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Brief paper

Optimal allocation of heterogeneous resources in cooperative control scenarios ☆

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ABSTRACT

This paper introduces a mathematical framework for the study of resource allocation problems involving the deployment of heterogeneous agents to different teams. In this context, the term heterogeneous is used to describe classes of agents that differ in the basic functions they can perform (e.g., one type of agent searches for targets while another type engages those targets). The problem is addressed in terms of optimization using concepts from economic theory and the proposed algorithm was designed to ensure asymptotic stability of the optimal solution.

1. Introduction

The field of cooperative control has the lofty aim of enabling a group of autonomous agents to either accomplish an objective in a more efficient manner than they could as individuals or do complex tasks that a single agent could never perform alone. One of the main goals of cooperative control is to establish organizational frameworks in which the capabilities of these groups are more than just those of their component agents, and there are a number of proposed applications for this type of research such as autonomous military robots and flexible manufacturing systems (Beard & McClain, 2003; Frazzoli & Bullo, 2004; King, Kuwata, Alighanbari, Bertuccelli, & How, 2005; Lum, Rysdyk, & Pongpunwattana, 2006; Moore & Passino, 2008; Scheutz, Schermerhorn, & Bauer, 2005). Nonetheless, all of these applications involve the key ingredients of either distributed action and/or distributed decision making and make use of the principle of *comparative advantage*. The concept of comparative advantage comes from the field of economics (Ricardo, 1996, originally published in 1817) and is essentially an observation of the fact that since people possess different natural talents it is to their advantage to specialize in the tasks they perform well and trade their services for those of others instead of attempting to meet all their needs themselves. By doing so, a society can produce much more than they would be able to without cooperating in this way. Just as human society consists of diverse individuals, many envisioned systems from the cooperative control field incorporate different types of agents and so they should be able to exploit those differences in order to improve the performance of the group. This, of course, depends on the development of proper algorithms for interaction between the agents, much in the same way that tools such as money and legally enforceable contracts provide a reliable basis for human economic interaction.

To date, many cooperative control problems have focused on groups of *homogeneous* agents (i.e., each agent has the same physical manifestation and/or computing ability) (Beard & McClain, 2003; Frazzoli & Bullo, 2004; Moore & Passino, 2008). In these setups the differences between the agents are entirely related to their current state (most commonly just their position in the environment) and thus an agent's comparative advantage for certain tasks may vary over time as its state changes. For instance, in a group of mobile robot agents the speed or ability of an agent to accomplish a certain task will usually depend on its distance to that task (relative to the other agents) and a common problem is to decide how the agents should be assigned to various tasks in order to optimize some performance measure (e.g., minimize total mission time, maximize rate of task completion, etc.). Other, more sophisticated, cooperative control scenarios will involve a group of *heterogeneous* agents in which the dynamics or capabilities of the individual agents may differ (King et al., 2005; Lum et al., 2006; Scheutz et al., 2005). Teams of military robots, for example, may consist of both ground and air vehicles and these may differ not only in their ability to maneuver through their environment but also in the functions they are able to

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perform. To date, it appears that most attempts to tackle problems involving heterogeneous agents have focused primarily on using combinatorial optimization techniques in order to generate agent-task assignments (Alighanbari, Kuwata, & How, 2003; King et al., 2005). While a valid approach for many problems, this sort of method suffers from the need for excessive computational power as the size of the problem increases.

An area of research involving heterogeneous agents that has received less attention than those already mentioned views the individual agents as *resources* which must be properly allocated to different tasks in order to maximize their effectiveness. In a control hierarchy this is the level above the one concerned with the actual execution of particular tasks (i.e., the level previously discussed) and is not so much about what to do with a certain group of agents as it is about creating these groups in the first place. This *team formation* problem has been addressed by some authors, but to make the problem mathematically tractable they are usually forced to resort to the simplifying assumption that each agent can be represented by a single relative *value* (Finke & Passino, 2007). For example, an agent that travels twice as fast or can accomplish tasks at twice the rate as another agent would have two times as much value. Alternatively, there may be a certain *benefit* associated with an agent-team assignment, but again for simplicity the fact that this benefit may depend on which other agents are assigned to that team is mostly ignored so that various algorithms for the standard assignment problem (Bertsekas & Tsitsiklis, 1997a) can be applied. So, while these simplifying assumptions may make sense for some problems, they fail to adequately address many important scenarios. Consider, for example, a *hunter-killer* scenario in which two types of military robots must work together to identify and destroy enemy targets. In this scenario the hunter robot would be a lighter, faster vehicle in charge of finding and confirming targets which a more heavily armed but less nimble killer robot would then attack. If the battlefield is large it will most likely be organized into smaller sectors and the goal would be to determine the proper assignment of hunter and killer vehicles for each sector (given a fixed number of available vehicles of each type). This problem can become very complex because of the coupling between all the factors involved. The correct number of hunter vehicles for a sector will depend on the rate potential targets appear, what fraction of those are actual enemy targets, and how many killer vehicles are in the sector to handle the identified targets. Conversely, the correct number of killer vehicles depends on how fast enemy targets are being identified which depends in turn on the rate potential targets appear and how many hunter vehicles are in the sector to investigate those targets.

These types of team formation problems have also been addressed in the field of behavioral ecology where, from an evolutionary perspective, the division of labor and the emergence of teams in an animal society may be viewed as the outcome of the allocation process of its working force (e.g., in honey bees where the hive must allocate its foraging workforce by determining the proportion of explorers, employed foragers, and resting bees (Anderson & Franks, 2001)). Thus, the team formation problem may be formulated as an optimization process, where each animal adopts a strategy (e.g., a type of task) that optimizes its fitness in the sense that a unilateral deviation from this strategy would result in fitness degradation, thereby relating to a notion called the ideal free distribution (IFD). The IFD characterizes an equilibrium distribution where all animals achieve equal fitness, and no animal can increase its fitness by unilateral deviation from one strategy to another. In particular, the IFD is optimal in the sense that it is a Nash equilibrium and an evolutionarily stable strategy (ESS) (Cressmann & Křivan, 2006).

In the past decades many models based on the IFD concept have been developed, trying to explain how different animal groups behave in different environments. In social foraging by honey bees, for instance, the hive achieves an IFD-like distribution with the allocation of foragers being approximately proportional to nectar source relative profitability despite the fact that each bee acts only on local knowledge of the available nectar sources (Seeley, 1995). Similar models which characterize the dynamic allocation of the labor force and how honey bees "organize" themselves through simple local rules have been introduced and validated (see Passino and Seeley (2008) and references therein). Understanding how optimal distribution patterns can be achieved by a group driven only by local rules allows us to overcome excessive computational power requirements which inhibit combinatorial optimization techniques to solve large-scale problems. Here, we develop a similar bio-inspired approach in that we define specific local rules which guarantee that the group as a whole achieves an optimal distribution. We will use a generic terminology for IFD concepts, one that is appropriate for biology and engineering. In what follows, habitats, food sources, resource sites, areas, etc. are referred to as tasks and the term resource is associated with entities capable of physical motion such as animals, vehicles, robots, or aircraft. Our framework can be viewed as a generalized IFD model which allows us to study not only the distribution of a set of resources over a given set of tasks, but also takes into account that multiple *types* of resources may influence every task differently.

2. Problem statement

2.1. Basic problem

This section uses the terms *tasks* and *resources*. Resources are the individual agents (which are of varying *type*) and tasks can be any well defined purpose to which a team of agents can be assigned. The tasks are numbered from 1 to n and the different resource types are numbered from 1 to m. This paper assumes that the number of resource units of each type is large enough to approximate the amount of a specific resource applied to a specific task as a continuous variable (as in Bertsekas and Tsitsiklis (1997b) and Burgess and Passino (1998)). Let $\mathcal{R} = [0, \infty)^m$ be the space of all possible resource combinations that could be applied to a task and let $r_i = [r_{i1}, \ldots, r_{im}]^\top \in \mathcal{R}$ be the *resource vector* applied to task i (where r_{ij} is the amount of a resource type j applied to task i). Let $\Delta_c \subset \mathcal{R}^n$ denote the $m(n-1)$ dimensional simplex defined by the equality constraint $\sum_{i=1}^n r_i = c$, where c is a vector $[c_1, \ldots, c_m]^\top \in \mathcal{R}$ and $c_j \geq 0$ denotes the total available amount of resource j. Let the performance of the application of resource allocation r_i to task i be given by the *utility function* $f_i : \mathcal{R} \to [0, \infty)$ and let the total utility function $f : \mathcal{R}^n \to [0, \infty)$ be equal to $\sum_{i=1}^n f_i$. Letting $r = [r_1^\top, \ldots, r_n^\top]^\top$, the objective is to maximize $f(r)$ subject to $r \in \Delta_c$.

2.2. Conditions on utility functions

Based on some reasonable assumptions common in economic theory, this paper will require each utility function f_i to satisfy three conditions. First, adding resources to a task must always increase the produced utility (i.e., for any $x, h \in \mathcal{R}$, $f_i(x+h) > f_i(x)$). Second, each f_i must be continuously differentiable on all of $\mathcal{R}$. Finally, each f_i satisfies a condition of *decreasing average returns with respect to increasing magnitudes of resource additions or exchanges*. This condition is a generalized version of the law of diminishing returns (Färe, 1980; Menger, 1979) and in words states that adding or exchanging resources in any fixed proportion results in a lower average utility increase as the magnitude of the resource change increases. Mathematically,

$$\frac{f_i(r_i + ah) - f_i(r_i)}{a} > \frac{f_i(r_i + bh) - f_i(r_i)}{b}, \quad \forall r_i \in \mathcal{R},$$
$$\forall h \in \mathbb{R}^m, \forall a, b \in \mathbb{R} : b > a > 0 \text{ and } r_i + ah, r_i + bh \in \mathcal{R}. \qquad (1)$$

To look at this requirement another way, use the algebraic substitutions $r = x$, $h = \frac{y-x}{b}$, and $\theta = 1 - \frac{a}{b}$, and rewrite (1) as $f_i(\theta x + (1-\theta)y) > \theta f_i(x) + (1-\theta)f_i(y), \forall\, x, y \in \mathcal{R}, \forall\, \theta \in (0, 1)$ to see that (1) is equivalent to saying each function f_i is strictly concave on $\mathcal{R}$. This is a useful property because it means that the total utility function is also strictly concave and thus must have a uniquely defined maximum on Δ_c.

2.3. Optimality conditions

For convenience, let $s_{ij}(r_i)$ be equal to the partial derivative of f_i with respect to resource j evaluated at the resource vector r_i. In economics, s_{ij} is referred to as the *marginal utility function* of task i with respect to resource j (the notation s_{ij} is used here to emphasize the connection to the *suitability* functions of Finke and Passino (2007)). Because of the conditions placed on the functions f_i, the functions s_{ij} are continuous on $\mathcal{R}$, strictly decreasing, and non-negative (so s_{ij} is bounded on $\mathcal{R}$ as well). To make the later analysis tractable, this paper assumes that each function s_{ij} is Lipschitz continuous on $\mathcal{R}$ (thus imposing an additional constraint on f_i). Since f is strictly concave it follows from Bertsekas (1995) that the optimal point of f given resource constraint c, denoted as $r^\star$, is the only point belonging to the set

$$\Delta_c^\star = \{r \in \Delta_c \mid \forall\, i, k \in \{1, \dots, n\}, \forall\, j \in \{1, \dots, m\}, \quad s_{ij}(r_i) < s_{kj}(r_k) \Rightarrow r_{ij} = 0\}. \tag{2}$$

In words this means that when $r = r^\star$ it must be the case that every task with a positive amount of a resource j will have the same marginal utility with respect to that resource and that this marginal utility must be at least as great as that for any task completely without resource j.

3. Discrete event system algorithm

In this section we present an algorithm to solve the problem of Section 2 based on a discrete event system (DES) framework. In this algorithm resources are transferred in discrete quantities and at discrete time instants. The algorithm is analyzed using the modeling methodology and the stability theories presented in Burgess and Passino (1998), albeit in abbreviated form. Resource transfers will happen at discrete points in time $t = 0, 1, 2, \dots$ and the algorithm is defined by the following two rules governing those transfers:

(1) Only one transfer of one particular resource will occur at each time step (a transfer of one resource type j from one source task i to one destination task k). For convenience let $\alpha(t)$ be the triplet (i, k, j) for the transfer occurring at time step t. For this work, $\alpha(t) = (i, k, j)$ must satisfy the following conditions at each time t:

$$s_{kj}(r_{kj}(t)) - s_{ij}(r_{ij}(t)) \geq s_{k'j'}(r_{k'j'}(t)) - s_{i'j'}(r_{i'j'}(t)),$$

for all $i', k' \in \{1, \dots, n\}$ and all $j' \in \{1, \dots, m\}$. (3)

Meaning the largest marginal utility difference in the system is addressed at each time step.

(2) Let $u_j(t) \geq 0$ denote the amount of resource j passed from task i to task k at time t, and let $u(t)$ be a vector in $\mathcal{R}$ with $u_j(t)$ as the jth entry and all other entries equal to zero. The size of $u_j(t)$ must satisfy

$$s_{ij}(r_i(t) - u(t)) \leq s_{kj}(r_k(t) + u(t)) \tag{4}$$

and one of the two following conditions:

$$s_{kj}(r_k(t) + u(t)) - s_{ij}(r_i(t) - u(t)) \leq \gamma_{ikj}(s_{kj}(r_k(t)) - s_{ij}(r_i(t)))$$

for some fixed $\gamma_{ikj} \in [0, 1)$ (5)

$$u_j(t) = r_{ij}(t). \tag{6}$$

Meaning that the marginal utility difference between task i and k w.r.t. resource type j cannot reverse sign and either that difference must be reduced by at least a certain proportion or task i must give all of its resource type j to task k.

Theorem 1. *The point $r^\star$ is an equilibrium of the DES model and has a region of asymptotic stability equal to Δ_c.*

Proof. To prove stability of this algorithm we will use the candidate Lyapunov function $V(r) = f(r^\star) - f(r) = \sum_{i=1}^n \left(f_i(r_i^\star) - f_i(r_i)\right)$. By definition $V(r)$ is strictly convex so it must be that $V(r) = 0$ if and only if $r = r^\star$ and so Theorem 7.12 of Miller and Michel (1982) guarantees the existence of a class $\mathcal{K}$ function of $r^\top r$ that bounds $V(r)$ from below. Also, since each f_i has continuous and bounded partial derivatives it must be that $V(r)$ is Lipschitz continuous, so there must exist some constant L allowing us to choose $Lr^\top r$ as an upper bound on $V(r)$. Thus $V(r)$ is positive definite and decrescent on Δ_c. Having confirmed the validity of $V(r)$ as a Lyapunov function, now fix t and analyze the difference in $V(r)$ between time steps letting $(i, k, j) = \alpha(t)$. Define $\Delta V(r(t))$ as $V(r(t+1)) - V(r(t)) = f_i(r_i(t)) - f_i(r_i(t+1)) + f_k(r_k(t)) - f_k(r_k(t+1))$. Since all marginal utility functions are Lipschitz continuous there exists a constant L such that $|s_{ij}(x) - s_{ij}(y)| \leq L|x-y|$. Because only one resource type is being exchanged per time step and because the individual utility functions are strictly increasing w.r.t. their arguments, simple geometric arguments result in bounds on the two basic terms of $\Delta V(r(t))$ as follows,

$$f_i(r_i(t)) - f_i(r_i(t+1)) = \int_{r_i(t)-u(t)}^{r_i(t)} s_{ij}(\sigma)\mathrm{d}\sigma < u_j(t)s_{ij}(r_i(t+1)) - \frac{[s_{ij}(r_i(t+1)) - s_{ij}(r_i(t))]^2}{2L} \tag{7}$$

$$f_k(r_k(t+1)) - f_k(r_k(t)) = \int_{r_k(t)}^{r_k(t)+u(t)} s_{kj}(\sigma)\mathrm{d}\sigma > u_j(t)s_{kj}(r_k(t+1)) + \frac{[s_{kj}(r_k(t)) - s_{kj}(r_k(t+1))]^2}{2L} \tag{8}$$

which means $\Delta V(r(t))$ can be bounded from above by a function of $r(t)$ and $r(t+1)$,

$$\Delta V(r(t)) < -\left(\frac{[s_{ij}(r_i(t+1)) - s_{ij}(r_i(t))]^2 + [s_{kj}(r_k(t)) - s_{kj}(r_k(t+1))]^2}{2L} + u_j(t)[s_{kj}(r_k(t+1)) - s_{ij}(r_i(t+1))]\right). \tag{9}$$

There are two cases to consider, (a) $u_j(t) = r_{ij}(t)$ or (b) $u_j(t) < r_{ij}(t)$. In the first case it is not guaranteed to have any particular decrease in $s_{kj} - s_{ij}$ between time step t and $t+1$. In the second case, the second term of (9) can be ignored because $s_{kj}(r_k(t+1)) - s_{ij}(r_i(t+1))$ could be equal to zero, so it is the case that

$$\Delta V(r(t)) < -\frac{[s_{ij}(r_i(t+1)) - s_{ij}(r_i(t))]^2 + [s_{kj}(r_k(t)) - s_{kj}(r_k(t+1))]^2}{2L} \tag{10}$$

Using the inequality $a^2 + b^2 \geq \frac{1}{2}(a+b)^2$, condition (5), and defining $\gamma = \max_{i,k,j} \gamma_{ikj}$, manipulation of (10) yields

$$\Delta V(r(t)) < -\frac{(1-\gamma)^2}{4L}[s_{kj}(r_k(t)) - s_{ij}(r_i(t))]^2. \tag{11}$$

In order to address the stability of the system, note that the inequality (9) shows that $V(r(t))$ is a non-increasing function w.r.t.

time and since V is bounded from below (by zero) there must exist a scalar $q \geq 0$ such that $V(r(t)) \to q$ as $t \to \infty$. If $q = 0$ then because V is continuous and zero only at the global optimum it must be that $r(t)$ converges to $r^\star$ and the theorem holds. Assuming $q > 0$, then because V is continuous, $r(t)$ converges to an ω-limit set $\Omega(r(t))$ which is a subset of the level set $S_q = \{x \in \Delta_c : V(x) = q\}$. Take any point $\bar{r}$ in $\Omega(r(t))$ (i.e., so that $V(\bar{r}) = q > 0$) and show that there exists a time index t such that $V(r(t)) < 0$, which contradicts the fact that $V(r(t)) \geq 0$ for all t and thus proves that $q = 0$ and $r(t) \to r^\star$. Now, since $\bar{r} \in \Omega(r(t))$ there exists, by definition, an infinite sequence of times $T \subset \mathbb{N}$ such that $\{r(t)\}_{t\in T} \to \bar{r}$. Since all the marginal utility functions are continuous it is also the case that

$$\{s_{kj}(r_k(t)) - s_{ij}(r_i(t))\}_{t\in T} \to s_{kj}(\bar{r}_k) - s_{ij}(\bar{r}_i) \in \mathbb{R} \tag{12}$$

for all task–task-resource combinations (i, k, j). Let us define a set $A = \arg\max_{(i,k,j)} s_{kj}(\bar{r}_k) - s_{ij}(\bar{r}_i)$ (i.e., the set of all task–task-resource combinations having the maximum positive marginal utility difference at $\bar{r}$). As a consequence of (12) and Rule 1 of the algorithm there must exist a time index τ such that $\alpha(t) \in A$ for all $t \in T \cap [\tau, \infty) = T_1$. Let us take one triplet $\bar{\alpha} \in A$ such that $T_2 = \{t \in T_1 : \alpha(t) = \bar{\alpha}\}$ is an infinite set. From this point on let the indices i, k, and j be those of the triplet $\bar{\alpha}$. Let T_2 be partitioned into two sets T_A and T_B such that for all $t \in T_A$ the condition (5) holds and for all $t \in T_B$ it does not. Consider first what happens at time indices in T_A. Since $\bar{r} \neq r^\star$ and because of the choice of $\bar{\alpha}$ it is the case that $s_{kj}(\bar{r}_k) - s_{ij}(\bar{r}_i) = \delta_A > 0$ and accordingly that $\{s_{kj}(r_k(t)) - s_{ij}(r_i(t))\}_{t\in T_A} \to \delta_A$. Thus there exists a time index τ_A such that $s_{kj}(r_k(t)) - s_{ij}(r_i(t)) \geq \frac{1}{2}\delta_A$ for all $t \in T_A \cap [\tau_A, \infty) = T'_A$. Since condition (5) holds for all $t \in T_A$, it must be that (11) applies for all $t \in T'_A$. Accordingly,

$$\Delta V(r(t)) < -\frac{(1-\gamma)^2}{4L}[s_{kj}(r_k(t)) - s_{ij}(r_i(t))]^2$$
$$\leq -\frac{(1-\gamma)^2}{4L}\left(\frac{\delta_A}{2}\right)^2 \tag{13}$$

which is to say that after time index τ_A, every resource transfer that occurs at times in T'_A results in a decrease in $V(r(t))$ that is bounded away from zero by a fixed constant. Finally, consider what happens at time indices in T_B. As stated, at these times condition (5) does not hold, and as a consequence of this it is the case that (a) $u_j(t) = r_{ij}(t)$ and (b) $s_{kj}(r_k(t)+u(t))-s_{ij}(r_i(t)-u(t)) > 0$. In this case $\Delta V(r(t))$ may be bounded from above by the second term of (9), i.e.,

$$\Delta V(r(t)) < -r_{ij}(t)[s_{kj}(r_k(t) + u(t)) - s_{ij}(r_i(t) - u(t))]. \tag{14}$$

Because (14) is a continuous function of $r(t)$ it is true that at times $t \in T_B$ it converges to the value $-\bar{r}_{ij}[s_{kj}(\bar{r}_k + \bar{u}) - s_{ij}(\bar{r}_i - \bar{u})]$, where $\bar{u}$ in this case is the $\mathcal{R}$ vector with $\bar{r}_{ij}$ as the jth entry and all other entries equal to zero. Since the above quantity is negative, for any $\delta_B > 0$ such that $-\delta_B > -\bar{r}_{ij}[s_{kj}(\bar{r}_k + \bar{u}) - s_{ij}(\bar{r}_i - \bar{u})]$ there exists a time index τ_B such that for all times $t \in T_B \cap [\tau_B, \infty) = T'_B$, $\Delta V(r(t)) \leq -\delta_B$. Now, since $V(r(t))$ is non-increasing at each time t, and since

$$\sum_{t\in T'_A\cup T'_B} \Delta V(t) < -\left(\sum_{t\in T'_A} \frac{(1-\gamma)^2}{4L}\left(\frac{\delta_A}{2}\right)^2 + \sum_{t\in T'_B} \delta_B\right) \tag{15}$$

is unbounded it must be that at some finite time index t it will be the case that $V(r(t)) < 0$. Since $V(r)$ is non-negative for all $r \in \Delta_c$, this is a contradiction. Hence $q = 0$ and $r(t) \to r^\star$. Because V is positive definite and decrescent on Δ_c and decreases monotonically to zero the system is asymptotically stable in that region (Burgess & Passino, 1998). □

4. Conclusions and future directions

This paper has introduced a new optimization problem formulation in cooperative control involving team formation using agents of more than one fundamental type. An algorithm was developed that can be used to solve this problem under certain assumptions and it was analytically proven that this algorithm converges to the optimal team formation. The assumptions of this paper are commonly made in problems of this type, but they are not very representative of real world scenarios, particularly the assumptions of zero communication delays and continuous resource variables. Thus the algorithm presented here is only a first step in a potentially large area of research. However, using a discrete resource model complicates the problem in that the optimal formation may not be reachable (see Bertsekas and Tsitsiklis (1997b) and Burgess and Passino (1998) for example) and because the concept of marginal utility becomes more complicated without the use of derivatives. It would also be prudent to consider models that involve decentralized decision making and the communication delays and or noise (e.g., estimation error) that occur in those scenarios, although adding these features greatly complicates the analysis of these models.

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