Experimental Studies in Nonlinear Discrete-Time Adaptive Prediction and Control

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***Abstract*—This paper presents implementation results using recently introduced discrete-time adaptive prediction and control techniques using online function approximators. We consider a process control experiment as our test bed, and develop a discrete-time adaptive predictor for liquid volume and a discrete-time adaptive controller for reference volume tracking. We use Takagi–Sugeno (TS) fuzzy systems as our function approximators, and for both prediction and control we investigate the use of a least-squares update of the fuzzy system's parameters.**

I. Introduction

Discrete-time adaptive predictors and controllers have been studied in which the system is linear with unknown parameters, or the system nonlinearities are known and linear in a set of unknown parameters (see, e.g., the classic work [1]). The work in [2]–[6] uses on-line function approximation to develop discrete time adaptive predictors and controllers for the case when the unknown coefficients do not enter linearly or when the form of the system dynamics is poorly understood. In the function approximation approach, a suitable parameterized nonlinearity such as a fuzzy system or neural network is tuned online to approximate the unknown portion of the plant dynamics. Using this approach, the focus is on function approximation rather than parameter estimation, and no knowledge of parameterized physical nonlinearities is assumed. Neural networks or fuzzy systems are especially good candidates for being used as tunable nonlinearities since they satisfy the universal approximation property (i.e., they can be tuned to approximate an arbitrary continuous function defined on a compact set to an arbitrary degree of accuracy, but the achievable accuracy depends on the size and form of the approximator structure), but other conventional approximator structures such as polynomials could also be used, as in [7]–[10].

Some neural or fuzzy predictor schemes have already been developed and applied [11]–[16], and similar schemes have been used for system identification [17] and signal processing [18], [19]. However, there is in general a definite need for additional experimental evaluations of prediction schemes that are developed to achieve stable operation. Similarly, for control there have been a variety of adaptive methods developed for discrete-time nonlinear systems. For instance, neural networks and fuzzy system based methods were studied in [20]–[25] for discrete-time systems. Reference [26] presents an adaptive control method for nonlinear discrete-time systems with input deadzone. In [27], [28] adaptive neural network controllers are presented for a class of single-input–single-output (SISO) strict-feedback discrete time nonlinear systems, and [29] considers the multiple-input–multiple-output (MIMO) case.

Moreover, conventional (i.e., nonfuzzy/neural) methods were studied in [30]–[40], among others. Reference [41] deals with adaptive control of a class of discrete-time parametric-strict-feedback nonlinear systems with additive white noise, where the control law is designed based on weighted least squares and on recursive adaptive predictors, whereas [42] presents adaptive inverse schemes for systems with multisegment piecewise-linear nonlinearities. In [43], the authors study methods for velocity estimation from discrete and quantized position samples using adaptive windowing. References [44] and [45] use linear matrix inequality (LMI) and H_∞ techniques for control of linear or nonlinear discrete time systems with delays.

Most of the studies using online approximation-based schemes, both in the prediction and in the control fields, are purely theoretical. While a few instances of new stable control or prediction methods validated by implementation exist (for example, see [12], [13], and [46]–[50]), it seems to hold true that the literature contains a large number of untested (or, rather, never implemented) methods whose practical value has not been evaluated except to the extent to which the respective assumptions needed for stable operation can be related to control practice.

Within this paper, experimental studies are conducted that apply the discrete-time adaptive prediction and control techniques developed in [2], [3]–[6] (hereafter referred to as the "companion work") to a process control experiment test bed. The experiments conducted here seek to determine, in the first place, whether update of an approximator where the tuned parameters enter nonlinearly may be more advantageous, in a practical situation, than a linear in the parameters update. Second, our study tries to find out how advantageous the addition of adaptation to an existing control system may be. Third, our general objective is to show how to pick the various parameters of the adaptive schemes to achieve good performance. In Section III, we develop an adaptive predictor system for liquid volume in the process control experiment. In Section IV, a discrete-time adaptive controller is applied to the problem of volume reference tracking. For convenience, the relevant prediction and control theorems used in the implementation

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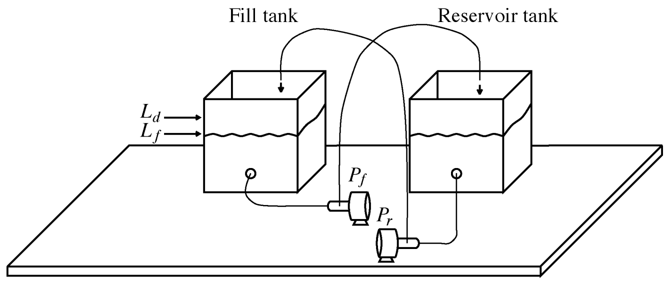

Fig. 1. Process control experiment.

will be given, without proof, together with the corresponding reference.

II. The Process Control Experiment

The process control experiment we use has been designed to resemble systems found in chemical processes where liquid volume control is performed. First we study prediction of a closed-loop system, where we use a simple, nonadaptive fuzzy controller as an illustration, and design a predictor of liquid volume. For control, we use the same fuzzy controller, and then allow it to adapt, to show the potential advantages of an adaptive system over a nonadaptive one. For a description of how a variety of control approaches work for this experiment, see [46].

A. Experimental Setup

The process control experiment, schematically shown in Fig. 1, consists of two tanks. The first one, called "fill tank" contains a liquid whose volume we wish to control. We denote the liquid volume by L_f and measure it in gallons. When full, the fill tank contains ten gallons of liquid. Note that rather than measuring liquid volume directly, we measure liquid level instead, since volume is directly proportional to level due to the tanks' geometry. The reference input, a desired liquid volume, is denoted by L_d. The second tank, also of ten gallons capacity, is called "reservoir tank," and contains the liquid that the controller will pump into and out of the fill tank in order to bring the liquid volume to its desired value.

There are two pumps that serve as system actuators. The first one is a variable rate direct current (dc) pump, denoted by P_r, which pumps liquid from the reservoir tank into the fill tank. The second one is an alternating current (ac) pump, denoted by P_f, which can only be turned on or off, and is used to remove liquid from the fill tank. The control input to the system is a single voltage u, where a sufficiently large positive value (of at most 10 V) will cause the dc pump P_r to transfer liquid into the fill tank, and any negative value (at least -10 Volts) will cause the ac pump P_f to turn on and remove liquid from the fill tank. Notice the asymmetry caused by the different operation of the pumps: The dc pump has a dead zone, above which the liquid flow is approximately a linear function of u (see the next section on modeling); the ac pump, on the other hand, is turned on to maximum power by any negative u, regardless of its magnitude. The combined behavior of the pumps when u is close to zero in magnitude make it very challenging to maintain volume at a steady value with small tracking error: the dead-band of P_r and the all-or-nothing functioning of P_f conspire to make the closed-loop system oscillate around the desired volume P_d.

The process control experiment can be modeled as a first order difference equation; nevertheless, the real plant is a complex nonlinear system, therefore well suited for testing of the potential of our chosen prediction and control techniques. In addition to the dead zone of the dc pump and the control asymmetry mentioned previously, both pumps have saturation nonlinearities which are difficult to characterize. Furthermore, the pumps introduce significant electrical noise and delays into the system. Finally, there is sensor noise, which occurs when liquid is pumped into and out of the fill tank and waves are produced in the liquid surface which, in turn, cause the level-measuring Styrofoam ball to oscillate.

B. Model

In this section, we develop an approximate mathematical model of the experiment. The purpose of this model is not to provide an accurate representation of the plant, but rather to illustrate qualitatively the complexity of the system, on one hand, and to provide a justification for the design choices made for the predictor and control systems of liquid volume, on the other.

The process control experiment may be represented by the first-order nonlinear difference equation

$$L_f(k+1) = L_f(k) + f_p\left(u(k)\right) \tag{1}$$

where k is a time index, u is a voltage input (ranging between -10 and 10 V) which drives the pumps P_f and P_r, and $f_p(u)$ represents the combined effects of the pumps P_f and P_r. Experimentally, we have found that f_p can be approximated by

$$f_p(u) = \begin{cases} 0.0333\tanh\left(8.5(u+0.045)\right) & \\ \quad -0.0056, & u < -0.05 \\ -\left(1.68u-(1.42)10^{-4}\right)^2 & \\ \quad +(6.63)10^{-5}, & -0.05 \le u < 0 \\ 0.0488\left(\frac{\tanh(3.6u-3.3)}{2}+0.5\right), & u \ge 0 \end{cases} \tag{2}$$

where $\tanh(t) = (e^t - e^{-t})/(e^t + e^{-t})$. This function is plotted in Fig. 2. It was derived from the experimentally determined characteristics of the combined dc and ac pumps (see [46] for details, as well as continuous time treatment of this plant). Indeed, for negative values of u the ac pump is activated in an approximately on-off manner. Similarly, the dc pump is turned on when u is positive, but only after a dead-band of approximately 1 V. The experimental characteristic is clearly piecewise continuous and noninvertible due to the several constant-valued sections it has (e.g., when $u < 0$). To get around this issue, we defined the approximation (2). Notice that (2) is invertible and differentiable everywhere. The invertibility of (2), in particular, is important for the development of the discrete time adaptive controller (note from the Appendix that the error dynamics have to be expressible in the form (27), whose existence depends on the invertibility of the function through which the control enters the system dynamics).

As mentioned before, we will not be overly concerned with the accuracy or lack thereof of the discrete-time representation

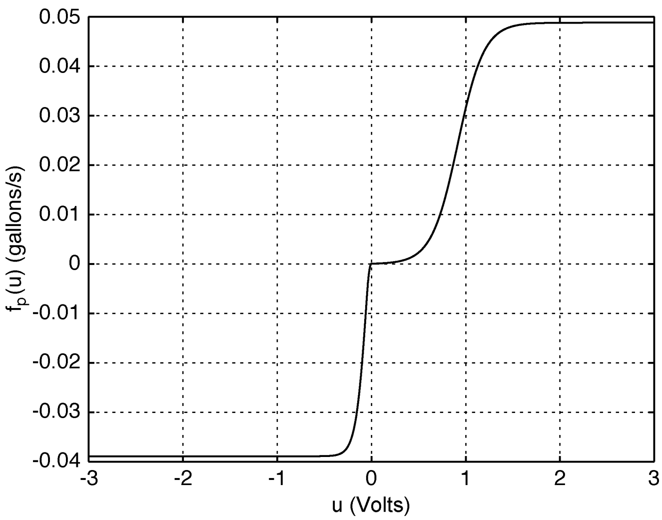

Fig. 2. Combined effect of pumps.

of the plant, since we would like, in the case of the adaptive predictor, to take care of the "modeling" work by adaptively approximating the plant dynamics. Similarly, the adaptive controller should be able to compensate for our lack of information about the plant. Nevertheless, the model provides us with an important parameter of the plant: Its relative degree, which is equal to one. In both prediction and control it is necessary to know the relative degree in order to specify the adaptation law; further, in the particular case of the predictor, the relative degree sets a limit to the maximum number of steps in the future for which we are able to predict the plant output.

C. Takagi–Sugeno (TS) Fuzzy Systems

To present the notation to be used in the remainder of the paper we will briefly overview the definition of TS fuzzy systems [51]. Let $\mathcal{F}(x, A)$ denote a fuzzy system with input x and parameter vector A. Then

$$\mathcal{F}(x, A) = \frac{\sum_{i=1}^{p} d_i \mu_i}{\sum_{i=1}^{p} \mu_i} \tag{3}$$

Here, singleton fuzzification of the input $x = [x_1, \ldots, x_n]^\top$ is assumed; the fuzzy system has p rules, and μ_i is the value of the membership function for the antecedent of the i^{th} rule given the input x. It is assumed that the fuzzy system is constructed in such a way that $\sum_{i=1}^{p} \mu_i \neq 0$ for all $x \in \mathbb{R}^n$. The parameter d_i is the consequent of the i^{th} rule, which in this work will be taken as a linear combination of Lipschitz continuous functions $z_k(x) \in \mathbb{R}$, $k = 1, \ldots, m-1$, so that

$$d_i = c_{i,0} + c_{i,1} z_1(x) + \cdots + c_{i,m-1} z_{m-1}(x), \qquad i = 1, \ldots, p. \tag{4}$$

Define $z = [1, z_1(x), \ldots, z_{m-1}(x)]^\top \in \mathbb{R}^m$, let

$$\zeta(x) = \frac{[\mu_1, \ldots, \mu_p]^\top}{\sum_{i=1}^{p} \mu_i} \tag{5}$$

and

$$C = \begin{bmatrix} c_{1,0} & \cdots & c_{p,0} \\ \vdots & \ddots & \vdots \\ c_{1,m-1} & \cdots & c_{p,m-1} \end{bmatrix}. \tag{6}$$

Then, the nonlinear equation that describes the fuzzy system can be written as

$$\mathcal{F}(x, A) = z^\top C \zeta \tag{7}$$

and $\partial \mathcal{F} / \partial C = z \zeta^\top$. The vector A contains those parameters of $\mathcal{F}$ that we are interested in tuning. We will consider two cases: A may contain the entries in matrix C, in which case the fuzzy system is said to be linearly parameterized and we have a linear in the parameters update; on the other hand, A may contain the centers and spreads of the input membership functions in addition to the entries of C, and in this case the fuzzy system is nonlinearly parameterized and, correspondingly, we have a nonlinear in the parameters update. In the remainder of this paper we will refer to the coefficients in C and in the input membership functions separately when we describe the way in which the fuzzy systems are updated, and it should be understood from the context that the vector A will contain the coefficients appropriate for either the linear or the nonlinear in the parameters updates, depending on the case at hand.

D. Nominal Control System

The controller used for volume reference tracking is the last piece to complete the system's description. Since the process control plant is a slow system, we use a sampling time of 1 s. We use a nonadaptive TS fuzzy controller, which serves a double purpose: For prediction, it provides us with data from the plant under closed-loop control; for control, it establishes a baseline of comparison, to be able to assess how much improvement the adaptation mechanism is bringing to our tracking objectives.

The fuzzy controller has only one input, the tracking error $e(k) = L_d(k) - L_f(k)$, with a gain of 5.0, and eleven Gaussian membership functions uniformly distributed over the interval $[-1, 1]$, all with spreads equal to 0.2. The outermost membership functions are saturated, and the output is computed using center average defuzzification. We set the system to have eleven rules, and we choose $z = [1, e]^\top$. The controller output can then be computed from (7), where, in order to avoid confusion with the predictor, we will use the notation C_{cont} to denote the matrix of the controller's consequent coefficients.

The design we settled on for the controller has consequent coefficients $\underline{c}_i^c = [((i-6)/5), 0]^\top$, $i = 1, \ldots, 11$, where $\underline{c}_i^c$ is the ith column of C_{cont}. As the reader will see in the next section, this controller performs poorly for the process control experiment. It presents a highly oscillatory behavior, poor tracking, and high sensitivity to measurement noise. From the point of view of prediction, this is a desirable feature, because it makes the prediction task more challenging (the closed-loop system behavior becomes more complex to predict due to poor performance of the controller); similarly, this fuzzy controller allows for a clear illustration of the improvements in performance that adaptation can provide: the controller has enough structure to do a much better control job, but its initial specification is not a

TABLE I
RULE BASE FOR THE PROCESS CONTROL PREDICTOR

	F_1^1	F_2^1	F_3^1	F_4^1	F_5^1	F_6^1	F_7^1	F_8^1	F_9^1	F_{10}^1	F_{11}^1
F_1^2	d_1	d_1	d_1	d_1	d_1	d_1	d_2	d_3	d_4	d_5	d_6
F_2^2	d_1	d_1	d_1	d_1	d_1	d_2	d_3	d_4	d_5	d_6	d_7
F_3^2	d_1	d_1	d_1	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
F_4^2	d_1	d_1	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9
F_5^2	d_1	d_1	d_2	$\boxed{d_3}$	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
F_6^2	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
F_7^2	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{11}
F_8^2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{11}	d_{11}
F_9^2	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{11}	d_{11}	d_{11}
F_{10}^2	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{11}	d_{11}	d_{11}	d_{11}
F_{11}^2	d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{11}	d_{11}	d_{11}	d_{11}	d_{11}

good choice. Adaptation, however, is able to exploit the potential of the TS fuzzy system much better than our original design, as will be shown in Section IV.

III. Discrete-Time Adaptive Prediction

A. Predictor Design Process

We notice that the process control model (1) has a relative degree of one. Therefore, our predictor will be able to provide us with $\hat{L}_f(k+1)$, the predicted liquid level one step ahead of the current time. Now, we would like not to have to make any assumptions on the plant dynamics: for instance, we do not want to be concerned with whether the system is feedback linearizable (the model (1) is not, since the input does not enter in an affine manner), or whether it has a particular functional structure; it is enough to know there exists some mapping between input and output that we will attempt to approximate. The direct adaptive prediction scheme (summarized in the Appendix) requires no explicit assumption on plant dynamics' structure, and is therefore the best choice in this case (note that if we knew with enough certainty that the plant's input–output equation is affine in the control, then the indirect scheme might be more appropriate, since then the plant's structure could be exploited more explicitly).

We use a TS fuzzy system to produce $\hat{L}_f(k+1)$. We choose the current level and control input, $L_f(k)$ and $u(k)$ as inputs to the fuzzy system, and we let $z^\top = [1, L_f(k), \ldots, L_f(k-9)]$ (we found it necessary to include all these past outputs in the predictor's consequent to achieve good performance). For both inputs we choose eleven Gaussian membership functions, initially uniformly distributed over the interval $[-1, 1]$, with equal spreads of 0.2 (we use scaling gains, 1/7 and 1/10 for $L_f(k)$ and $u(k)$, respectively, to normalize the inputs to this interval). We saturate the outermost membership functions, and the output is computed using center average defuzzification. For the inferences, we utilize the rule-base shown in Table I. The labels F_i^j denote the i^{th} fuzzy set for the j^{th} input, where $i = 1$ corresponds to the leftmost, and $i = 11$ to the rightmost fuzzy set. Each entry of the table corresponds to one output function d_i, $i = 1, \ldots, 11$, where $d_i = z^\top \underline{c}_i^p$, and $\underline{c}_i^p$ is the ith column of C_{pred}, which will denote the matrix containing the consequent coefficients of the predictor fuzzy system. We initialize C_{pred} with 0.1 everywhere except the first row, for which we choose $[1, 0.8, \ldots, 0.2, 0, -0.2, \ldots, -0.8, -1]$ (see below for some remarks on the initialization of C_{pred}). As an example, consider the rule for d_3 that is inside of a box in Table I,

$$\textbf{If } L_f(k) \text{ is } F_3^1 \textbf{ and } u(k) \text{ is } F_5^2 \textbf{ Then } d_3 = z^\top \underline{c}_3^p$$

where we evaluate the **and** operation using minimum.

Nonlinearity of the system and the measurement noise seemed to require a rapidly converging predictor. For this reason, although computationally intensive, we decided to use the least-squares update law for our adaptive predictors, since the sampling time of 1 second used in implementation is long enough to allow for computation. Furthermore, the update is applied both to the matrix C_{pred} of output coefficients *and* to the input membership functions' centers and spreads.

Another important design issue is the bound for the representation error of the predictor fuzzy system (please see the Appendix), which in turn affects the size of the time-varying discontinuous dead-zone used in the least-squares update law. Unfortunately, there is currently no clear analytic way of choosing this bound, especially when dealing with a complex physical system. Therefore, we again resorted to a pragmatic approach: We treated this bound as a parameter to be tuned. In general one would want to have an approximator with a large enough structure, so that the representation error is small. In our case, the representation error could be larger than what it would be if we only adjusted C_{pred}, since the input centers and spreads appear nonlinearly in the update scheme. However, we adopted the point of view that the added structural richness of the fuzzy system would in fact improve its approximation capabilities, and therefore *reduce* the representation error rather than enlarge it. The results we obtained seemed to corroborate our pragmatic design strategy: we found that letting the representation error bound $\bar{W}_y$ be as small as 0.001 yielded good results, as shown in the next section. Following these guidelines, we found that it is indeed more advantageous to update *both* the matrix C_{pred} *and* the centers and spreads of input membership functions, as the reader will see in the next section. For completeness, we use standard projection to keep the predictor's parameters bounded within a suitably large region. We found that the parameters never approached the boundary of this region, so that the projection is never actually activated. The same is the case with the adaptive controller in the next section.

The theoretical background for the discrete-time techniques we are using here establishes that transient performance of the

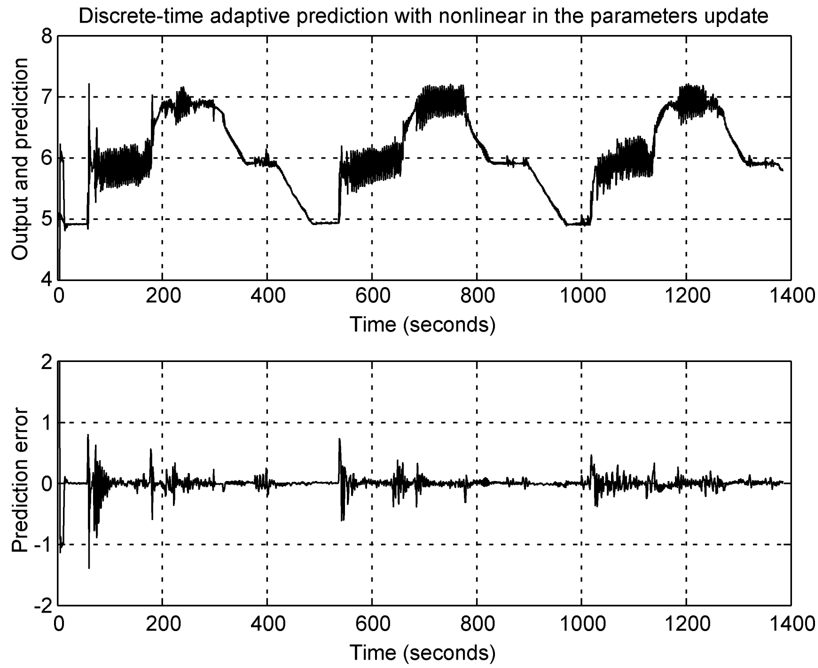

Fig. 3. Prediction with update of C_{pred} and input membership functions.

predictor (or the controller, as in Section IV) can be greatly improved if the initial choices for the coefficients of the approximator, a TS fuzzy system in this case, are close to the ideal values. Unfortunately, these ideal values may not be unique, and it may be difficult to guess them. This situation may force the designer to resort, yet again, to pragmatics and intuition. In our case, the size of the vector z made the initial choice of $C_{\rm pred}$ a challenging design issue. Our rationale was that, since we could not determine which, if any, of the past values of liquid volume, or the control input, should have a greater impact on the approximator output, the best guess (under such a lack of information) would be to set all coefficients initially equal, but with a small magnitude. Some testing of this strategy proved successful, and we settled for the value 0.1, as mentioned before. Note, however, that although effective in this case, a simple strategy like this one will most likely not generalize well.

B. Experimental Results

We chose a reference liquid volume that takes the values 5, 6, 7, and 6 gallons periodically, each value for a duration of 60 s. Fig. 3 shows the results for one step ahead prediction where the matrix $C_{\rm pred}$ and the centers and spreads of input membership functions are updated; the top plot contains the actual (dashed line) and predicted (solid line) liquid levels, and the bottom plot presents the prediction error. Notice that the error is relatively large during the first half of the first cycle, and then decreases rapidly.

Fig. 4 shows prediction results when only the coefficients in $C_{\rm pred}$ are updated, what corresponds to having a linear in the parameters update. One should observe that in this case, too, prediction is successful, but the predictor has noticeably more problems, especially when the liquid volume is not too oscillatory (e.g., between times 300 and 500 hundred s).

Subjective observations such as these are best analyzed and corroborated numerically. To do so, we note that the first 20 s of both the linear and the nonlinear in the parameters prediction errors exhibit a rather large transient, in particular in relation with the behavior thereafter. For this reason, we ignore the first 20 s of the data and examine the first and second order statistics of the square errors. Table II shows the results.

Clearly, the nonlinear in the parameters update of the predictor yields more desirable performance. The mean square error is reduced by about 41% when a nonlinear in the parameters update is employed. Likewise, the standard deviation is smaller by about 20% with respect to the linear in the parameters update. Thus, we conclude that, although both configurations yield an acceptable performance, in this case a nonlinear in the parameters update of the predictor is preferable because of its improved prediction ability.

IV. Discrete-Time Adaptive Control

A. Controller Design Process

Next, we consider discrete-time adaptive control for the process control experiment. We will make use of the fuzzy controller detailed in Section II, with the purpose of illustrating how adaptation may be able to improve the performance of an existing control design, especially when one has to deal with a complex physical plant for which an adequate design based purely on intuition and experience may not suffice.

Before proceeding, we need to decide on the kind of adaptive controller to use: Indirect or direct. Indirect adaptive control relies on the plant being feedback linearizable. However, the control u does not enter the plant in an affine way; rather, it enters nonlinearly, as can be seen in (1), which invalidates the indirect choice. Therefore, we apply the direct adaptive approach in the companion work, for which existence of an ideal

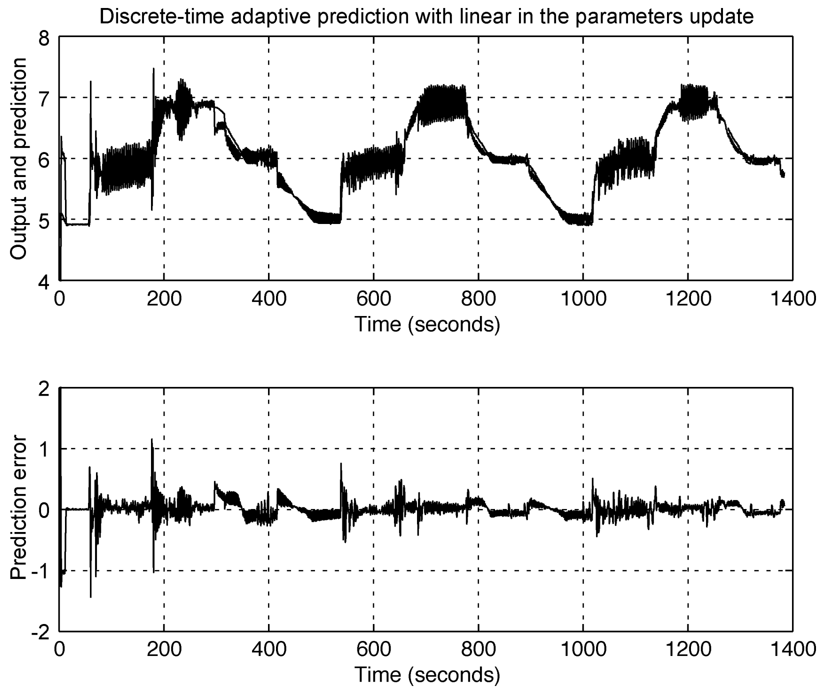

Fig. 4. Prediction with update of C_{pred} only.

TABLE II
MEAN AND STANDARD DEVIATION OF THE SQUARE PREDICTION ERRORS

	μ	σ
Nonlinear update	0.0144	0.0760
Linear update	0.0246	0.0942

control $u^*(L_f(k), L_d(k+1))$ is assumed such that the tracking error dynamics $(e(k) = L_d(k) - L_f(k))$ can be expressed as $e(k+1) = -\theta(L_f(k))[u(k) - u^*(k)] + \nu(k)$, where $\theta(L_f(k))$ is bounded away from zero by known constants, and $\nu(k)$ is an unknown, bounded function.

In order to put the error dynamics in the required form of (27), notice that we can let $f_p(u(k)) = u(k) + \beta(u(k))$ for some $\beta(u(k))$. Now, let $u^*(k) + \beta(u^*(k)) = -L_f(k) + L_d(k+1)$. With these choices,

$$e(k+1) = L_d(k+1) - L_f(k) - u(k) - \beta(u(k)) \quad (8)$$
$$= -[u(k) - u^*(k)] + \beta(u^*(k)) - \beta(u(k)) \quad (9)$$

so that, with $\theta(L_f(k)) = 1$ and $\nu(k) = \beta(u^*(k)) - \beta(u(k))$, the error dynamics are in the appropriate form. Note that since (2) is invertible for all u, we may select an ideal control that feedback linearizes the plant dynamics, (1). In particular, let $u^*(k) = f_p^{-1}(-L_f(k) + L_d(k+1))$. We observe that, since

$$u^*(k) + \beta(u^*(k)) = f_p(u^*(k)) = -L_f(k) + L_d(k+1) \quad (10)$$

we can guarantee the existence of u^* such that the error dynamics satisfy the required form. The function $\beta(u(k))$ is continuous but unbounded for unbounded u, but given the practical constraint that u must lie between -10 and 10 V, a bound exists for $\nu(k)$ within this interval. Note, finally, that since $\theta(x(k)) = 1$ we are not constrained to applying gradient update, but are also able to use least-squares for direct adaptive control (please refer to the companion work for details). As we did in the prediction case, we will again resort to least-squares update for our control purposes, because the faster error convergence is a very desirable feature in control applications. Least-squares is computationally expensive, but our fuzzy system is small enough to allow real-time implementation for our given sampling rate.

B. *Experimental Results*

We will now allow the fuzzy controller described in Section II-D to adapt, in order to find out how, if at all, adaptation is able to improve the closed-loop performance. Fig. 5 shows, with greater detail, the tracking performance of the nonadaptive fuzzy controller used in the previous section. The top plot contains the reference and the real liquid volumes; the second, the tracking error; and the lower plot, the control input. Observe, in the first place, that the control u is bounded between -1 and 1, which is due to the fact that the output membership functions (the first row of the matrix of coefficients, C_{cont}) are distributed between -1 and 1 (please refer to Section II for a description of the controller) and a gain of 1 is used for the output of the fuzzy system. Note that the controller is highly oscillatory, in particular when the reference volume increases (a result of the asymmetry of the actuators and of the measurement noise). Observe, also, that even when the control input is smoother, the tracking error tends to be a nonzero constant. In general, this design presents several shortcomings, like low noise rejection, poor steady-state tracking, and chattering.

Fig. 6 shows the tracking results when least-squares adaptation is used for the discrete-time system. Here, we only consider the case where a nonlinear in the parameters update is performed, i.e., update of the output coefficients in matrix C_{cont}, as well as the centers and spreads of the input membership functions. Note how steady-state chattering is greatly reduced, even during the first period, and then is even further minimized

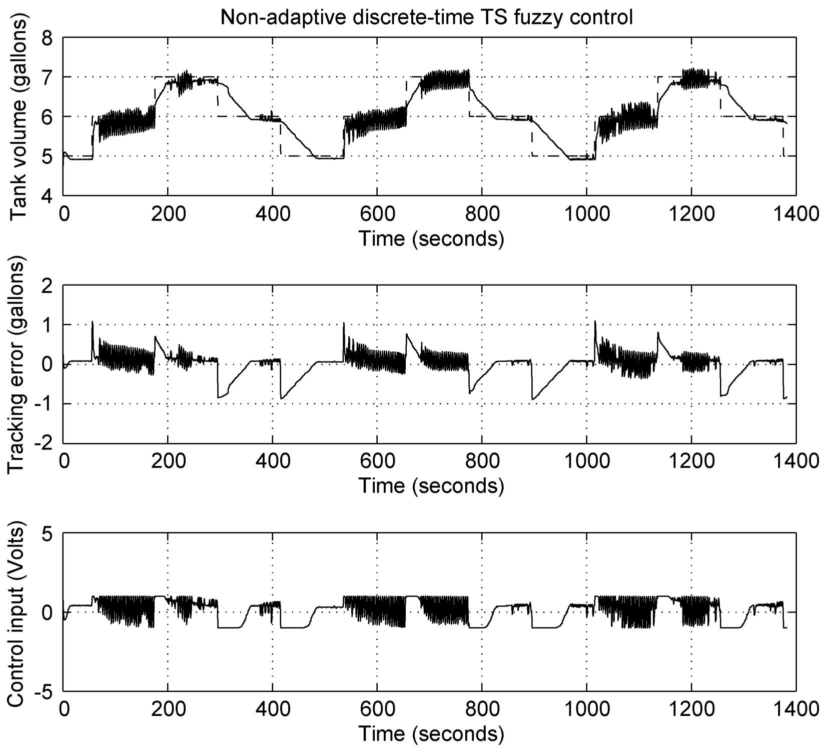

Fig. 5. Nonadaptive fuzzy controller for the process control experiment.

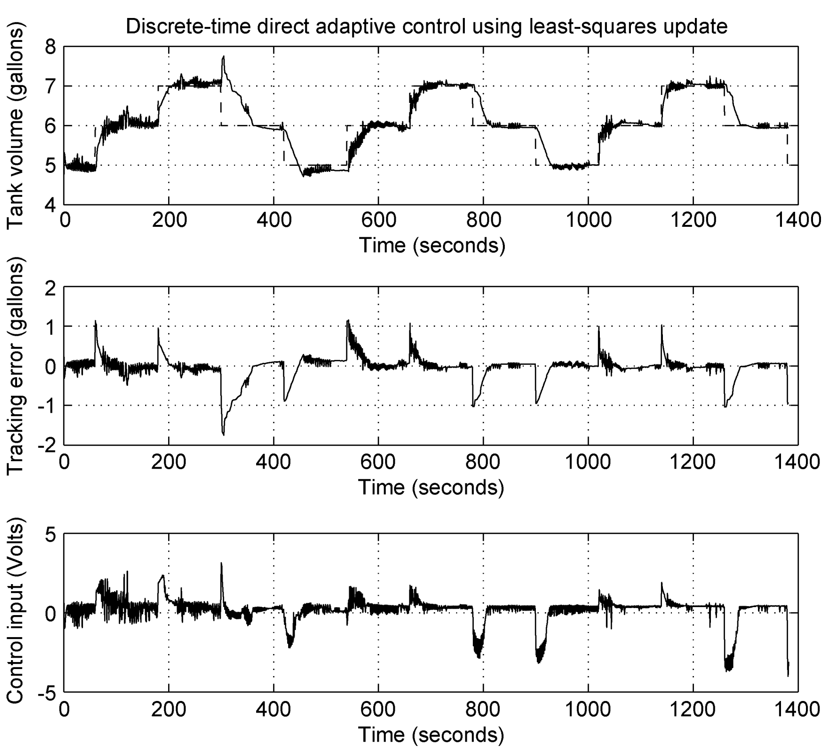

Fig. 6. Direct discrete-time adaptive control for the process control experiment.

as adaptation proceeds. Similarly, the overall magnitude of the error is reduced with respect to Fig. 5, and tends to become smaller with time. In general, tracking performance appears to be improved by means of the adaptation mechanism.

Again, we would like to obtain an objective measure to confirm these observations. Here, we will use the absolute value of the tracking error, rather than the square error, in order to remove the disproportionate bias introduced by the instances when the error is larger than one in magnitude. Table III summarizes the results. In this case, the results are not as clearly manifest from these statistics as in the prediction case. The mean absolute error is reduced by about 20% by the use of adaptation,

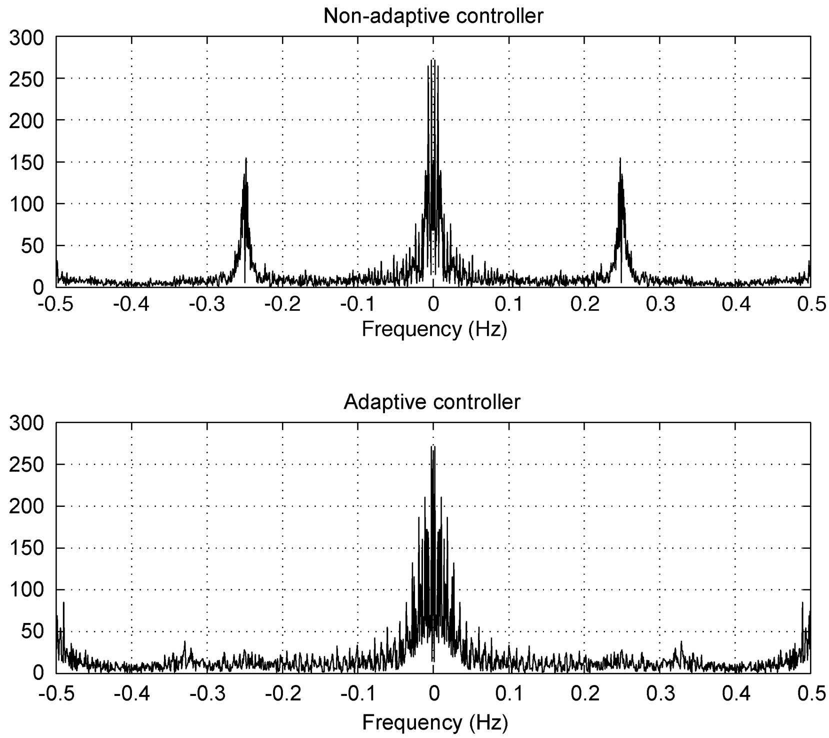

Fig. 7. Frequency contents of the control signals.

TABLE III
MEAN AND STANDARD DEVIATION OF THE ABSOLUTE VALUE OF THE TRACKING ERRORS

	μ	σ
No adaptation	0.2212	0.1922
Adaptation	0.1789	0.2643

but the standard deviation increases. In order to obtain further insight into the performance of the two controllers, we analyze the frequency content of the control signals produced by each one.

Examining Fig. 7 one notes that both controllers are comparable at low frequencies; however, the nonadaptive controller exhibits a peak at about 0.25 Hz which does not appear in the adaptive counterpart. Given the slowness of the plant dynamics (recall that the sampling period is one second), this peak occurs at a relatively high frequency, and it is reflected in the frequency content of the plant output (not shown for brevity). The effects of this peak are evident upon examination of the output for the nonadaptive case in Fig. 5. One observes large "patches" of rapidly oscillating liquid level (the chattering referred to above). These patches are noticeably reduced in the adaptive control case. Clearly, the lower frequency content of the adaptive controller is beneficial not only in that average tracking is improved, but also in reducing mechanical stress to the actuators and the plant itself. Using adaptation on the original design does, then, yield a clear and concrete benefit (keeping in mind that coming up with a better nonadaptive design for this plant is not necessarily straightforward).

V. CONCLUDING REMARKS

In this paper, we presented implementation results for the discrete-time prediction and control techniques in the companion work. We investigated application of the techniques to one experimental test bed, a process control system, for which we constructed direct adaptive prediction and control systems. For the predictor, our results suggest that, in some cases, a nonlinear in the parameters update may be more advantageous than a linear one, in the sense that a smaller mean square error is achieved. The control case illustrates how the addition of adaptation may result in an improved tracking performance, especially when the controller design task is complicated by the presence of complex nonlinearities and significant levels of measurement noise. The average tracking error of the adaptive controller is reduced with respect to the nonadaptive case, and moreover, the adaptive controller displays a significantly more band limited frequency content, thus potentially reducing mechanical wear of the plant.

APPENDIX
DIRECT ADAPTIVE PREDICTION AND CONTROL SUMMARIES

The direct adaptive prediction and control theorems we use are for systems which may be placed in the form described by

$$y(k+d) = f\left(x(k), u(k)\right) \tag{11}$$

where $f(\cdot)$ is smooth, $x(k)$ is the state vector, $u(k)$ is the (scalar) input, $y(k)$ is the (scalar) output, and $d \geq 0$ is the delay between the input and output. The state vector may be, in general, of the form $x(k) = [y(k), y(k-1), \ldots, y(k-n), u(k-1), u(k-2), \ldots, u(k-m)]^\top$ for some $n \geq 0$ and $m \geq 0$.

Direct Adaptive Predictor:

The direct adaptive predictor [2], [6] will use past and current inputs and outputs of the plant to determine how the plant will behave in the future. To develop the direct adaptive predictor, we first express (11) as

$$y(k+q) = f\left(x(k-d+q), u(k-d+q)\right) \quad (12)$$
$$= y_u\left(x(k-d+q), u(k-d+q)\right) + y_k(k) \quad (13)$$

where we wish to predict the system output at time $k+q$ where $q \leq d$. The signal $y_k(k)$ is used to allow for the possibility of incorporating some a priori knowledge of what the system output will be at time $k+q$ (the subscript " k" on y_k stands for "known;" it is not a time index), while $y_u(x(k-d+q), u(k-d+q))$ represents the unknown portion of the plant dynamics which will be approximated online. All the results to follow also hold if $y_k(k) = 0$ for all k. Using an ideally adjusted fuzzy system or neural network, the plant dynamics may be expressed as

$$y(k+q) = \mathcal{F}_y\left(x(k-d+q), u(k-d+q), A_y^*\right) + w_y(k+q) + y_k(k) \quad (14)$$

Here, $w_y(k)$ is the error in representing the discrete time system with an approximator of finite size, where $0 \leq |w_y(k)| < W_y < \infty$ and $k \geq 0$. Here, A_y^* is a vector of ideal coefficients that are chosen which ensure that $\sup_k |w_y(k)|$ will be minimized. This may be expressed as

$$A_y^* = \arg\min_{A_y \in \Omega_y}\left[\sup_{x \in S_x, u \in S_u} |\mathcal{F}_y(x, u, A_y) - y_u(x, u)|\right] \quad (15)$$

where S_x and S_u are the compact sets in which x and u are contained, respectively. The compact convex set Ω_y is the region which contains feasible parameter sets for A_y^*. The predictor of the output of the system at time $k+q$ is

$$\hat{y}(k+q) = \mathcal{F}_y\left(x(k-d+q), u(k-d+q), A_y(k)\right) + y_k(k) \quad (16)$$

and we assume that a standard "projection" method is used to ensure that $A_y(k) \in \Omega_y$, $k \geq 0$.

The parameter error is described by $\phi(k) = A_y(k) - A_y^*$, so that using (14) and (16) the prediction error becomes

$$\begin{aligned} e(k+q) &= y(k+q) - \hat{y}(k+q) \\ &= \mathcal{F}_y\left(x(k-d+q), u(k-d+q), A_y^* - A_y(k)\right) \\ &\quad + w_y(k+q) \end{aligned}$$

where $|w_y(k)| \leq W_y$. Also, since $\zeta(x(k-d+q), u(k-d+q), A_y(k)) = \partial\mathcal{F}(x, u, A_y)/\partial A_y$

$$\begin{aligned} e(k+q) =& -\phi^\top(k)\zeta\left(x(k-d+q), u(k-d+q), A_y(k)\right) \\ & + w_y(k+q) + \delta(x, u, A, A^*) \\ =& -\phi^\top(k)\zeta\left(x(k-d+q), u(k-d+q), A_y(k)\right) \\ & + \bar{w}_y(k+q) \end{aligned} \quad (17)$$

where $0 \leq |\bar{w}_y(k)| < W_y + \delta(x, u, A, A^*)$. The term $\delta(x, u, A, A^*)$ is bounded by $L|\phi|^2$ for some $L > 0$. Define $\bar{W}_y$, $W_y \leq \bar{W}_y$ such that if $|\phi|^2$ is a bounded sequence, then $0 \leq |\bar{w}_y(k)| < \bar{W}_y$ for all k. The $\bar{w}_y(k)$ term is used here instead of just $w_y(k)$ since for multilayered neural networks (or fuzzy systems with adjustable input membership functions), $\bar{w}_y(k)$ is a result of both the ideal approximation error and the linearization in terms of the parameters. The linearization term is on the order of $|\phi|^2$ which can be shown to be bounded. If approximation schemes for which the adjustable parameters appear linearly are used, then the $\delta(x, u, A, A^*)$ term vanishes. The assumptions required when using the direct adaptive predictor are summarizd as follows.

Assumption 1: The plant output is defined by (12), with states $x(k) \in S_x$ and input $u(k) \in S_u$. An approximator structure is defined such that there exists some $A_y^* \in \Omega_y$ so that if $|\phi|^2$ is a bounded sequence, then $|\bar{w}_y(k)| \leq \bar{W}_y$ where $\bar{W}_y > 0$.

Note that when using a predictor, we assume that the plant states and control signal are bounded (some controller may be needed to achieve this). Otherwise, the compact sets S_x and S_u used in the definition of the ideal parameters [see (15)] may not be bounded, requiring an infinitely large approximator (since the approximator must have only a finite approximation error over the whole space).

The direct adaptive predictor will use a discontinuous dead zone, defined as

$$D_d(x, \epsilon) = \begin{cases} x, & \text{if } x > \epsilon \\ 0, & \text{if } |x| \leq \epsilon \\ x, & \text{if } x < -\epsilon \end{cases} \quad (18)$$

where $\epsilon > 0$ is the size of the dead zone. In order to implement the least squares adaptation, we define an error signal, $\bar{e}_\epsilon$, as follows:

$$\bar{e}_\epsilon(k) = D_d\left(e(k), \epsilon_d(k)\right) \quad (19)$$

where

$$\epsilon_d(k) = \sqrt{2}\bar{W}_y\sqrt{1 + \zeta^\top(k-d)P(k-d-q)\zeta(k-d)} \quad (20)$$

Finally, we define $a(k)$ as

$$a(k) = \begin{cases} 0, & \text{if } |e(k)| \leq \epsilon_d(k) \\ 1, & \text{otherwise} \end{cases}. \quad (21)$$

Theorem 1: Given the error dynamics (17) with appropriate assumptions satisfied, then the adaptation law

$$\begin{aligned} A(k) &= A(k-q) \\ &\quad + \frac{\kappa P(k-d-q)\zeta(k-d)\bar{e}_\epsilon(k)}{1 + a(k)\zeta^\top(k-d)P(k-d-q)\zeta(k-d)} \\ P(k) &= P(k-q) \\ &\quad - \frac{a(k+d)P(k-q)\zeta(k)\zeta^\top(k)P(k-q)}{1 + a(k+d)\zeta^\top(k)P(k-q)\zeta(k)} \end{aligned} \quad (22)$$

will ensure that

1) $\lim_{k\to\infty}(\bar{e}_\epsilon(k)/\sqrt{1 + \zeta^\top(k-d)P(k-d-q)\zeta(k-d)}) = 0$;
2) $\lim_{k\to\infty} |\phi(k) - \phi(k-q)| = 0$.

Additionally, if $|\zeta(k-d)| \leq C_1 + C_2 \max_{j=0}^{k} |e(j)|$, for $0 \leq C_1, C_2 \in \mathbb{R}$ some finite constants, then

3) $\lim_{k\to\infty} \bar{e}_\epsilon(k) = 0$.

Remark 1: The condition $|\zeta(k-d)| \leq C_1 + C_2 \max_{j=0}^{k} |e(j)|$ is satisfied if the plant states and control signal are bounded, so that ζ is bounded when A is bounded, which is satisfied because of result (2) in the theorem.

Direct Adaptive Control:

The control objective is to have the plant output, $y(k)$, follow some reference signal, $r(k)$. We define the tracking error to be

$e(k) = r(k) - y(k)$. For the direct adaptive control approach [2], [3], [5], we will not place strict requirements upon the dynamics of the plant. Instead, we require that there exist some $u^*(x(k), r(k+d))$ (where we assume that $r(k+d)$ is known) that is continuous in its arguments such that the error dynamics may be expressed as

$$e(k+d) = -\theta\left(x(k)\right)\left[u(k) - u^*(k)\right] + \nu(k)$$

where $\theta(x(k))$ is such that

$$0 < \theta_0 \leq \left|\theta\left(x(k)\right)\right| \leq \theta_1$$

where θ_0 and θ_1 are known constants related to the plant dynamics, and $\sup_k |\nu(k)| \leq \mathcal{V}$ for some known $\mathcal{V} < \infty$.

To develop the error dynamics for the direct adaptive control case first express u^* as

$$u^*\left(x(k), r(k+d)\right) = u_u\left(x(k), r(k+d)\right) + u_k(k) \tag{23}$$

where $u_u(x(k), r(k+d))$ and $u_k(k)$ represent the unknown and known portions of the ideal control law, respectively. It should be noted that the signal $u_k(k)$ is not required for implementation and we may assign $u_k(k) = 0$, $k \geq 0$. We may represent $u_u(x(k), r(k+d))$ as

$$u^*(k) = \mathcal{F}_u\left(x(k), r(k+d), A_u^*\right) + u_k(k) + w_u(k) \tag{24}$$

where

$$A_u^* = \arg\min_{A_u \in \Omega_u}\left[\sup_{x \in S_x, r \in S_r}\left|\mathcal{F}_u(x, r, A_u) - u_u(x, r)\right|\right] \tag{25}$$

and Ω_u is the convex compact set of allowable controller parameters, S_x is the compact set through which the state may travel (we do not restrict its size *a priori*), and S_r is the space through which the reference input may travel (we assume that S_r is bounded). The current estimate of the ideal controller is given by

$$u(k) = \mathcal{F}_u\left(x(k), r(k+d), A_u(k)\right) + u_k(k) \tag{26}$$

where the current parameter set $A_u(k)$ will be updated online and a projection algorithm can be used to make sure that $A_u(k) \in \Omega_u$.

Defining the parameter error as $\phi(k) = A_u(k) - A_u^*$, the output error dynamics may be expressed as

$$\begin{aligned} e(k+d) = &-\theta\left(x(k)\right)\phi^\top(k)\zeta\left(x(k), r(k+d), A_u(k)\right) \\ &+\theta\left(x(k)\right)\bar{w}_u(k) + \nu(k) \end{aligned} \tag{27}$$

where $0 \leq |\bar{w}_u(k)| < W_u + \delta_u(x, r, A_u, A_u^*)$. The term $\delta_u(x, r, A_u, A_u^*)$ is again bounded by $L|\phi|^2$. Define $\bar{W}_u$, $W_u \leq \bar{W}_u$ such that if $|\phi|^2$ is bounded, then $0 \leq |\bar{w}_u(k)| < \bar{W}_u$ for all k.

Assumption 2: There exists some $u^*(x(k), r(k+d))$ that is continuous in its arguments such that the error dynamics may be expressed as $e(k+d) = -\theta(x(k))[u(k) - u^*(k)] + \nu(k)$, where $0 < \theta_0 \leq |\theta(x(k))| \leq \theta_1$, where θ_0 and θ_1 are known constants, and $\sup_k |\nu(k)| \leq \mathcal{V}$ for some known $\mathcal{V} < \infty$. An approximator structure is defined such that there exists some A_u^* so that if $|\phi|^2$ is bounded, then $|\bar{w}_u| \leq \bar{W}_u$ where $\bar{W}_u > 0$. The reference model has bounded output such that $|r(k)| \leq R$ for $R \in \mathbb{R}$ and $r(k+d)$ is known.

The requirement that $r(k+d)$ be known is needed due to the delay between the plant input and output. If the desired output trajectory is defined by $\bar{r}(k)$, then we may let $r(k+d) = \bar{r}(k)$ so that the plant output will converge to $r(k) = \bar{r}(k-d)$. This implies that the plant output will actually converge to a delayed version of the desired trajectory. For many applications this will be suitable.

Similar to the predictor, we define the error signal

$$\bar{e}_\epsilon(k) = D_d\left(e(k), \epsilon_d(k)\right) \tag{28}$$

where now

$$\epsilon_d(k) = \sqrt{2}(\theta_1\bar{W}_u + \mathcal{V})\sqrt{1 + \zeta^\top(k-d)P(k-d-q)\zeta(k-d)} \tag{29}$$

Finally, $a(k)$ is defined as in (21).

The direct adaptive control theorem using least squares is very similar to Theorem 1. We state it for completeness.

Theorem 2: Given the error dynamics (27) with appropriate assumptions satisfied, then the control law (26) together with adaptation law (22) will ensure that

1) $\lim_{k\to\infty}(\bar{e}_\epsilon(k)/\sqrt{1 + \zeta^\top(k-d)P(k-d-q)\zeta(k-d)}) = 0$;
2) $\lim_{k\to\infty}|\phi(k) - \phi(k-q)| = 0$.

Additionally, if $|\zeta(k-d)| \leq C_1 + C_2 \max_{j=0}^k |e(j)|$, for $0 \leq C_1, C_2 \in \mathbb{R}$ some finite constants, then

3) $\lim_{k\to\infty} \bar{e}_\epsilon(k) = 0$.

Remark 2: For the direct adaptive controller, the condition $|\zeta(k-d)| \leq C_1 + C_2 \max_{j=0}^k |e(j)|$ is satisfied automatically when a fuzzy system with adjustable output membership functions is used as defined in (5) since for that case $|\zeta| \leq 1$. With A bounded (where A is a vector of adjustable weights), we have $|\zeta|$ bounded so $|\zeta(k-d)| \leq C_1 + C_2 \max_{j=0}^k |e(j)|$ is also satisfied for feed forward neural networks with saturating activation functions. Since we use projection for each case we know that A is bounded.

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