Stable Cooperative Vehicle Distributions for Surveillance

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A mathematical model for the study of the behavior of a spatially distributed group of heterogeneous vehicles is introduced. We present a way to untangle the coupling between the assignment of any vehicle's position and the assignment of all other vehicle positions by defining general sensing and moving conditions that guarantee that even when the vehicles' motion and sensing are highly constrained, they ultimately achieve a stable emergent distribution. The achieved distribution is optimal in the sense that the proportion of vehicles allocated over each area matches the relative importance of being assigned to that area. Based on these conditions, we design a cooperative control scheme for a multivehicle surveillance problem and show how the vehicles' maneuvering and sensing abilities, and the spatial characteristics of the region under surveillance, affect the desired distribution and the rate at which it is achieved. [DOI: 10.1115/1.2767656]

1 Introduction

Examples of cooperative control scenarios include the distributed decision-making systems for a network of autonomous vehicles tasked with a search and rescue operation, a surveillance and attack mission, or a flexible manufacturing system. Resource allocation concepts lie at the heart of the significant technological advances in the design, understanding, and operation of such cooperative control systems. In particular, resource allocation strategies have been used to solve constrained optimization problems where a group of vehicles must distribute themselves among several areas of interest so that a predefined cost function is optimized (e.g., to maximize the probability of finding a target or to minimize the expected waiting time to service tasks/targets). One of the main challenges in developing allocation models for cooperative control problems is to capture the time-varying, distributed, and spatially coupled properties, which are inherent to the group of vehicles and the region being considered. In particular, the spatially distributed nature of cooperative control problems implies that allocation algorithms must be distributed across multiple moving vehicles, and even though these vehicles may only sense local information about their immediate surroundings, they must still cooperate in order to accomplish as a group a *global* common objective [1]. Furthermore, the presence of uncertainty in the vehicles' sensing abilities implies that the information driving their actions may be inaccurate and perhaps outdated as well. A vehicle may know the results of its own local actions but may not know what actions other vehicles should perform. In particular, when a vehicle gets assigned to an area to perform tasks, the benefit of assigning all other vehicles to this area generally decreases since the same vehicle may usually perform several tasks in the same vicinity and with relatively little additional cost (i.e., generally, the utility of a group of vehicles in an area decreases with an increasing number of agents). Despite recent progress, the challenge of how to untangle this space and time dependent "multivehicle to multiarea (-task)" coupling to provide a high performance cooperative behavior, especially in the presence of only local inaccurate information about the surroundings (e.g., when the effects of uncertainty noticeably affect the underlying vehicle dynamics and trajectories), remains an open and important problem in cooperative control. This is the problem we study here.

Recent studies trying to overcome the effects of uncertainty in cooperative control scenarios include work on (i) the synchronization of information shared between vehicles [2], (ii) the problem of dynamic reassignment of tasks among vehicles that communicate their information asynchronously and with random finite delays [3], (iii) the problem of assignment of mobile vehicles to stationary targets in the presence of communication imperfections [4], (iv) task load balancing approaches to control a group of vehicles when there are network delays [5], (v) the "persistent area denial (PAD)" problem where vehicles use both actual and predicted information about pop-up targets in order to decide where to move [6,7], and (vi) surveillance problems where vehicles must provide "patrol" coverage for an area significantly larger than their communication radius [8]. Like in Refs. [6–8], vehicle allocation models, which allow for spatial uncertainties, usually assume that different targets may appear in a predefined region of interest, but at unpredictable locations and random instants of time. They may then remain stationary, move around, or disappear again after some time. While vehicle allocation models need to explicitly capture the effects of such uncertainties, cooperative control strategies must provide the vehicles with the ability to appropriately adjust their actions so that they can still exploit some benefit from cooperation.

Here, we develop vehicle allocation strategies for cooperative control systems where spatial uncertainties dominate the problem (e.g., when target pop-up locations cannot be predicted because they may not depend on past events or where vehicles may have *some* prior knowledge about the terrain, but not in a detailed form as the classical search-theoretic rate of return (ROR) map [9] or other such maps used in map-based approaches). In particular, we develop scalable allocation strategies for surveillance, which guarantee that despite the effects of uncertainty, a desired vehicle distribution among several predefined areas of interest is still achieved (e.g., so that a global common objective can be met), without significantly constraining the individual decision-making abilities of the vehicles (e.g., so that vehicle route planing [10] and receding horizon [11], map-based [12], or other coordination schemes may still be used by vehicles located within the same area to benefit from their spatial proximity).

Our approach is inspired by how some animals seem to optimally distribute and reallocate themselves in nature. In particular, we use a concept from ecology known as the "ideal free distribution" (IFD) [13] to capture the multivehicle to multiarea coupling described above. The word "ideal" refers to the assumption that animals have perfect sensing abilities for simultaneously determining area "suitability" (assumed to be a correlate of Darwinian fitness) for all areas and that each animal will move to maximize its fitness. "Free" indicates that animals can move at no cost and instantaneously to any area regardless of their current location. If

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an animal perceives one area as more suitable, it moves to this area in order to increase its own fitness. This movement will, however, reduce the new area's suitability, both to itself and other animals in that area. The IFD is an animal distribution (an optimal equilibrium point) where no animal can increase its fitness by unilateral deviation from one area to another (i.e., when the IFD is achieved, all areas achieve equal suitability levels). Here, we use the IFD concept to describe a desired vehicle distribution where the proportioning of the vehicles allocated to each area matches the importance of being assigned to that area.

For cooperative surveillance scenarios, achieving the IFD implies that vehicles monitor a set of areas so that all areas are covered uniformly according to their priorities. We let every vehicle have a certain "capacity," which is assumed to be a correlate of its "competitive" strength capabilities, its sensing ability (e.g., a vehicle may have noisy sensors), its maneuvering ability (e.g., its speed or turn radius), or other vehicular characteristics that affect the coverage of an area. We allow vehicles to differ in their capacity and study how differences in their capacities affect the desired distribution. A description of our earlier work in this area is provided in Ref. [14].

The main contributions of this paper are as follows. In Sec. 2, we describe a cooperative surveillance problem and a particular surveillance scenario where vehicles must distribute themselves among several predefined areas of interest. In Sec. 3, we develop a discrete model that captures individual motion dynamics of a group of vehicles across the different regions. We untangle the multivehicle to multiarea coupling by establishing a wide class of cooperative control strategies that will lead to an *emergent* behavior of the group that is a "type of IFD" (which later, for simplicity, we will refer to as an IFD). By this, we mean one of many possible IFD realizations that are, in some sense, close to an IFD that is achieved under the original assumptions in Ref. [13]. Here, we must consider a wide class of distributions since the sensing noise and discretization that quantify the agent capacity both generally make it impossible to achieve perfect suitability equalization, which is demanded by the original IFD concept. We show how an "invariant set" of spatially distributed discrete vehicles can represent the IFD and use a Lyapunov stability analysis of this set to illustrate that there is a wide class of resulting vehicle movement trajectories across the areas that still achieve a desirable distribution. By achieving the desired distribution, we show a way to untangle the multiagent to multitask assignment coupling to provide a good cooperative behavior, even when agents are highly constrained on what to sense, may only sense noisy quantities, and differ in their individual capacities (this is the most important and novel contribution of this paper). Finally, in Sec. 4, we design an IFD-based cooperative surveillance strategy and present simulation results that illustrate the performance of such a strategy and the impact of embedded cooperative sensing strategies on achieving the IFD. As we show for the cooperative surveillance problem, and unlike in Ref. [15], here, we design individual control laws that are, in fact, suitable for real implementation in a group of discrete vehicles that share information over a common communication network.

2 Cooperative Surveillance Problem

Given a group of networked autonomous vehicles, one of the main challenges of multivehicle surveillance problems is to ensure that even when the vehicles' sensing and maneuvering abilities are highly constrained, they can nonetheless cooperatively monitor and track the state of some region they try to cover. Such surveillance missions include, for instance, deploying autonomous vehicles to monitor forest fires, patrol border lines, or suppress adversary targets. They usually require that vehicles persistently visit different locations in a predefined region of interest and perform one or several tasks, once a location is reached (e.g., on a target located there). While performing these tasks, vehicles may detect critical changes within their surroundings and must cooperatively adjust their actions to counteract such changes. We now describe the particular surveillance scenario we consider here.

First, we assume that vehicles may not be able to follow straight-line trajectories between targets, either because these trajectories are not feasible (e.g., flying or water vehicles usually require a minimum turn radius) or because they must reach their targets at certain approach angles (e.g., to obtain sufficient confidence about the successful completion of a task). In particular, assume that the ith vehicle obeys a Dubins model given by $\dot{x}_1^i = v\cos(\theta^i)$, $\dot{x}_2^i = v\sin(\theta^i)$, and $\dot{\theta}^i = \omega u^i$, where x_1^i is its horizontal position, x_2^i is its vertical position, v is its (constant) velocity, θ^i is its orientation, ω is its maximum angular velocity, and $-1 \leq u^i \leq 1$ is the steering input. Then, the minimum turn radius imposed by the dynamics of this model is $T = v/\omega$. We assume that vehicles will either travel on the minimum turning radius or on straight lines.

Second, we assume that the region under surveillance can be partitioned into smaller nonoverlapping areas. In particular, we assume that the region can be divided into N equal-size square $\ell \times \ell$ areas. For example, a fully connected region represents a region where vehicles can sense and travel to any other area regardless of their current location; a ring is one where the connectedness of the areas is characterized by a closed loop (e.g., the perimeter of a central area); and a line represents a region where vehicles can sense and move to adjacent neighboring areas but with limits on each end (e.g., the border of some area). Moreover, we assume that new targets continually pop up at different points throughout the entire region to be covered according to some stochastic process. We let R_i characterize the (average) rate of appearance of pop-up targets in area i, and assume it is constant but unknown to the vehicles. We assume that pop-up target locations in area i are known only to vehicles currently in i and that they stay exposed until they are visited by some vehicle. When a vehicle starts approaching a target, the target is considered to be "attended," and a vehicle may visit a new target only after the target being approached has been reached. While a vehicle approaches a target located in a particular area, it is considered to be monitoring that area. Once the target is reached, the vehicle may perform various tasks such as classification, engagement, or verification of the target, and it is then ignored for the rest of the mission.

We define the "suitability level" of an area as the (average) *rate of appearance of unattended targets* (i.e., targets that have appeared but are not being or have not been attended by any vehicle). Assume that each area can be characterized by a suitability function and that the suitability function of area i strongly depends on the number of vehicles x_i monitoring that area. Figure 1 shows simulation results for two classes of suitability functions for different intra-area vehicle coordination strategies and target pop-up rates R_i. The left plot assumes that vehicles monitoring the *same* area coordinate in order to decide which targets within that area to attend to (i.e., after a target is reached, a vehicle approaches the closest target that is not being approached by any other vehicle). The right plot assumes that vehicles monitoring the same region do not coordinate and they randomly approach any target located within the area they are monitoring. Note that an intra-area coordination strategy leads to a higher rate of targets being attended by the vehicles and, consequently, to a faster decrease in suitability level as the number of vehicles monitoring that area increases. However, note that coordination gains per additional vehicle decrease as the number of vehicles monitoring the area increases.

Finally, we assume that the goal of a surveillance mission is to make the proportion of vehicles visiting a set of predefined areas match the relative importance of monitoring each area. In particular, a successful surveillance strategy must concentrate more vehicles at areas with higher suitability levels so that vehicles achieve an emergent distribution across these areas that results in

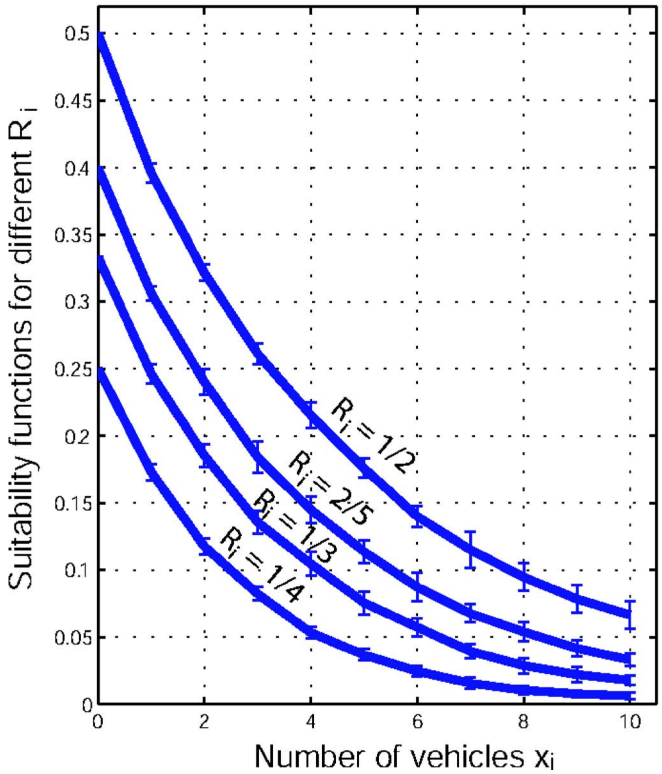

Fig. 1 Suitability functions for an area with $\ell=2.5$ km and vehicles at speed $v=15$ m/s; $T=100$ m with (left) and without (right) intra-area coordination strategies. Each data point represents 60 simulation runs with varying target pop-up locations. The error bars are sample standard deviations from the mean.

all areas attaining similar suitabilities (i.e., similar rates of appearance of unattended targets). This vehicle distribution goal must be achieved in spite of vehicle sensing, communication, and motion constraints (the combination of which requires a decentralized vehicle guidance strategy, with each vehicle making independent decisions). In particular, note that due to the stochastic nature of the problem (for instance, the randomness in the location where targets appear or in the time instants they are found), and regardless of other possible sensing and moving constraints (whether the region is modeled as a fully connected region, a ring, or a line), no vehicle can perfectly know the value of the suitability level of any area, including the ones it is monitoring (e.g., since R_i is unknown to the vehicles for all $i=1,\ldots,N$). The vehicles only have noisy perceptions about the suitability levels of the area they are monitoring and its adjacent areas. We now develop a model that captures the dynamics of a group of vehicles deployed in the described scenario. We then use this model to define a class of strategies that guarantees the achievement of the surveillance distribution goal.

3 Model, Strategy, and Analysis

The model we introduce here is built on a directed graph, so that the graph topology defines the interconnections between the various areas within some region of interest (e.g., to characterize the different types of regions a group of vehicles may try to cover). Thus, in what follows we refer to areas as "nodes." The term "agent" is associated with entities capable of physical motion such as vehicles, robots, or aircrafts. A common approach in modeling a group behavior is to assume the existence of a large number of agents. Under such an assumption, the total number of agents at a node can be adequately represented by a continuous variable. Such an approach was used in Ref. [15]. Here, we extend the model in Ref. [15] to allow for a finite number of discrete agents. As in Ref. [15], we assume that agents may move and distribute themselves over N available nodes and let $H=\{1,\ldots,N\}$. Moreover, we define the suitability of node $i \in H$ as $s_i(x_i)$, where x_i represents the state of node i. However, we do not require the existence of a large number of agents, and we assume instead that x_i is described with a discrete variable. This allows us to capture individual agent characteristics by taking into account, for example, different agent capacities. Hence, here, we assume that $x_i \in \mathbb{R}_+=[0,\infty)$ represents the total agent capacity at node i, which results from multiple discrete agents being present at that node. The total agent capacity employed in a mission stays constant, so that $C=\sum_{i=1}^{N} x_i$ is fixed. Let $\varepsilon_c \in \mathbb{R}_+$ be the minimum agent capacity required to be present at any node $i \in H$ (i.e., either so that all suitability functions are well defined at any state or as an additional constraint on the state of a node). We assume that $C > N\varepsilon_c$ so that the total agent capacity employed in a mission is strictly larger than the combined minimum agent capacity of all nodes. Note that the precise value of ε_c will depend on the lowest agent capacity of any agent and the minimum number of agents allowed at any node. In fact, we assume that the total agent capacity can be partitioned into discrete blocks. Each block represents a particular agent, and its size is assumed to be a correlate of its capacity. We assume that the largest capacity of any agent is given by a constant $\bar{x}>0$, and the smallest capacity of any agent is given by a constant $\underline{x}$, so that $\bar{x} \geq \underline{x} > 0$. In particular, if $\bar{x}=\underline{x}=1$ then all agents have equal capacities and x_i represents, e.g., the total number of agents monitoring node i as in the surveillance scenario described in Sec. 2. In general, we assume the following.

- *Node suitability changes relate to total node agent capacity changes*. We assume that for all $s_i(x_i)$, $i \in H$, there exist constants $\underline{c}_i, \bar{c}_i \in \mathbb{R}$, $\underline{c}_i, \bar{c}_i > 0$, such that

$$-\underline{c}_i \leqslant \frac{s_i(y_i)-s_i(z_i)}{y_i-z_i} \leqslant -\bar{c}_i \quad (1)$$

for any $y_i, z_i \in [\varepsilon_c, C]$, $y_i \neq z_i$. Thus, $s_i(x_i)$ is a strictly monotonically decreasing function in its argument $x_i \in [\varepsilon_c, C]$, so that as the total agent capacity in node i increases, the suitability of the node decreases. Moreover, we assume that $\lim_{x_i \to \infty} s_i(x_i) = 0$ for all $i \in H$.

- *Strictly positive suitability*. We assume that the functions $s_i(x_i) > 0$ for all $i = 1, \ldots, N$, and all $x_i \in [\varepsilon_c, C]$. For this condition to hold, the suitability functions can typically be shifted vertically if necessary so that the suitability levels remain strictly positive for any agent capacity. Hence, we think of suitabilities as *relative* suitabilities.

3.1 Environmental Constraints on Agent Sensing and Motion. The interconnection of nodes is described by a directed graph, (H, A), where $A \subset H \times H$. If $(i,j) \in A$, this represents that an agent at node i can sense its *neighboring* node j and can move from i to j. If $(i,j) \in A$, agents at node i can sense the total agent capacity at node j, x_j. However, unlike in Ref. [15], we do not assume that agents have perfect sensor capabilities to measure its own suitability level or the suitability levels of its neighboring nodes. In particular, for agents at node i, where $(i,j) \in A$, "sensing node j" implies that agents at node i know $s_j(x_j) + w$, where w is the "sensing noise" that can change over time randomly, but $-\underline{w} \leqslant w \leqslant \bar{w}$ for known constants $\underline{w}, \bar{w} \geqslant 0$. Let $s_j^i(x_j) = s_j(x_j) + w$ denote the *perception* (i.e., the noisy measured value) by agents at node i of the suitability level of node j with total agent capacity x_j. In some cases, one might want to assume that w depends on x_i. For instance, $s_j^i(x_j) = s_j(x_j) + w(x_i)$ with $\underline{w} = \bar{w}$, and $|w(x_i')| > |w(x_i'')| \geqslant 0$ for $x_i'' > x_i'$ represents sensing conditions where a larger agent capacity at node i results in a better suitability perception of its neighboring node j (e.g., due to better sensing capacities of the individual agents, agreement strategies among different agents at the same node that improve their individual sensing abilities, or averaging strategies which compensate for the error present in individual suitability assessments). Other sensing conditions may require that $s_j^i(x_j) = s_j(x_j) + w^{ij}$, where w^{ij} is the sensing noise present when agents at node i measure the suitability level of node j in order to represent that different nodes may be measured with different accuracies. Here, we simply assume that if $w(k)$ is the sensing noise present in an agent's perception at time k, then it may be the case that $w(k_1) \neq w(k_2)$ for $k_1 \neq k_2$, which produces a general framework to represent that the sensing capabilities of the agents may change over time (e.g., as agents discover their surroundings, their ability to assess the suitability levels of neighboring nodes may change).

Note that an agent's perception about the suitability level of a neighboring node may differ from its actual value by at most $\max\{\underline{w}, \bar{w}\}$. Also, note that given a node $\ell \in H$, and two neighboring nodes i, j such that $(\ell, i) \in A$ and $(\ell, j) \in A$ with $s_i(x_i) > s_j(x_j)$, if $s_i(x_i) - s_j(x_j) > 2\max\{\underline{w}, \bar{w}\}$, then the measured values of the suitability levels of nodes i and j by agents at node ℓ are such that $s_i^\ell(x_i) > s_j^\ell(x_j)$, regardless of the sensing noise w present during the measurements. In other words, if $s_i(x_i) - s_j(x_j) > 2\max\{\underline{w}, \bar{w}\}$, then the two sets of all possible measured values of the suitability levels of the corresponding nodes i and j, given $s_i(x_i)$ and $s_j(x_j)$, do not overlap. Conversely, note that these sets may only overlap if $0 < s_i(x_i) - s_j(x_j) \leqslant 2\max\{\underline{w}, \bar{w}\}$. Moreover, if $(j,i) \in A$, then $|s_i^j(x_i) - s_i(x_i)| \leqslant \max\{\underline{w}, \bar{w}\}$ and, therefore, $|s_i^j(x_i) - s_j(x_j)| \leqslant 3\max\{\underline{w}, \bar{w}\}$. Finally, since $|s_j^j(x_j) - s_j(x_j)| \leqslant \max\{\underline{w}, \bar{w}\}$, we obtain $|s_i^j(x_i) - s_j^j(x_j)| \leqslant 4\max\{\underline{w}, \bar{w}\}$, regardless of the noise w present during the measurement. Let us define $W = 4\max\{\underline{w}, \bar{w}\}$ as the maximum difference between the measured suitability value of a neighboring node and the perception of the suitability level of the node where the sensing agents are located, given that the actual suitability levels of both nodes i and j are close enough (i.e., they do not differ by more than $2\max\{\underline{w}, \bar{w}\}$).

We assume that for every $i \in H$, there must exist some $j \in H$, $i \neq j$, such that $(i,j) \in A$, and there must exist a path between any two nodes in order to ensure that every node is connected to the graph. We also assume that if $(i,j) \in A$, then $(j,i) \in A$, so that if an agent is at i and can move to j (sense the suitability at j), agents at j can also move from j to i (sense the suitability at i). An agent at node i can only directly move to node j if $(i,j) \in A$. However, if $(i,j) \notin A$, it may in some situations be possible for an agent to (indirectly) move to j by passing through a series of other nodes. If $(i,j) \in A$, then $i \neq j$; however, agents at i know the value of $s_i^i(x_i)$, are assumed to know x_i, and are already at i so they do not need to move to get to it.

Let $\mathbb{R}_{\varepsilon_c} = [\varepsilon_c, \infty)$ and $\boldsymbol{\mathcal{X}} = \{x \in \mathbb{R}_{\varepsilon_c}^N : \Sigma_{i=1}^N x_i = C\}$ be the simplex over which the x_i dynamics evolve. Let $x(k) = [x_1(k), x_2(k), \ldots, x_N(k)]^T \in \boldsymbol{\mathcal{X}}$ be the state vector, where $x_i(k)$ represents the total agent capacity at node i at time index $k \geqslant 0$. Constraints on our model below will ensure that $x(k) \in \boldsymbol{\mathcal{X}}$ for all $k \geqslant 0$. Let $I(x) = \{i \in H : x_i > \varepsilon_c, x \in \boldsymbol{\mathcal{X}}\}$ represent the set of nodes at state x, such that each node $i \in I(x)$ is occupied by a certain number of agents, which results in the total agent capacity at node i exceeding the value of ε_c. Similarly, let $U(x) = H - I(x)$ represent the set of nodes at state x whose total agent capacity equals the minimum agent capacity ε_c. The size of the set $I(x)$ is denoted by N_I. Let

$$M = \max_i \{s_i(x_i) - s_i(x_i + \bar{x}) : \quad \text{for all } x_i \in [\varepsilon_c, C]\} \quad (2)$$

for all $i \in H$. In other words, M is the maximum change in suitability that could occur by having an agent of maximum capacity leave any node. Figure 2 shows an example of a system with $N = 3$ nodes and perfect sensing capabilities so that $\underline{w} = \bar{w} = 0$. Note that a horizontal band of width $M > 0$ crossing at least one s_i curve represents an IFD state for some total agent capacity C. As the total agent capacity C increases, the band moves toward the x axis, representing that the average suitability of all nodes at the IFD state decreases with increasing total agent capacity.

For a general graph topology, the best we can generally hope to do with local information only and a distributed decision-making strategy is to distribute agent capacities in such a way that the suitability levels between any two connected nodes remain within M. In particular, we can guarantee that $|s_i(x_i) - s_j(x_j)| \leqslant M$ for all $(i,j) \in A$ such that $i, j \in I(x)$ at the desired distribution. Note that the value of M depends on the particular shape of all the suitability functions (i.e., the suitability function of any node is bounded by Eq. (1)), the total agent capacity C, and the largest capacity of any agent $\bar{x}$. In particular, note that since Eq. (1) applies for all $i \in H$ and any $y_i, z_i \in [\varepsilon_c, C]$, if we let $y_i = x_i$ and $z_i = x_i + \bar{x}$, we can bound M by $\bar{x}\min_i\{\bar{c}_i\} \leqslant M \leqslant \bar{x}\max_i\{\underline{c}_i\}$. Similarly, let $m = \min_i\{s_i(x_i) - s_i(x_i + \underline{x}) : \text{for all } x_i \in [\varepsilon_c, C]\}$ for all $i \in H$. Note that Eq. (1) guarantees that $M, m > 0$.

3.2 Agent Sensing, Coordination, and Motion Requirements. Let $\mathcal{E}$ be a set of events and let $e_{\alpha(i,k)}^{i,p(i)}$ represent the event that one or more agents move from node $i \in H$ to neighboring nodes $\ell \in p(i)$ at time k, where $p(i) = \{j : (i,j) \in A\}$. Note that the movement of agents from node i to neighboring nodes decreases x_i since node i reduces its total agent capacity and consequently increases $s_i(x_i)$. Let $\alpha_\ell(i,k)$ denote the total agent capacity of the agents that move from node $i \in H$ to node $\ell \in p(i)$ at time k. Let the list $\alpha(i,k) = [\alpha_j(i,k), \alpha_{j'}(i,k), \ldots, \alpha_{j''}(i,k)]$ such that $j < j' < \cdots < j''$ and $j, j', \ldots, j'' \in p(i)$ and $\alpha_j \geqslant 0$ for all $j \in p(i)$ represent the total agent capacity of the agents that move to all neighboring nodes of node i; the size of the list $\alpha(i,k)$ is $|p(i)|$ and

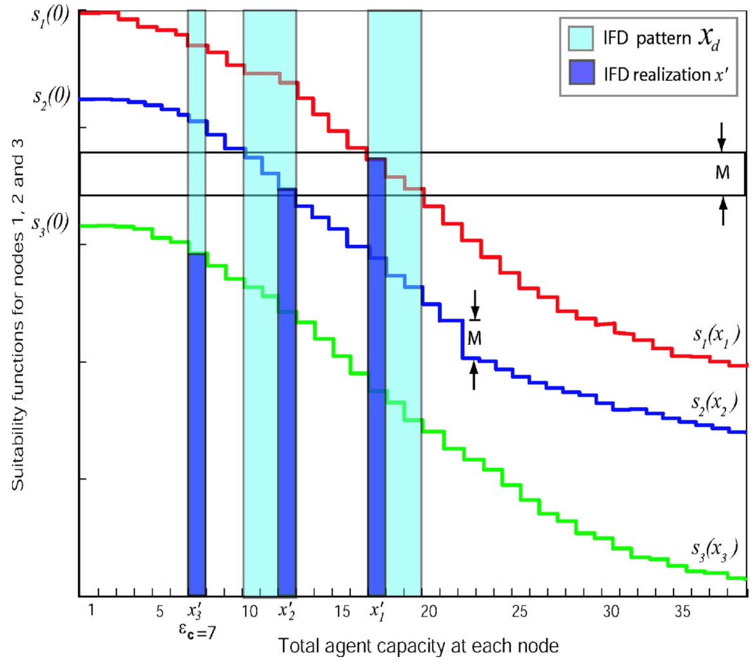

Fig. 2 Suitability functions $s_i(x_i)$ for three fully connected nodes with $\bar{x}=\underline{x}=1$, $\underline{w}=\bar{w}=0$, $\varepsilon_c=7$, and $C=36$. Under perfect sensing conditions, the IFD distribution is reached when all agents are distributed in such a way that at state x neighboring nodes i such that $i\in I(x)$ have suitability levels that do not differ by more than M. After the IFD is reached, there is no movement of agents between nodes. For the example shown in the plot, while agents distribute themselves over nodes 1 and 2, node 3 remains at the minimum agent capacity ε_c at the desired distribution. Node $i=3$ is called a *truncated* node. The suitability level $s_3(\varepsilon_c)$ is too low to be chosen by any agent. Note also that there may exist different distributions of the total agent capacity that correspond to neighboring suitability levels of nodes $i\in I(x)$ differing by at most M. The light-colored vertical bands represent all possible distributions of agent capacity for which the IFD pattern is achieved. We denote the set of all such distributions by $\mathcal{X}_d$ and will describe it mathematically in Sec. 3.3. The dark-colored vertical bars illustrate a particular distribution $x'=[7,12,17]^T$, and its resultant suitability levels satisfy the IFD pattern (note that $x'=[7,11,18]^T$ and $x'=[7,10,19]^T$ would also result in suitability levels that satisfy the IFD pattern).

remains constant for all time $k\geq 0$ for all $i\in H$ since the topology of the graph (H,A) is assumed to be time invariant (i.e., $\alpha(i,k)\in\mathbb{R}_C^{|p(i)|}$ for all k, where $\mathbb{R}_C=[0,C]$). Let $\{e_{\alpha(i,k)}^{i,p(i)}\}$ represent the set of *all* possible combinations of how agents can move from node i to its neighboring nodes for all k. Let the set of events be described by $\mathcal{E}=\mathcal{P}(\{e_{\alpha(i,k)}^{i,p(i)}\})-\{\emptyset\}$ ($\mathcal{P}(\cdot)$ denotes the power set). Notice that each event $e(k)\in\mathcal{E}$ is defined as a *set*, with each element of $e(k)$ representing the transition of possibly multiple agents among neighboring nodes in the graph. Multiple elements in $e(k)$ represent the simultaneous movements of agents, i.e., migrations out of multiple nodes.

An event $e(k)$ may only occur if it is in the set defined by an "enable function," $g:\boldsymbol{\mathcal{X}}\to\mathcal{P}(\mathcal{E})-\{\emptyset\}$. State transitions are defined by the operators $f_e:\boldsymbol{\mathcal{X}}\to\boldsymbol{\mathcal{X}}$, where $e\in\mathcal{E}$. We now specify g and f_e for $e(k)\in g[x(k)]$, which define the agents' sensing and motion.

If for a node $i\in H$, $s_j^i(x_j)-s_i^i(x_i)\leq M$ for all $(i,j)\in A$, then $e_{\alpha(i,k)}^{i,p(i)}\in e(k)$ such that $\alpha(i,k)=(0,\ldots,0)$ is the only enabled event. Hence, agents at the most suitable node that they know of do not move. Note also that this does not then allow for a "swap" of equal numbers of agents between two nodes i and j, $(i,j)\in A$ such that $s_j^i(x_j)-s_i^i(x_i)\leq M$. The reason that such swaps are not possible (or allowed) is that the agents do not necessarily share information (e.g., via communications) that would allow them to achieve an equal number of agents switching between nodes without changing the corresponding suitability levels. In other words, we do not assume that there is coordination between individuals at different nodes.

Moreover, if for node $i\in H$, $s_j^i(x_j)-s_i^i(x_i)>M$ for some j such that $(i,j)\in A$, then the only $e_{\alpha(i,k)}^{i,p(i)}\in e(k)$ are the ones with $\alpha(i,k)=[\alpha_j(i,k):j\in p(i)]$ such that

(i) $x_i(k)-\Sigma_{\ell\in p(i)}\alpha_\ell(i,k)\geq\varepsilon_c$

(ii) $s_i^i[x_i(k)-\Sigma_{\ell\in p(i)}\alpha_\ell(i,k)]<\max_j\{s_j^i[x_j(k)]:j\in p(i)\}-W$

(iii) if $\alpha_j(i,k)>0$ for some $j\in p(i)$, then $\alpha_{j^*}(i,k)\geq\underline{x}$ for some $j^*\in\{j:s_j^i[x_j(k)]\geq s_\ell^i[x_\ell(k)]\,\forall\,\ell\in p(i)\}$

(iv) $\alpha_j(i,k)=0$ for any $j\in p(i)$ such that $s_i^i[x_i(k)]>s_j^i[x_j(k)]$ and $x_j(k)=\varepsilon_c$

Condition (i) guarantees that at any node there is at least a ε_c agent capacity. It is required so that conditions (ii) and (iii) are well defined at all times. To interpret conditions (ii) and (iii), it is useful to note that reducing (increasing) the total agent capacity at a node always increases (decreases) the suitability at that node. The two conditions constrain how agents can move based on their capacities and in terms of node suitabilities. Note that agents may also move from higher suitability nodes to lower suitability nodes as long as all conditions are satisfied. Condition (ii) states that *after* agents move from node i to other nodes, the perception of the suitability level of node i due to some agents leaving is strictly less than the highest perception among neighboring nodes minus W before agents started moving. This prevents there being many agents moving from node i that stationary agents in node i try to achieve higher suitability than all its neighbors (of course, additional agents could move to the highest suitability node reducing its suitability far enough, so that node i actually becomes the highest suitability node at time $k+1$). Without condition (ii), there could be a sustained oscillation of agent movements between nodes. Condition (iii) implies that if the perception of the suitability of node i differs by more than M with the perception of some of its neighboring nodes, and at some point in time some agents move to neighboring nodes, then at least one agent must move to the neighboring node perceived with the highest suitability. Without condition (iii), some high suitability node could be ignored by the agents and the IFD distribution might not be achievable. As we will show below, condition (ii), together with condition (iii), guarantees that the highest suitability node strictly monotonically decreases over time. Condition (iv) states that if agents at node i perceive a less suitable neighboring node to have only the minimum agent capacity, they do not move to that node. Without condition (iv), some agents would still be free to move to nodes with lower suitability levels, which in an optimal distribution ought to only have the minimum agent capacity required, ε_c. Consequently, the desired distribution would not be maintained.

Finally, if $e(k) \in g[x(k)]$, $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$, then $x(k+1)=f_{e(k)}[x(k)]$, where $x_i(k+1)$ equals $x_i(k)$ plus

$$\sum_{\{j:i \in p(j), e^{j,p(j)}_{\alpha(j,k)} \in e(k)\}} \alpha_i(j,k) - \sum_{\{j:j \in p(i), e^{i,p(i)}_{\alpha(i,k)} \in e(k)\}} \alpha_j(i,k)$$

Note that if $x(0) \in \boldsymbol{\mathcal{X}}, x(k) \in \boldsymbol{\mathcal{X}}, k \geq 0$ (i.e., $\boldsymbol{\mathcal{X}}$ is invariant).

Next, let $\mathcal{E}^{\mathbb{N}}$ denote the set of all infinite sequences of events in $\mathcal{E}$. Let $E_v \subset \mathcal{E}^{\mathbb{N}}$ be the set of valid event trajectories for the model (i.e., the ones that are physically possible). Event $e(k) \in g[x(k)]$ is composed of a set of what we will call "partial events." Define a partial event of type i to represent the movement of $\alpha(i,k)$ agents from node $i \in H$ to its neighbors $p(i)$ so that conditions (i)–(iv) are satisfied at time k. A partial event of type i will be denoted by $e^{i,p(i)}$ and the occurrence of $e^{i,p(i)}$ indicates that some agents located at node $i \in H$ move to other nodes. Partial events must occur according to the "allowed" event trajectories. The allowed event trajectories define the degree of asynchronicity of the model at the node level. We define two possibilities for the allowed event trajectories.

(i) For allowed event trajectories $E_i \subset E_v$, assume that each type of partial event occurs infinitely often on each event trajectory $E \in E_i$. The assumption is met if at each node all agents do not ever stop trying to move (e.g., if each agent persistently tries to move to neighboring nodes). This corresponds to assuming "total asynchronism" [16].

(ii) For allowed event trajectories $E_B \subset E_v$, assume that there exists $B>0$, such that for every event trajectory $E \in E_B$, in every substring $e(k'), e(k'+1), e(k'+2), \ldots, e[k'+(B-1)]$ of E, there is the occurrence of every type of partial event (i.e., for every $i \in H$, the partial event $e^{i,p(i)} \in e(k)$, for some $k, k' \leq k \leq k'+B-1$). This corresponds to assuming "partial asynchronism" [16].

Let E_k denote the sequence of events $e(0), e(1), \ldots, e(k-1)$, and let the value of the function $X[x(0), E_k, k]$ denote the state reached at time k from the initial state $x(0)$ by application of the event sequence E_k.

3.3 Emergent Agent Distribution. The set

$$\boldsymbol{\mathcal{X}}_d = \{x \in \boldsymbol{\mathcal{X}}: \forall\ i \in H \quad \text{either} |s_i(x_i) - s_j(x_j)| \leq M + W\ \forall\ j \in p(i): x_j \neq \varepsilon_c \text{ and } s_i(x_i) > s_j(x_j)\ \forall\ j \in p(i): x_j = \varepsilon_c \text{ or } x_i = \varepsilon_c\} \tag{3}$$

is an invariant set that represents all possible distributions of the total agent capacity C at the IFD since $|s_i(x_i)-s_j(x_j)| \leq M+W$ for $x \in \mathcal{X}_d$ for all $i,j \in I(x)$ such that $(i,j) \in A$. Note that $\mathcal{X}_d$ is invariant since for any $x \in \mathcal{X}_d$ any node $i \in I(x)$, one of two cases must be true. If node $i \in I(x)$ has no neighboring nodes $j \in p(i)$ such that $x_j = \varepsilon_c$, then it differs in suitability from its neighboring nodes by at most $M+W$. Therefore, according to the definition of the enable function g, there is no agent movement between nodes $i,j \in I(x)$, which differ in suitability by at most $M+W$ [$\alpha_j(i,k)=0$ for all $k \geq 0$ for all $i,j \in I(x)$ when $x \in \mathcal{X}_d$]. If for node $i \in I(x)$, there exist neighboring nodes $j \in p(i)$ such that $x_j = \varepsilon_c$, then condition (iv) guarantees that $\alpha_j(i,k)=0$ for all $k \geq 0$ for all $j \in U(x)$ when $x \in X_d$. Therefore, at state $x \in \mathcal{X}_d$, there is no movement from agents from nodes $i \in H$ such that $x_i > \varepsilon_c$ to nodes at the minimum agent capacity. Hence, it is always true that $\alpha(i,k) = (0, \ldots, 0)$ for all $i \in I[x(k)]$ when $x(k) \in \mathcal{X}_d$. Furthermore, any node $i \in U[x(k)]$ when $x(k) \in \mathcal{X}_d$ has the minimum agent capacity, so that condition (i) does not allow agents to move and change the distribution $x \in \mathcal{X}_d$. Hence, $\alpha(i,k) = (0, \ldots, 0)$ for all $i \in H$ when $x(k) \in \mathcal{X}_d$. Recall, however, that there exist many different agent distributions that belong to $\mathcal{X}_d$. Any agent distribution, such that the distribution of the total agent capacities $x \in \mathcal{X}_d$, is called an IFD realization. Note that according to the definition of M, a set of suitability functions where $\bar{c}_i \geq 0$ for all $i \in H$ represents a region where the migration of agents to any node has a large effect on its suitability level. Furthermore, note that for this set of suitability functions, a large value of $\bar{x}$ means that at the IFD, the difference in suitability levels between any two neighboring nodes may be large. Moreover, if $\bar{c}_i$ increases for larger values of the argument of $s_i(x_i)$, an increase in the total agent capacity (e.g., due to a larger number of agents, or agents with better capabilities) implies steeper suitability curves, which in turn results in an increase in the value of M. Larger values of M imply, in general, a larger set of possible IFD realizations.

Note that according to the definition of $\mathcal{X}_d$, it is possible for unconnected nodes (i.e., ones such that $(i,j) \notin A$) in the set $I(x)$ to have suitabilities that differ by more than M when the distribution is achieved. This could happen if two nodes i,j such that $i,j \in I(x)$ with high suitability levels when $x \in \mathcal{X}_d$ are separated by a node with a minimum agent capacity (e.g., in a region represented by a line topology of the graph (H,A)). However, any two nodes that are linked according to the graph (H,A) (i.e., ones such that $(i,j) \in A$) and belong to the set $I(x)$ must have suitability levels that differ at most by $M+W$ at the desired distribution. Hence, depending on the graph's connectivity, there could be isolated groups of nodes where only nodes belonging to the same group have suitability levels that differ by at most $M+W$ (i.e., dividing the region of different groups). Moreover, note that the formation of group nodes depends on the total agent capacity employed in the mission, the initial distribution $x(0)$, and the random agent migration between nodes.

THEOREM 1 *(Stability for a fully connected region, any total agent capacity). Given a fully connected graph* (H,A), $\varepsilon_c > 0$, *any number of agents with total agent capacity* C, *and agent motion conditions* (i)–(iv), *the invariant set* $\mathcal{X}_d$ *is asymptotically stable in the large with respect to* E_i *and exponentially stable in the large with respect to* E_B.

Note that asymptotic/exponential stability *in the large* implies that for any initial distribution of agents, the invariant set will be achieved. This result provides general sufficient conditions on when a distribution satisfying the IFD pattern is achieved. However, the size of $\mathcal{X}_d$ is not necessarily 1, since there are many possible IFD realizations that may be achieved. Theorem 1 guarantees that under the above stated sensing and motion conditions, one of them will be reached.

Moreover, notice that Theorem 1 requires $\varepsilon_c > 0$ because if $\varepsilon_c = 0$ at a truncated node i, then $s_i(x_i)$ equals infinity for certain suitability functions (e.g., $s_i(x_i) = a_i/x_i$). The proof of Theorem 1 considers the emergence of different node groups when the region is modeled by a fully connected topology. Node groups emerge as agents distribute themselves over the nodes if the total agent capacity is small enough. The dynamic emergence of node groups is considered in the proof of Theorem 1.

Exponential stability of the invariant set $\mathcal{X}_d$ means that all agents are guaranteed to converge to $\mathcal{X}_d$ at a certain rate. Define

$$\rho(x,\boldsymbol{\mathcal{X}}_d) = \min\{\max\{|x_1 - x_1'|, \ldots, |x_N - x_N'|\}:x' \in \boldsymbol{\mathcal{X}}_d\} \tag{4}$$

Notice that $\max\{|x_1-x_1'|, \ldots, |x_N-x_N'|:x' \in \mathcal{X}_d\}$ measures the maximum difference among all nodes between the current total agent capacity at a node and the total agent capacity it ought to have to achieve the IFD pattern. Hence, $\rho(x,\mathcal{X}_d)$ measures the maximum difference in agent capacity between the current agent capacity distribution and a *particular* IFD distribution that minimizes the maximum difference. Exponential stability implies that motions can be overbounded by an exponential function, so that

$$\rho\{X[x(0),E_k,k],\boldsymbol{\mathcal{X}}_d\} \leq \zeta e^{-\alpha k}\rho[x(0),\boldsymbol{\mathcal{X}}_d] \tag{5}$$

for some $\alpha > 0$ and some $\zeta > 0$ for all E_k and $k \geq 0$ such that $E_k E \in E_B$. The parameter α affects the rate at which the IFD is achieved. If the initial distribution of the total agent capacity $x(0)$ is further away from $\mathcal{X}_d$, Eq. (5) shows that it can take longer to achieve the IFD. Note also that it sets a bound for all possible trajectories of x, regardless of the particular IFD realization that is achieved.

Finally, note that unlike in Ref. [15], here, we are able to quantify the rate at which agents abandon nonworthy nodes in order to achieve an IFD realization. Moreover, here, we are able to guarantee that such a realization is achieved even though agents possess only noisy assessments of the state of their immediate surroundings.

THEOREM 2 *(Stability for a not fully connected region, but sufficient total agent capacity). Given any* (H,A), $\varepsilon_c \geq 0$, *and agent motion conditions* (i)–(iv), *there exists a constant* $C > N\varepsilon_c$ *such that if the total agent capacity employed in the mission is at least* C, *then the invariant set* $\mathcal{X}_d$ *is asymptotically stable in the large with respect to* E_i. *and exponentially stable in the large with respect to* E_B.

Note that Theorem 2 considers a general interconnection topology, which allows us to consider less restrictive agent sensing and motion abilities.

Note also that the model we introduce here may be viewed as a dual representation of the one introduced in Ref. [5], where each agent is associated with a "load function." There, the load level of an agent depends on the location of that agent and that of the targets it plans to visit. The results in Ref. [5] are an extension of the load balancing [16] theorems in Refs. [17 and 18] to the "discrete nonvirtual load" case with sensing and travel delays. Theorem 2 is an extension to the case when the "discrete virtual load" is a nonlinear function of the state with no delays.

4 Application

Suppose that we wish to design a multi-vehicle guidance strategy to enable a group of vehicles to perform surveillance of some region, the problem described in Sec. 2. As we mentioned in Sec. 2 our goal is to make the proportion of vehicles visiting a set of predefined areas match the relative importance of monitoring each area. Note that both classes of suitability functions in Fig. 1 are admissible for the theoretical development in Sec. 3 (i.e., both satisfy Eq. (1)). However, since our focus is on the relative proportioning of area monitoring and not intra-area coordination, we use the no intra-area coordination approach in the remainder of the paper. Note that conceptually similar results to those below are obtained for specific intra-area coordination strategies.

To define the perception by any vehicle about the suitability level of a particular area, we use a system identification approach to determine a parametrized model of the *expected* suitability function of that area, $\hat{s}_i(x_i)$. In particular, for the no intra-area coordination approach and, according to Fig. 1, for a fixed turn radius T, the expected suitability functions are of the form

$$\hat{s}_i(x_i) = \hat{R}_i - \hat{r}(v,\ell)x_i \tag{6}$$

for all $i \in H$, where $\hat{R}_i$ is the expected target pop-up rate for area i (targets/s) and $\hat{r}(v,\ell)$ is the expected rate of targets being attended by each vehicle moving at speed v in an area of size $\ell \times \ell$ (targets/s/vehicle). Here, we will assume that for any vehicle monitoring area i, x_i and x_j for all $j \in p(i)$ are known (i.e., the total agent capacity monitoring areas i and j are known). Note that knowing the value of x_i in Eq. (6) is a reasonable assumption, particularly when vehicles can rely on sensing information about the status of other vehicles monitoring the same and neighboring areas (e.g., via individual limited-range communication networks/radars). A vehicle's perception about the suitability level of an area will depend on how the different parameters in Eq. (6) are affected by its limited sensing and maneuvering capabilities.

First, note that for a vehicle traveling in straight-line trajectories between targets, $\hat{r}(v,\ell)$ is directly proportional to its speed and inversely proportional to the size of the area. In fact, it can be shown that such a vehicle reaches uniformly distributed targets at a rate $\hat{r}(v,\ell) \approx 1.9179 v/\ell$. As ℓ increases, a vehicle will on average need more time to reach a target, and as v increases, it will on average need less. When more complex vehicle dynamics and trajectories are taken into account (e.g., when vehicles obey a Dubins model), although the simple mathematical relationship between $\hat{r}(v,\ell)$ and v and ℓ does not generally hold, the same qualitative behavior can be expected. In particular, simulation results show that while maneuvering constraints on the vehicles (i.e., an increasing minimum turn radius) may diminish the expected rate $\hat{r}(v,\ell)$ for all vehicles in an area, the expected suitability function *shape* stays the same as in Eq. (6).

Furthermore, note that knowing the value of $\hat{R}_i$ in Eq. (6) usually requires that vehicles estimate the number of targets that have appeared in that area in a time window divided by the length of that window; thus, note that to estimate $\hat{R}_i$ vehicles may have to sense area i over a long period of time in order to obtain an accurate estimation. In particular, the error induced in assessing $\hat{R}_i$ depends largely on the length of the estimation window being used and the rate at which targets appear in area i, R_i. Here, we assume that vehicles have good sensing capabilities and use a large enough window in estimating the rate of appearance of targets (e.g., so that vehicles monitoring area i can ultimately obtain $\hat{R}_i$ and $\hat{R}_j$ for all $j \in p(i)$ within 10% of R_i and R_j, respectively). We validated via simulations that this is possible.

We define the perception by a vehicle located over area i about the suitability level of a neighboring area j as $s_j^i(x_j) = \hat{s}_j(x_j)$, and this will be used in the movement rules defined in Sec. 3.2. To consider the error being induced by a vehicle's perception about the suitability level of an area (i.e., its sensing noise), we assume that variations in $\hat{R}_j$ are independent of variations $\hat{r}(v,\ell)$. We also assume that such deviations are bounded within one standard deviation so that for some constants $\underline{w}$, $\bar{w} \geq 0$, $|s_j^i(x_j) - s_j(x_j)| = |\hat{s}_j(x_j) - s_j(x_j)| \leq \max\{\underline{w},\bar{w}\}$, and W is defined as in Sec. 3.1. As the mission progresses, vehicles decide to move from one area to another only if conditions (i)–(iv) are satisfied (i.e., when a vehicle reaches a target, it verifies the conditions and tries to move to the neighboring area with the highest suitability level).

Figure 3 shows two typical different achieved IFD realizations for 20 vehicles in a region divided into four areas and where a line topology is used. While the plots illustrate that good vehicle surveillance distributions are achieved, different IFD realizations can emerge due to the discrete nature of vehicle capabilities (compare left and right plots).

Next, using ideas from Ref. [16], we define two cooperative sensing strategies to try to reduce the effects of the sensing noise w on the mission performance. In particular, we assume that every

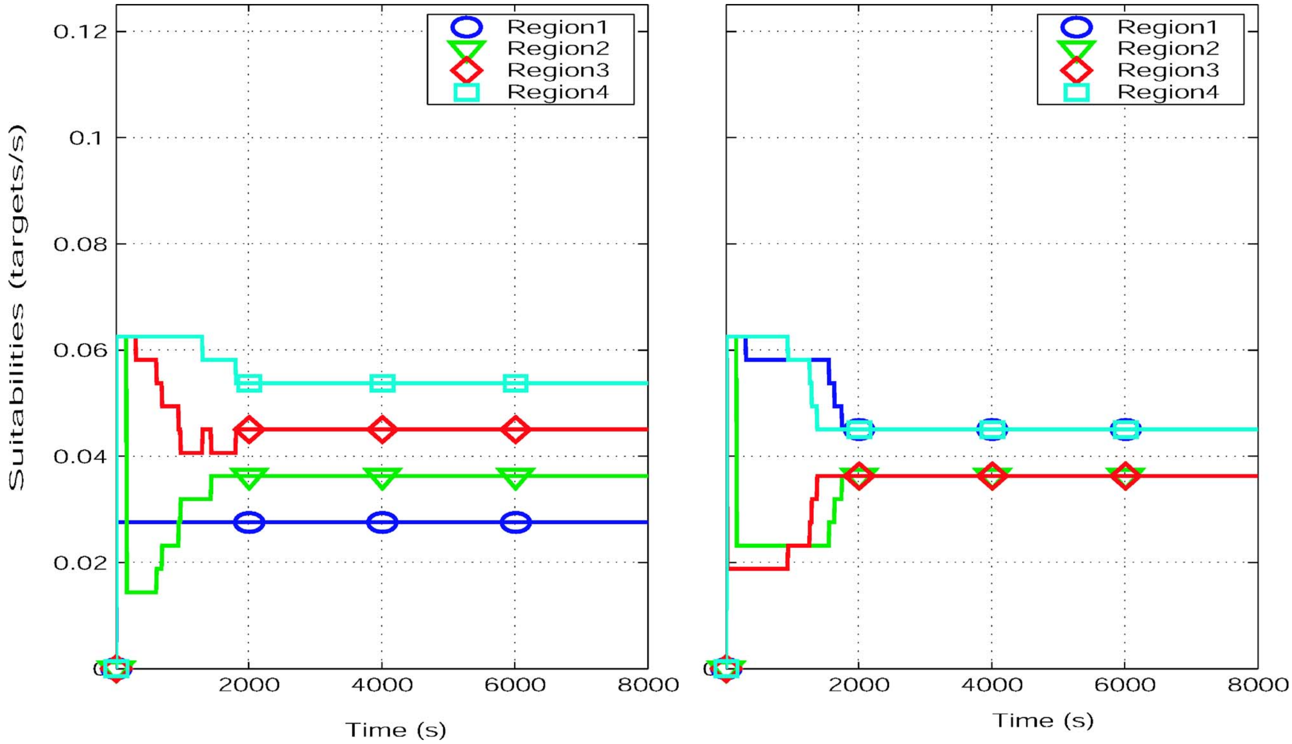

Fig. 3 Two possible IFD realizations for vehicles traveling at constant speed $v=15$ m/s and $T=800$ m deployed in a region divided into four areas with $\ell=2.5$ km and connected by a line topology

vehicle that is able to measure the suitability level of an area will cooperate with other vehicles within their area and the areas it is connected to by sharing with them its own perception about that area. We first implement a synchronous averaging strategy, where at any time k, all vehicles in an area and its connected areas may exchange their current perceptions about neighboring areas, and any vehicle evaluating conditions (i)–(iv) uses the average value of all sensing vehicles in order to define its current perception about an area. Note that such an approach generally requires a fast and synchronized communication network. Hence, if communication delays between the vehicles are too long or not all sensing vehicles update their perceptions about neighboring areas at the same time or rate, a synchronous algorithm is not desirable. Hence, we define an asynchronous agreement algorithm, where those vehicles able to measure the suitability of area i try to reach a common value by exchanging their perceptions and combining them by forming convex combinations. In other words, a vehicle's perception about a neighboring area i is influenced to different extents by all vehicles that can sense that area (e.g., depending on how outdated the received information might be). Note that vehicles may actually exchange perceptions several times until they eventually agree on a common estimate (an agreed upon value) that is guaranteed to lie between the maximum and the minimum perception of all sensing vehicles [16]. Figure 4 shows an example of the typical different IFD realizations for these two strategies and the no-coordination case (i.e., where vehicles just use their own perception to evaluate conditions (i)–(iv)). Note that the ultimate distribution has less variation when cooperative sensing is used. Finally, the Monte Carlo runs in Fig. 5 show that when the ultimate distribution has less variation, vehicles require more time to achieve it. More time is required since the distribution that can be achieved is more even.

5 Concluding Remarks

We introduce an asynchronous formulation of a multiagent system, which captures the essential characteristics of a group of agents and a predefined set of spatially distributed areas, including the interconnections between areas (via the topology of a graph) and the agents' maneuvering and sensing abilities (via the state of the nodes of a graph). In particular, we study how the differences in individual abilities affect the optimal distribution of the agents across these areas. Using a Lyapunov approach, we derive stability conditions under which an IFD realization is achieved, even when the agents' motion and sensing abilities are highly constrained. By considering the presence of sensing noise, we remove the requirements (as in Ref. [15]) that agents must perfectly assess the suitability levels of their immediate surroundings. We show that although the presence of sensing noise may increase the difference in suitability levels among neighboring nodes at the desired distribution, a global distribution pattern will still emerge in a fully connected topology, regardless of the total abilities of the agents. However, since fully connected topologies are rarely applicable, we present similar results that show that under stronger conditions on the total abilities of the agents, an IFD realization can still be achieved for a general topology under only minimal restrictions on the graph topology.

Finally, we show how the theory presented here is useful in designing cooperative control strategies for multiagent surveillance problems. Via simulations, we show how the abilities of the agents affect the achievement of the IFD. In particular, we show the impact of sensing noise on the difference in suitability levels among neighboring nodes at the IFD. Via Monte Carlo runs, we show that distributed sensing strategies offer the potential to mitigate the effects of noise but increase the time at which the IFD distribution is achieved. Taking into account how agent sensing and travel delays affect achievable distributions and their stability properties remains a future research direction, as is the implementation of the proposed strategies in a cooperative robotics experimental testbed.

Acknowledgment

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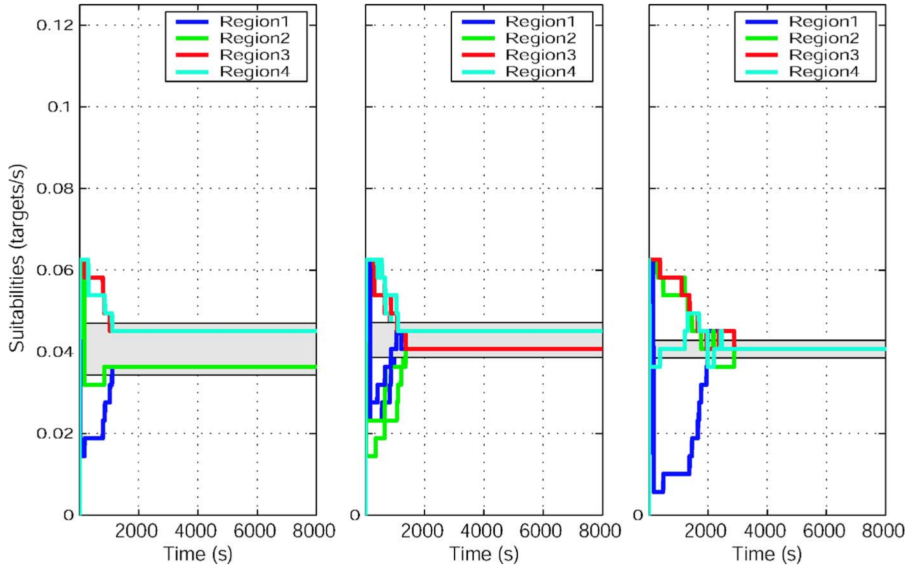

Fig. 4 Effects of implementing a synchronous and partially asynchronous iterative methods to try to reduce the effects of the sensing noise w on the mission performance with 20 vehicles at constant speed v=15 m/s and T=800 m; no cooperative sensing (left), agreement strategy (middle), averaging strategy (right)

3154). We gratefully acknowledge the help from A. Sparks and C. Schumacher at AFRL.

Appendix A: Proof of Theorem 1

Due to space constraints, we present a compact version of the proofs. For further details, the reader is directed to the authors' websites where an extended version can be found.[1]

Let $x' = [x'_1, \ldots, x'_N]$ and choose $\rho(x, \mathcal{X}_d)$ as in Eq. (4). Also,

$$V(x) = \begin{cases} \max_i\{s_i(x_i)\} - 1/N \sum_{j \in H} s_j(x_j) + 1/N \sum_{j \in U(x)} s_j(\varepsilon_c) & x \notin \mathcal{X}_d \\ 0 & x \in \mathcal{X}_d \end{cases} \tag{A1}$$

Note that the definition of $V(x)$ allows us to consider truncated nodes at the IFD. Since $U(x)$ is not known a priori, we do not explicitly know $V(x)$. However, we do not need to know its explicit form. We only need to know that it satisfies certain mathematical conditions. It can be shown that for the choices of $\rho(x, \mathcal{X}_d)$ and $V(x)$, there exist two constants $c_1 > 0$ and $c_2 > 0$ such that

$$c_1 \rho(x, \mathcal{X}_d) \leq V(x) \leq c_2 \rho(x, \mathcal{X}_d) \tag{A2}$$

(for details see the extended version of this appendix) Now, in order to show that $\mathcal{X}_d$ is asymptotically stable in the large with respect to E_i, we must show that for all $x(0) \notin \mathcal{X}_d$ and all E_k, such that $E_k E \in E_i[x(0)]$, $V[X[x(0), E_k, k]] \to 0$ as $k \to \infty$ (i.e., $V \to 0$ along all possible motions of the system). If $x(k) \notin \mathcal{X}_d$, then there must exist some node with the highest suitability among all nodes (there might actually be more than 1). In a fully connected graph (H, A), there must also exist another node i such that $(i, j^*) \in A$, $x_i(k) \neq \varepsilon_c$, and $s_{j^*}[x_{j^*}(k)] - s_i[x_i(k)] > M + W$, and node j^* has the highest suitability level among all nodes. Moreover, since $|s^{i'}_{j^*}[x_{j^*}(k)] - s_{j^*}[x_{j^*}(k)]| \leq \max\{\underline{w}, \bar{w}\}$ for all $i' \in H$ such that $j^* \in p(i')$, it applies to node i, and since $s^i_i[x_i(k)] \leq s_i[x_i(k)] + \max\{\underline{w}, \bar{w}\}$, we know that if $s^i_{j^*}[x_{j^*}(k)] \geq s_{j^*}[x_{j^*}(k)]$, then $s^i_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] \geq s_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] \geq s_{j^*}[x_{j^*}(k)] - [s_i[x_i(k)] + \max\{\underline{w}, \bar{w}\}] > M + W - \max\{\underline{w}, \bar{w}\} \geq M$ since $W = 4\max\{\underline{w}, \bar{w}\}$. If $s^i_{j^*}[x_{j^*}(k)] < s_{j^*}[x_{j^*}(k)]$, then since $s^i_{j^*}[x_{j^*}(k)] \geq s_{j^*}[x_{j^*}(k)] - \max\{\underline{w}, \bar{w}\} > 0$, and $s^i_i[x_i(k)] \leq s_i[x_i(k)] + \max\{\underline{w}, \bar{w}\}$, we know that $s^i_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] > \{s_{j^*}[x_{j^*}(k)] - \max\{\underline{w}, \bar{w}\}\} - s^i_i[x_i(k)] \geq (s_{j^*}[x_{j^*}(k)] - \{s_i[x_i(k)] + \max\{\underline{w}, \bar{w}\}\}) - \max\{\underline{w}, \bar{w}\} > (M + W) - 2\max\{\underline{w}, \bar{w}\} \geq M$ since $W = 4\max\{\underline{w}, \bar{w}\}$. Hence, it must be true that the perception of agents at node i about the suitability of node j^* is such that $s^i_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] > M$. In other words, agents at

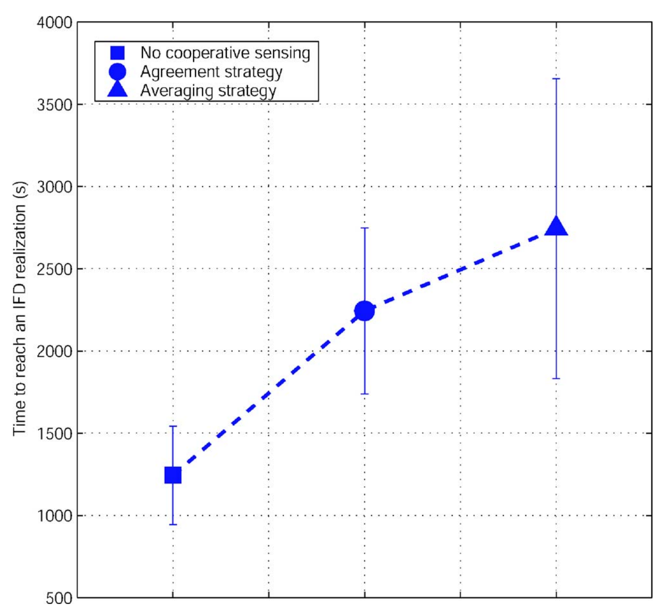

Fig. 5 Effects of implementing a synchronous and partially asynchronous iterative methods on reaching an IFD realization; no cooperative sensing (square), agreement strategy (circle), averaging strategy (triangle). Each data point represent 50 simulation runs with varying target pop-up locations. The error bars are sample standard deviations from the mean.

[1] www.ece.osu.edu/~passino/kmp-pubs.html

node i perceive the suitability of node j^* to be higher than the perception of their own suitability by more than M. However, note that they might still perceive a different node to have even higher suitability than node j^*. Hence, we consider two possible cases: either one of the highest suitability nodes j^* is perceived as being the highest neighboring node by agents at some node i with $x_i(k) \neq \varepsilon_c$ and $s^i_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] > M$ or not.

Case 1. If one of the highest suitability nodes $j^* \in \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]\}$ for all $\ell \in p(i)$ because of the restrictions imposed by E_i, we know that all the partial events are guaranteed to occur infinitely often, and according to condition (iii) on $e(k) \in g[x(k)]$, each time partial event $e^{i,p(i)}$ occurs, the suitability of node j^* is guaranteed to decrease by at least m. Moreover, according to condition (ii) on $e(k) \in g[x(k)]$, $s^i_i[x_i(k+1)] < s^i_{j^*}[x_{j^*}(k)] - W \leq s_{j^*}[x_{j^*}(k)]$. Therefore, $s_i[x_i(k+1)] \leq s^i_i[x_i(k+1)] + \max\{\underline{w}, \bar{w}\} < \{s^i_{j^*}[x_{j^*}(k)] - 4\max\{\underline{w}, \bar{w}\}\} + \max\{\underline{w}, \bar{w}\} \leq s_{j^*}[x_{j^*}(k)]$ since $s_{j^*}[x_{j^*}(k)] \geq s^i_{j^*}[x_{j^*}(k)] - \max\{\underline{w}, \bar{w}\}$. In fact, because the system is composted of a finite number of agents, each with a constant agent capacity, we know that there is a constant $\delta_1 > 0$ such that $s_i[x_i(k+1)] \leq s_{j^*}[x_{j^*}(k)] - \delta_1$. Hence, if node i would become the highest suitability node at time $k+1$, its suitability level would still have decreased by δ_1 compared to the highest suitability level at time k.

Case 2. Next, note that if $j^* \notin \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]\}$ for all $\ell \in p(i)$, that is, if node j^* is *not perceived* as being one out of possibly few nodes with the highest suitability, then the restrictions imposed by E_i, together with condition (iii) on $e(k) \in g[x(k)]$, do not guarantee that each time partial event $e^{i,p(i)}$ occurs, $\alpha_{j^*}(i,k) \geq \underline{x}$. However, there must exist a node $j' \in \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]\}$ for all $\ell \in p(i)$, which is perceived as being the one with the highest suitability. Notice that if j' is perceived by agents at node i as being the node with the highest suitability, then it must be the case that $s_{j^*}[x_{j^*}(k)] - s_{j'}[x_{j'}(k)] \leq 2\max\{\underline{w}, \bar{w}\}$. Moreover, since $j^* \in p(j')$ in a fully connected topology, then $|s^{j'}_{j^*}[x_{j^*}(k)] - s_{j^*}[x_{j^*}(k)]| \leq \max\{\underline{w}, \bar{w}\}$, so we know $s^{j'}_{j^*}[x_{j^*}(k)] - s_{j'}[x_{j'}(k)] \leq 3\max\{\underline{w}, \bar{w}\}$ and $s^{j'}_{j^*}[x_{j^*}(k)] - s^{j'}_{j'}[x_{j'}(k)] \leq W$. Notice that the restrictions imposed by E_i, together with condition (ii) on $e(k) \in g[x(k)]$, guarantee that there is no agent movement from node j' to j^*, so the suitability level of node j' cannot increase. Furthermore, condition (iii) on $e(k) \in g[x(k)]$ guarantees that each time partial event $e^{i,p(i)}$ occurs, $\alpha_{j'}(i,k) \geq \underline{x}$. Therefore, eventually, the suitability level of node j' decreases so that $s_{j^*}[x_{j^*}(k)] - s_{j'}[x_{j'}(k)] > 2\max\{\underline{w}, \bar{w}\}$ and, therefore, $s^i_{j^*}[x_{j^*}(k)] > s^i_{j'}[x_{j'}(k)]$. Note that even when node j^* is not perceived as being the highest suitability node so that condition (iii) only guarantees that $\alpha_{j'}(i,k) \geq \underline{x}$, condition (ii) on $e(k) \in g[x(k)]$ still guarantees that $s_i[x_i(k+1)] < s^i_{j'}[x_{j'}(k)] - 3\max\{\underline{w}, \bar{w}\} \leq s_{j'}[x_{j'}(k)] < s_{j^*}[x_{j^*}(k)]$.

Since there are only a finite number of nodes that may be mistakenly perceived by agents at node i as having the highest suitability level (i.e., Case 2 can only apply a finite number of times), eventually $j^* \in \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]\}$ for all $\ell \in p(i)$, and at least one of the highest suitability nodes will be perceived properly (i.e., Case 1 applies). Moreover, as long as $s_{j^*}[x_{j^*}(k)] - s_i[x_i(k)] > M + W$ for some $(i,j^*) \in A$, the agents at node i will be $s^i_{j^*}[x_{j^*}(k)] - s^i_i[x_i(k)] > M$ and, eventually, $j^* \in \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]\}$ for all $\ell \in p(i)$. Therefore, regardless of how many highest suitability nodes there are, sooner or later some agents will move one of the highest suitability nodes. Hence, it is inevitable that eventually the overall highest suitability level will decrease (i.e., Case 1 can only occur a finite number of times without the overall highest suitability level decreasing). Hence, for every $k \geq 0$, there exits $k' > k$ such that $V(k') > V(k'+1)$ as long as $x(k') \notin \mathcal{X}_d$. Hence, $\mathcal{X}_d$ is asymptotically stable in the large with respect to E_i.

We now assume that the allowed event trajectories are E_B, and we show that $\mathcal{X}_d$ is exponentially stable in the large with respect to E_B. We first specify the constant $\delta > 0$, which quantifies the decrease in the highest suitability value of any neighboring node of node i (or its own, if it would become the highest suitability node of all its neighbors). Note that for a fully connected graph (H,A), the highest suitability value of any neighboring node of node i corresponds to the highest suitability level of *all* nodes (except the node with the highest suitability itself). However, in specifying δ, we consider a general graph topology (H,A) since its definition is also used in the proof of Theorem 2.

For a given set of suitability functions and a set of agents, if $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$ and $\alpha_j(i,k) > 0$ for some $j \in p(i)$, then according to condition (iii), $a_{j^*}(i,k) \geq \underline{x}$ for some node $j^* \in \{j: s^i_j[x_j(k)] \geq s^i_\ell[x_\ell(k)]$ for all $\ell \in p(i)\}$. Therefore, according to the definition of m, $s_{j^*}[x_{j^*}(k+1)] \leq s_{j^*}[x_{j^*}(k)] - m$ if no agent would leave node j^* at time k (note that if j^* is the highest suitability node in the graph, then according to condition (ii) $\Sigma_{\ell \in p(j^*)} \alpha_\ell(j^*,k) = 0$ at time k). Therefore, considering m in the definition of δ takes into account the case when the highest suitability node of all neighboring nodes of node i may remain the same, but its suitability level $s_{j^*}[x_{j^*}(k)]$ decreases in value.

Next, there also exists a constant $\delta_1 > 0$, such that if $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$ and $\alpha_j(i,k) > 0$ for some $j \in p(i)$, then $s_i[x_i(k+1)] \leq s_{j^*}[x_{j^*}(k)] - \delta_1$, where $j^* \in p(i)$ and $s^i_{j^*}[x_{j^*}(k)] \geq s^i_j[x_j(k)]$ for all $j \in p(i)$. In other words, if agents at node i decide to move to some node j, the suitability at node i at time $k+1$ is less than the highest perception of the suitability levels of all neighboring nodes of node i at time k by at least δ_1. In particular, note that according to the definition of $f_{e(k)}[x(k)]$, we know that whether agents arrive at node i at time $k+1$ or not $s_i[x_i(k+1)] \leq s_i[x_i(k) - \Sigma_{\{\ell \in p(i), e^{i,p(i)}_{\alpha(i,k)} \in e(k)\}} \alpha_\ell(i,k)]$ since s_i is a strictly monotonically decreasing function and $x_i(k+1) \geq x_i(k) - \Sigma_{\{\ell \in p(i), e^{i,p(i)}_{\alpha(i,k)} \in e(k)\}} \alpha_\ell(i,k)$. Moreover, according to condition (ii), we also know that for all $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$, $s^i_i[x_i(k) - \Sigma_{\ell \in p(i)} \alpha_\ell(i,k)] < \max_j\{s_j[x_j(k)]: j \in p(i)\} - W$. Hence, if $j^* \in \max_j\{s^i_j[x_j(k)]: j \in p(i)\}$ and $\alpha_{j^*}(i,k) \geq \underline{x}$, then since $W = 4\max\{\underline{w}, \bar{w}\}$ and $s_{j^*}[x_{j^*}(k)] \geq s^i_{j^*}[x_{j^*}(k)] - \max\{\underline{w}, \bar{w}\}$, $s_i[x_i(k+1)] \leq s^i_i[x_i(k+1)] + \max\{\underline{w}, \bar{w}\} < s^i_{j^*}[x_{j^*}(k)] - W + \max\{\underline{w}, \bar{w}\} \leq s_{j^*}[x_{j^*}(k)]$ so that $\delta_1 > 0$ for any agent movement. Note that defining δ_1 considers the case when agents moving away from node i may cause its suitability to become the highest suitability of all its neighboring nodes.

Finally, there is also a constant $\delta_2 > 0$, such that if $(i,j) \in A$ and $s_i[x_i(k)] \neq s_j[x_j(k)]$, then $|s_i[x_i(k)] - s_j[x_j(k)]| \geq \delta_2$ for all $k \geq 0$. The value of δ_2 depends on the particular set of suitability functions and a given set of agents. Defining δ_2 takes into account the case when agents moving from node i to the neighboring node among the highest suitability (together with other possible agent migrations from other nodes) cause another node that did not necessarily receive any agents to become the highest suitability node of node i at time $k+1$.

Let $\delta = \min\{m, \delta_1, \delta_2\}$. Next, fix a time $k \geq 0$. If $x(k) \notin \mathcal{X}_d$, we know there must exist at least one pair of nodes $i,j \in H$ such that $(i,j) \in A$, $s_j[x_j(k)] - s_i[x_i(k)] > M + W$, and $x_i(k) \neq \varepsilon_c$. Let $H^*_i(k) = \{j: s_j[x_j(k)] \geq s_{j'}[x_{j'}(k)], j, j' \in p(i)\} \subset H$ be the set of neighboring nodes of node i with the highest suitability levels. Since $s_j[x_j(k)] - s_i[x_i(k)] > M + W$, note that for any $j' \in H^*_i(k)$, $s_{j'}[x_{j'}(k)] - s_i[x_i(k)] > M + W$, so that $s^i_{j'}[x_{j'}(k)] - s^i_i[x_i(k)] > M$. Hence, according to the restrictions imposed by E_B, there is some time k_1, $k \leq k_1 < k + B$ such that $e^{i,p(i)}_{\alpha(i,k_1)} \in e(k_1)$. In other words, agents at the node with lower suitability, but not the least agent capacity ε_c, must try to move to other nodes at least once every B time steps. Note also that there are only a finite number of nodes that may *mistakenly* be perceived as having the highest suitability level and that condition (ii) on those nodes guarantees that no agents may move away from such nodes since its actual suitability level must differ from the highest suitability level of all neighboring nodes of node i by less than $2\max\{\underline{w},\overline{w}\}$ (in fact, agents do not leave any node whose perception of its own suitability level does not drop below the highest neighboring suitability perception by at least W). Hence, if we let $\lceil\cdot\rceil$ denote the ceiling function (which gives the smallest integer greater than or equal to its argument) and agents move to but do not leave any particular node mistakenly perceived as having the highest suitability level, then after $B\max\{\lceil(2/m)\max\{\underline{w},\overline{w}\}\rceil,1\}$ time steps we can guarantee that it will no longer be perceived as being the highest suitability node by any agents at node i, unless it has actually become 1. Hence, for a fully connected topology, conditions (ii) and (iii) on $e(k)\in g[x(k)]$, along with the definition of δ, imply that there exist a time k_2, $k\leq k_2 < B\max\{N\lceil W/2m\rceil,1\}$, such that (a) $|H_i^*(k_2+1)| \leq |H_i^*(k_2)|-1$ and $s_q[x_q(k_2+1)]=s_j[x_j(k_2)]$ for all $q\in H_i^*(k_2+1)$ and all $j\in H_i^*(k_2)$ or (b) $s_q[x_q(k_2+1)]\leq s_j[x_j(k_2)]-\delta$ for all $q\in H_i^*(k_2+1)$ and all $j\in H_i^*(k_2)$ since condition (ii) guarantees that all neighboring nodes that are properly perceived as having the highest suitability level do not increase in suitability value. In other words, after $B\max\{N\lceil W/2m\rceil,1\}$ time steps, either the number of best neighboring nodes decreases by 1 (e.g., the number of nodes $j\in H_i^*(k)$ with the highest suitability) or the suitability level of the best neighboring node decreases by at least δ. Since there might be at most $N-1$ nodes with the highest suitability, and for a fully connected topology the suitability level of the best neighboring node of node i corresponds to the highest suitability in the entire graph, we conclude that as long as $x\notin \mathcal{X}_d$, the highest suitability in the entire graph decreases by at least δ every $NB\max\{N\lceil W/2m\rceil,1\}$ steps, so we obtain that the highest suitability in the entire graph is guaranteed to decrease according to $\max_i\{s_i[x_i(k)]\}-\max_i\{s_i[x_i(k+NB\max\{N\lceil W/2m\rceil,1\})]\}\geq\delta$.

Finally, note that according to Eq. (A1), $V[x(k)]$ equals the difference between the maximum suitability level among all nodes and the average suitability level of all nodes with more than the minimum agent capacity. Therefore, since the maximum suitability level among all nodes overbounds $V[x(k)]$, guaranteeing that the maximum suitability level decreases at least by $\delta>0$ together with the bounds on $V[x(k)]$ satisfies sufficient conditions for exponential stability to the invariant set $\mathcal{X}_d$ [17].

Appendix B: Proof of Theorem 2

In the proof of Theorem 2, we do no longer assume a fully connected topology of the graph (H,A). Instead, we assume a general graph topology, and we only require that every node is connected to some other node in the graph. Notice that the proof of Theorem 1 made no assumption on the total agent capacity C, so that some of the previous results will be used in this section. We focus on how topological characteristics of the region of interest affect the rate of convergence to the desired distribution. In particular, we show that $\mathcal{X}_d$ is still exponentially stable with respect to E_B, even when we do not assume a fully connected topology.

It can be shown that for a large enough agent capacity C, for $\mathcal{X}_d$ to be invariant, it requires that $U(x)=\emptyset$ for all $x\in\mathcal{X}_d$. Moreover if we choose $\rho(x,\mathcal{X}_d)$ as defined in Eq. (4) and let

$$V(x)=\begin{cases}\max_i\{s_i(x_i)\}-(1/N)\sum_{j\in H}s_j(x_j) & x\notin\mathcal{X}_d\\ 0 & x\in\mathcal{X}_d\end{cases}$$

it can also be shown that the bounds in Eq. (A2) still hold for this particular definition of $V(x)$ (again, for details see the extended version of this appendix). For similar reasons as in the proof of Theorem 1, we can also establish asymptotic stability of $\mathcal{X}_d$ with respect to E_i. In particular, as long as $x\notin\mathcal{X}_d$, we know that there must exist a pair of nodes $(i,j)\in A$, where j has the highest suitability level of all neighboring nodes of node i and $x_i\neq\varepsilon_c$, such that $s_j[x_j(k)]-s_i[x_i(k)]>M+W$ and hence $s_j^i[x_j(k)]-s_i^i[x_i(k)]>M$. Note that for a general graph (H,A), at time k, node $j\in H_i^*(k)$ may *not* be the node with the highest *overall* suitability level in the entire graph. However, as in the proof of Theorem 1, because of the restrictions imposed by E_i, we know that all the partial events are guaranteed to occur infinitely often, and according to condition (iii) on $e(k)\in g[x(k)]$, if no agents move away from node j (for example, if node j is actually the highest suitability node in the entire graph), its suitability level is guaranteed to eventually decrease by at least m (since Case 2 in the proof of Theorem 1 can only occur a finite number of times and, eventually, node j must be perceived as having the highest suitability level of all neighboring nodes of node i, so that $\alpha_j(i,k)\geq\underline{x}$). Note that if there exists another node j', such that $j'\in p(j)$, $j'\notin p(i)$, and $s_{j'}^j[x_{j'}(k)]-s_j^j[x_j(k)]>M$, then the suitability level of the neighboring node $j\in p(i)$ may actually increase, given that agents from node j may move to j' as long as conditions (i–iv) on node j are satisfied. In particular, condition (ii) on node j guarantees that even if agents actually move away from node j, its suitability level is strictly less than the highest suitability level of all *its* neighboring nodes. Since every node is connected to some other node in the graph, there must exist a path from node i to any of the possibly few highest suitability nodes in the entire graph. Condition (ii) on an overall highest suitability node j^* guarantees that agents do not move away from that node because even if agents located at that node perceive another neighboring node as having a higher suitability level (due to the sensing noise w), the difference in suitability perceptions is less than W. Hence, condition (ii) on any of the highest suitability nodes in the graph (which ensures that no agents move away from such a node at time k) and condition (ii) on any of its neighboring nodes (which ensures that the suitability level of such nodes at time $k+1$ remains strictly less than the maximum suitability at time k) guarantee that $\max\{s_i[x_i(k)]\}$ is nonincreasing. Therefore, since the overall suitability level is nonincreasing, the suitability level of node j remains strictly below the highest suitability level of the entire graph. In particular, if agents move away from node j, eventually, either node j becomes the highest suitability node of the entire graph or agents stop moving away from node j (because its suitability level differs by at most M from all its neighboring nodes). Either way, the suitability level of node j eventually decreases.

Finally, since the highest neighboring suitability level eventually decreases in value, and there are a finite number of highest neighboring nodes, then, eventually, either the highest suitability node of the entire graph decreases or the maximum path length between a pair of nodes $(i,j)\in A$ such that $s_j^i[x_j(k)]-s_i^i[x_i(k)]>M$ and $x_i\neq\varepsilon_c$, to a maximum suitability node in the entire graph, decreases. Since there can only be a finite number of overall highest suitability nodes, and there are at most $(N-1)$ links between any two nodes, eventually, the overall highest suitability level must decrease. Hence, for every $k\geq 0$, there exists $k'>k$ such that $V(k')>V(k'+1)$ as long as $x(k')\notin\mathcal{X}_d$. Hence, $\mathcal{X}_d$ is asymptotically stable in the large with respect to E_i.

To establish exponential stability of $\mathcal{X}_d$ with respect to E_B for Theorem 2, we take into account how the topology of a general graph (H,A) affects the rate of convergence to the desired state.

Again, as long as $x \notin \mathcal{X}_d$, we know that there must exist a pair of nodes $(i,j) \in A$, where j has the highest suitability level of all neighboring nodes of node i and $x_i \neq \varepsilon_c$, such that $s_j^i[x_j(k)] - s_i^i[x_i(k)] > M$. Note that there might be several such pairs, and let us refer to node i as the closest one to some highest suitability in the *entire* graph. Fix a time k and let $H^*(k) \subset H$ to be the set of nodes such that $H^*(k) = \{i : s_i[x_i(k)] \geq s_j[x_j(k)], j \in H\}$. In other words, the set $H^*(k)$ holds all nodes with the highest suitability in the entire graph. Let $L(k)$ represent the maximum number of links between node i and any node $j^* \in H^*(k)$.

For $x \notin \mathcal{X}_d$, if there exists some node j' such that $(j', j^*) \in A$, $x_{j'} \neq \varepsilon_c$, and $s_{j^*}^{j'}(x_{j^*}) - s_{j'}^{j'}(x_{j'}) > M$ for all $j^* \in H^*(k)$ (e.g., in a fully connected topology considered in Theorem 1), then according to the restrictions imposed by E_B, agents try to move to neighboring nodes at least once every B time steps, and conditions (ii) and (iii) on $e(k) \in g[x(k)]$ guarantee that after $NB \max\{N\lceil W/2m \rceil, 1\}$ time steps the maximum suitability in the entire graph decreases by at least δ. Note that node j' must not be unique for all $j^* \in H^*(k)$. However, for a general graph topology, it could be that some nodes connecting to any of the highest suitability nodes $j^* \in H^*(k)$ have local suitability perceptions that differ by at most M from their perception of $s_{j^*}[x_{j^*}(k)]$, so that the restrictions imposed by E_B together with condition (iii) do not guarantee that at least some agents located at neighboring nodes of $j^* \in H^*(k)$ move to the highest suitability node in the entire graph. Consequently, the maximum suitability in the entire graph is not guaranteed to decrease after $NB \max\{N\lceil W/2m \rceil, 1\}$ steps (i.e., since there is no direct neighbor where agents are guaranteed to come from). The restrictions on E_B along with conditions (ii) and (iii) only guarantee that all best *neighboring* nodes of node i decrease by at least δ after $NB \max\{N\lceil W/2m \rceil, 1\}$ steps, provided that there is no agent migration from neighboring nodes of node i to other nodes. Moreover, note that even if agents move away from node j, its suitability $s_j[x_j(k)]$ cannot increase by more than NM since node i is the closest node to $j^* \in H^*(k)$, which differs in suitability level by more than $M+W$ from its neighboring node. However, after $\lceil (2NM/\delta)NB \max\{N\lceil W/2m \rceil, 1\} \rceil$ we know that all neighboring nodes of node i have to decrease its suitability by more than $2M$ and/or there must exist a pair of nodes (i', j') such that $(i', j') \in A$ and $s_{j'}^{i'}[x_{j'}(k')] - s_{i'}[x_{i'}^{i'}(k')] > M$ such that the maximum path length $L(k') \leq L(k) - 1$ for $k' \geq k + \lceil 2(M/\delta)N^2 B \max\{N\lceil W/2m \rceil, 1\} \rceil$.

Since $L(k) \leq N$ (the maximum span between any two nodes) we know that after $N\lceil 2(M/m)NB \max\{N\lceil W/2m \rceil, 1\} \rceil$ steps, either (a) $|H^*(k+1)| \leq |H^*(k)| - 1$ and $s_q[x_q(k+1)] = s_j[x_j(k)]$ for all $q \in H^*(k+1)$ and all $j \in H^*(k)$ or (b) $s_q[x_q(k+1)] \leq s_j[x_j(k)] - \delta$ for all $q \in H^*(k+1)$ and all $j \in H^*(k)$. In other words, either the number of best suitability nodes in the entire graph decreases by at least 1, or the suitability of the best node decreases by at least δ.

Again, because $|H^*(k)| \leq N-1$, we can conclude that for $x(k) \notin {}'\mathcal{X}_d$, $\max_i\{s_i[x_i(k)]\} - \max_i\{s_i[x_i(k + N^3 \lceil 2(M/\delta)NB \max \times \{N\lceil W/2m \rceil, 1\} \rceil)]\} \geq \delta$ which again, together with the bounds on $V[x(k)]$, satisfies sufficient conditions for exponential stability to the invariant set $\mathcal{X}_d$ [17].

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