+\sum_{i=1}^{m-2}\sum_{j=1}^{R}\left[\left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\phi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\phi_i^j}}\right)^{\top}\right)\right.\right.$$
$$\times\gamma_{\phi_i^j}\zeta_{\phi_i^j}\left(\frac{\partial\alpha_{m-1}}{\partial x_i}+\frac{\partial\alpha^s_{m-1}}{\partial x_i}\right)$$
$$+\left(\sum_{l=2}^{m-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\psi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\psi_i^j}}\right)^{\top}\right)$$
$$\left.\left.\times\gamma_{\psi_i^j}\zeta_{\psi_i^j}\left(\frac{\partial\alpha_{m-1}}{\partial x_i}+\frac{\partial\alpha^s_{m-1}}{\partial x_i}\right)x_{i+1}\right]\right) \tag{59}$$

We can make the last term of (59) equal to zero with the choice (23). In this way the mth step is completed.

The argument continues similarly up to the nth step, where the expression for $\dot z_n$ is the same as (55), with $m=n$ and the term $\psi^c_m z_{m+1}$ missing. To determine the stability properties of the closed loop system, consider the Lyapunov function candidate

$$V=V_{n-1}+\tfrac12 z_n^2+\tfrac12\sum_{j=1}^{R}\left(\frac{1}{\gamma_{\phi_n^j}}\Phi^{\top}_{\phi_n^j}\Phi_{\phi_n^j}+\frac{1}{\gamma_{\psi_n^j}}\Phi^{\top}_{\psi_n^j}\Phi_{\psi_n^j}\right) \tag{60}$$

Using (59) and the control law (25) it is easy to show that

$$\dot V\le-\sum_{i=1}^{n}c_iz_i^2+\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\Phi^{\top}_{\phi_i^j}\hat\theta_{\phi_i^j}+\frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\Phi^{\top}_{\psi_i^j}\hat\theta_{\psi_i^j}\right)$$
$$+\sum_{i=1}^{n}(n+1-i)\left(\frac{d^2_{\phi_i}}{4k_{i1}}+\frac{d^2_{\psi_i}}{4k_{i2}}\right)$$
$$+\sum_{i=1}^{n}\sum_{j=1}^{R}\left[\Phi^{\top}_{\phi_i^j}\left(\tau_{\phi^j_{i(n+1-i)}}-\frac{1}{\gamma_{\phi_i^j}}\dot{\hat\theta}_{\phi_i^j}\right)\right.$$
$$\left.+\Phi^{\top}_{\psi_i^j}\left(\tau_{\psi^j_{i(n+1-i)}}-\frac{1}{\gamma_{\psi_i^j}}\dot{\hat\theta}_{\psi_i^j}\right)\right]$$
$$+\sum_{i=1}^{n-1}\sum_{j=1}^{R}\left[\left(\sum_{l=2}^{n} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\phi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\phi_i^j}}\right)^{\top}\right)\right.$$
$$\times\left(\gamma_{\phi_i^j}\tau_{\phi^j_{i(n+1-i)}}-\dot{\hat\theta}_{\phi_i^j}\right)$$
$$+\left(\sum_{l=2}^{n} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\psi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\psi_i^j}}\right)^{\top}\right)$$
$$\left.\times\left(\gamma_{\psi_i^j}\tau_{\psi^j_{i(n+1-i)}}-\dot{\hat\theta}_{\psi_i^j}\right)\right]$$
$$+z_n\left(\kappa_n+\sum_{i=1}^{n-2}\sum_{j=1}^{R}\left[\left(\sum_{l=2}^{n-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\phi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\phi_i^j}}\right)^{\top}\right)\right.\right.$$
$$\times\gamma_{\phi_i^j}\zeta_{\phi_i^j}\left(\frac{\partial\alpha_{n-1}}{\partial x_i}+\frac{\partial\alpha^s_{n-1}}{\partial x_i}\right)$$
$$+\left(\sum_{l=2}^{n-1} z_l\left(\frac{\partial\alpha_{l-1}}{\partial\hat\theta_{\psi_i^j}}+\frac{\partial\alpha^s_{l-1}}{\partial\hat\theta_{\psi_i^j}}\right)^{\top}\right)$$
$$\left.\left.\times\gamma_{\psi_i^j}\zeta_{\psi_i^j}\left(\frac{\partial\alpha_{n-1}}{\partial x_i}+\frac{\partial\alpha^s_{n-1}}{\partial x_i}\right)x_{i+1}\right]\right) \tag{61}$$

The last term is made equal to zero with the choice (23) for κ_n. At this point we can at last cancel the uncertain terms in (61) with the adaptation laws (24). Finally, notice that, by completing squares

$$\Phi^{\top}_{\phi_i^j}\hat\theta_{\phi_i^j}=-\Phi^{\top}_{\phi_i^j}\left(\Phi_{\phi_i^j}-\theta^*_{\phi_i^j}\right)\le-\frac{|\Phi_{\phi_i^j}|^2}{2}+\frac{|\theta^*_{\phi_i^j}|^2}{2} \tag{62}$$

for $i=1,\dots,n$ and $j=1,\dots,R$. Hence, we obtain the inequality

$$\dot V\le-\sum_{i=1}^{n}c_iz_i^2-\tfrac12\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\left|\Phi_{\phi_i^j}\right|^2+\frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\left|\Phi_{\psi_i^j}\right|^2\right)+W_i \tag{63}$$

with

$$W_i=\sum_{i=1}^{n}(n+1-i)\left(\frac{d^2_{\phi_i}}{4k_{i1}}+\frac{d^2_{\psi_i}}{4k_{i2}}\right)+\tfrac12\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\left|\theta^*_{\phi_i^j}\right|^2+\frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\left|\theta^*_{\psi_i^j}\right|^2\right) \tag{64}$$

a measure of the representation error and ideal parameter vector sizes. Note that if

$$\sum_{i=1}^{n}c_iz_i^2\ge W_i \tag{65}$$

or

$$\tfrac12\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\left|\Phi_{\phi_i^j}\right|^2+\frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\left|\Phi_{\psi_i^j}\right|^2\right)\ge W_i \tag{66}$$

then $\dot V\le 0$. Moreover, letting†

† Please see Remark 5 for the reason to include v in the definition of c_0.

$$\left.\begin{aligned} c_0 &= \min_{1\le i\le n, v\in \mathrm{R}^q}(c_i) \\ \sigma_0 &= \min_{1\le i\le n, 1\le j\le \mathrm{R}}\left(\sigma_{\phi_i^j}, \sigma_{\psi_i^j}\right) \end{aligned}\right\} \quad (67)$$

it holds that

$$-\sum_{i=1}^{n} c_i z_i^2 \le -c_0 \sum_{i=1}^{n} z_i^2$$

$$-\tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{\sigma_{\phi_i^j}}{\gamma_{\phi_i^j}}\left|\Phi_{\phi_i^j}\right|^2 + \frac{\sigma_{\psi_i^j}}{\gamma_{\psi_i^j}}\left|\Phi_{\psi_i^j}\right|^2\right)$$

$$\le -\sigma_0 \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{1}{\gamma_{\phi_i^j}}\left|\Phi_{\phi_i^j}\right|^2 + \frac{1}{\gamma_{\psi_i^j}}\left|\Phi_{\psi_i^j}\right|^2\right) \quad (68)$$

Then, letting $\beta_i = \min(2c_0, \ \sigma_0)$, if

$$V = \tfrac{1}{2}\sum_{i=1}^{n} z_i^2 + \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{R}\left(\frac{1}{\gamma_{\phi_i^j}}\left|\Phi_{\phi_i^j}\right|^2 + \frac{1}{\gamma_{\psi_i^j}}\left|\Phi_{\psi_i^j}\right|^2\right) \ge V_0$$

$$\text{with} \quad V_0 = \frac{W_i}{\beta_i} \quad (69)$$

it holds that $\dot{V} \le 0$, and all signals in the closed loop are bounded. Moreover

$$\dot{V} \le -\beta_i V + W_i \quad (70)$$

which implies

$$0 \le V(t) \le \frac{W_i}{\beta_i} + \left(V(0) - \frac{W_i}{\beta_i}\right) \mathrm{e}^{-\beta_i t} \quad (71)$$

so that both the transformed states and the parameter error vectors converge to a bounded set. Finally, we conclude from (71) that the plant state X_n converges to the residual set

$$\mathcal{D}_i = \left\{ X_n \in \mathrm{R}^n : \sum_{i=1}^{n} z_i^2 \le \frac{2W_i}{\beta_i} \right\} \quad (72)$$

Then the proof is completed. □

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