A Numerical Method for Studying Modulation Effects in Radar Observations of the Sea Surface

Guangdong Pan and J. T. Johnson

Abstract— A numerical method for performing detailed investigations of modulations effects in sea backscattering is presented. The method is based on a combination of numerical hydrodynamic and electromagnetic codes, as well as repeated simulations as the spectral content of the sea surface is varied. The results obtained can be useful in separating "tilt" and hydrodynamic modulations in computed sea backscattering cross sections, and help to provide some insight into the need for "three scale" modeling of sea surface returns. Sample results are presented to illustrate the basic process.

I. INTRODUCTION

Radar observations of the sea surface are well known to provide information on sea waves that are much larger than the electromagnetic wavelength, even though the dominant scattering process involves Bragg scattering from sea waves whose wavelengths are on the order of the electromagnetic wavelength. Radars with spatial resolutions that resolve individual "long" sea waves observe modulations of measured power as a function of sea long wave properties. Two basic mechanisms are used to explain these modulations. The first is a "tilt" modulation in which sea long wave slopes cause a change in apparent incidence angle for the radar; sea "facets" tilted toward the radar by long waves produce larger returns, while those tilted away produce smaller returns. The second mechanism is a hydrodynamic modulation in which the amplitude of Bragg scattering short waves in the sea surface varies with position on the long wave due to non-linear hydrodynamic interactions; this variation causes modulations of radar backscattered returns as well. In the absence of hydrodynamic forcing and dissipation terms, short wave amplitudes are typically enhanced at the crest of long waves and decreased in the troughs, so that the hydrodynamic modulation is in phase with the long wave height.

While both of the effects produce non-linear relationships between either the sea surface height or slope and the observed radar cross section, it is widely accepted that these influences can be approximated as linear under the assumption of small long wave sea surface slopes. The linear approximation results in a description of variations in the measured radar cross section through the "tilt modulation transfer function" (TMTF) and the "hydrodynamic modulation transfer function" (HMTF) [1]–[3], which are linearly combined to predict the total radar cross section modulation.

It is often assumed that the TMTF can be well approximated by the two-scale theory of sea backscattering, and analytical

expressions for the TMTF have been presented in the literature. This assumption ideally allows radar measurements to be used to infer the HMTF directly, since the known TMTF can be subtracted from measured data. However, estimates of the HMTF so achieved have been shown to be problematic, in particular since they are found to vary with the radar polarization used in their inference [4]–[6]. Since the HMTF is a purely hydrodynamic quantity, the radar polarization should have no influence on the result obtained.

To address this issue, a "three scale" model has been proposed [5]. In this model, returns from sea "facets" tilted over sea surface long waves are modeled using a two-scale rather than Bragg scattering model. In particular, returns from facets are impacted by the slope variance of "intermediate waves" that are not resolved by the radar system but remain large compared to Bragg scatterers. Any hydrodynamic modulations of the intermediate waves by the longer waves or of the Bragg waves by the intermediate waves may also be important. Discrepancies in radar measurements of the HMTF have been explained using this approach.

This paper expands these studies further by proposing a numerical method that allows detailed investigations of the source of modulations in radar observations of the sea surface. The technique is based on a combination of numerical methods for electromagnetics and hydrodynamics (as previously considered in [7]–[10]) but further uses repeated simulations as the surface spectral content is varied and use a deterministic long wave to investigate modulation effects. Sample results using the procedure are shown to illustrate the impact of the intermediate scale waves.

The next section describes the numerical method, and sample results are presented in III. Section IV presents final conclusions.

II. NUMERICAL APPROACH

The simulations described utilize sea surface profiles rough in one dimension only in order to reduce computational requirements. However many of the concepts described here can also be applied in simulations using surfaces rough in two dimensions. Surface profiles are also modeled as perfectly electrically conducting to reduce computational requirements.

A. Combined hydrodynamic/electromagnetic simulations

The techniques used for computing electromagnetic scattering from a time evolving sea surface are identical to those described in [7]–[10], and are only briefly reviewed here.

The Ohio State University, Department of Electrical and Computer Engineering and ElectroScience Laboratory, 1320 Kinnear Road, Columbus, OH 43210, USA. Email: johnson.1374@osu.edu, pan.88@osu.edu

The nonlinear ocean surfaces utilized are modeled as incompressible and inviscid fluid surfaces of infinite depth; surface tension effects are also neglected. The surface elevation is denoted as $z = \eta(x,t)$ and the surface velocity potential as $\phi(x,t)$, where x, z are the horizontal and vertical space coordinates, respectively, and t represents time. The evolution of these two quantities is computed using the Watson-West method [11] as described in [12]. The simulation retains terms up to the 2nd order in the slope expansion. In the results to be shown, hydrodynamic computations were performed to generate a single set of surface realizations; examinations of the impact of varying surface spectral content were performed only in the electromagnetic scattering process.

Electromagnetic backscattering fields for both HH and VV polarizations are computed using the procedures described in [7]–[10], which require solution of a discretized integral equation. Efficient solution is accomplished through use of the "Method of Ordered Multiple Interactions" (MOMI) [13] iterative algorithm in combination with the spectral acceleration method [14], [15].

B. Surfaces considered

To emphasize modulation effects in the study, surfaces were initialized as a single sinusoidal "long" wave added to a spectrum of stochastic short waves. Short waves were produced as realizations of a Gaussian random process with a band-limited "Pierson-Moskowitz" (PM) spectrum [8]. To provide a wide separation between the "long" and "short" waves, the short wave spectrum was truncated so that the largest short wave had a wavelength 31 times shorter than that of the long wave. The shortest short wave was chosen to be approximately 3 times shorter than the electromagnetic wavelength. In the simulations performed, the wind speed parameter of the Pierson-Moskowitz spectrum was chosen as 3 m/s, but had negligible impact on the surface, as all short waves were short compared to the long wave cutoff frequency of the spectrum.

This initial condition was then allowed to time evolve using the hydrodynamic simulation over a time duration of twice the period of the long wave. Scattering computations were performed forty times during the surface time evolution. The results to be shown used a small long wave steepness ak of 0.05 (a is the long wave amplitude and k is the long wave wavenumber) in order to ensure the stability of the hydrodynamic simulation.

To reduce computational requirements and to allow a large set of surface realizations to be used, a moderate surface size in terms of the electromagnetic wavelength was used. A length of $L_s = 2\pi$ meters was used at electromagnetic frequency 6.1 GHz, so that the surface length is 128 electromagnetic wavelengths. This choice results in the "short" wave spectrum containing length scales from approximately 4 times to one third of the electromagnetic wavelength. The surface was discretized into 2048 points, which is consistent with the hydrodynamic and electromagnetic sampling requirements. Results were computed for 1600 realizations in the datasets shown in order to obtain sufficient convergence to provide reasonable modulation estimates.

C. Local scattering using an antenna pattern

Examining modulation effects requires that a particular "facet" of the long wave be resolved in the scattering computation. While this can be accomplished by using the range resolution achieved in a swept frequency electromagnetic simulation [16], resolution of a particular surface facet can also be accomplished by simulating a narrow antenna footprint on the long wave. The latter method was selected here in order to avoid the requirements for repeated computations at multiple electromagnetic frequencies. While the footprint simulated is clearly very small (i.e. less than 1 m typically), the results nevertheless provide information on the modulation mechanisms of interest.

The antenna footprint model utilized is the "tapered" incident field of [17] that has been widely used in rough surface scattering studies. It has been shown that this incident field provides a reasonable model of localized scattering from a surface so long as the spot size of the antenna footprint remains reasonably large compared to the electromagnetic wavelength, with this requirement becoming more severe as low grazing angles of incidence are approached. In this study, the incidence angle is limited to a maximum of 60 degrees, so that moderate footprint sizes are acceptable. Simulations are performed using the tapering parameter $g = 5.33$ electromagnetic wavelengths, so that a 3 dB footprint size of around 6.3 wavelengths results. While this is small compared to typical rough surface scattering simulations, numerous tests showed that reasonable scattering results were achieved up to incidence angle 60 degrees. This choice results in the antenna footprint occupying approximately 5 percent of the long sea wave wavelength, so that local tilt and hydrodynamic modulation impacts can be resolved.

D. Modulation studies

The key method used in this study is a comparison of scattered power time histories as the spectral content of the surfaces used in the scattering simulation is varied. Figure 1 illustrates the basic concept; plot (a) is a basic illustration of the tapered antenna footprint that produces locally resolved scattering, while plot (b) illustrates simulations using the complete surface, including the sinusoidal long wave and the stochastic short waves. Plot (c) illustrates a similar electromagnetic computation using hydrodynamic surfaces identical to those in plot (b) except that the long wave portion of the surface is removed; note hydrodynamic modulation effects on short waves are identical to those in part (b) since the modulated short waves surfaces are identical. Plot (d) further illustrates that the simulations of part (c) can be repeated in an identical fashion but with additional portions of the short waves surfaces (i.e. the intermediate waves) removed by passing the surface through additional high pass filters. These variations allow the impact of tilt, hydrodynamic, and intermediate wave modulations to be distinguished more clearly.

E. MTF Fitting

Scattering computations provide an array of complex field backscattered returns $f_{pp}(t_j, r_m)$ for each time t_j, surface
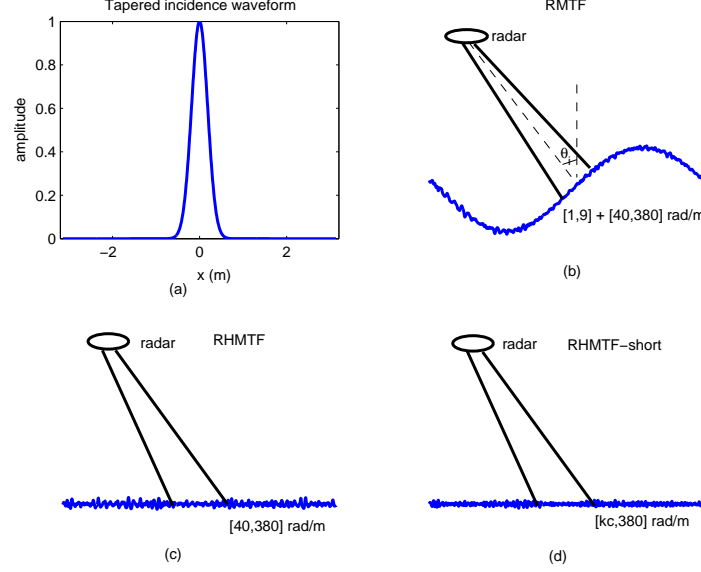
Fig. 1.   Illustration of the method used to study modulation effects

realization r_m, and polarization pp. Ensemble averaged cross sections are then computed as a function of time using

$$\sigma_{pp}(t_j) = \langle |f_{pp}(t_j, r_m) - \langle f_{pp}(t_j, r_m)\rangle|^2\rangle, \qquad (1)$$

to obtain the time varying polarized RCS $\sigma_{pp}(t_j)$. Figure 2 presents an example of these results for 25 degrees incidence angle and for the entire surface spectrum. The sinusoidal modulations associated with the long wave profile as it moves through the antenna footprint are clear in this Figure.
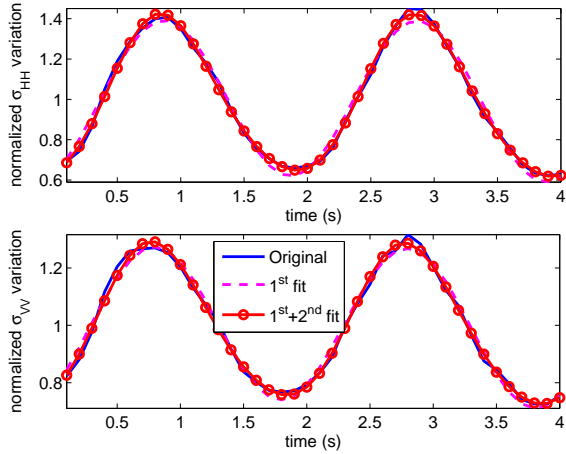

Fig. 2.   Time histories of backscattered radar cross sections normalized by the mean over time. Scattering from complete surfaces at incidence angle 25 degrees.

A least squares procedure is utilized to fit $\sigma_{pp}(t_j)$ as [12]

$$\begin{aligned}\frac{\sigma_{pp}(t_i)}{\overline{\sigma}_{pp}} =\ & c_0 + c_{11}\sin(-c_g(k_B)t) + c_{12}\cos(-c_g(k_B)t) \\ & + c_{21}\sin(\Phi(t)) + c_{22}\cos(\Phi(t)) \\ & + c_{31}\sin(2\Phi(t)) + c_{32}\cos(2\Phi(t)),\end{aligned} \qquad (2)$$

where $\overline{\sigma}_{pp}$ denotes the average of $\sigma_{pp}(t_i)$ over time, $\Phi(t) = \omega_L t$, $\omega_L = \sqrt{gk}$ is the long wave radian frequency, and $c_g(k_B)$ is the group velocity of the short Bragg waves with $k_B = 2k_i\sin(\theta_i)$. Terms involving the latter are a minor effect that produces modulations over longer time scales. The amplitude and phase of the first-order MTF coefficients are then

$$R_1 = \sqrt{c_{21}^2 + c_{22}^2}, \qquad (3)$$
$$\psi_1 = \arctan(c_{22}/c_{21}). \qquad (4)$$

Second order terms in the long wave phase are small and are ignored in what follows. Modulation coefficients R_1 are normalized by the $ak = 0.05$ product when shown in the following.

III. RESULTS AND DISCUSSIONS

A. Intermediate wave effects on "short" wave surface scattering

A key effect for examination is the level of modulation observed when only the "short wave" surface is utilized as a function of the proportion of the intermediate scale surface included in the scattering simulation. The upper plot of Figure 3 plots the normalized modulation coefficient component that is in phase with the long wave height (i.e. $R_1\cos\psi_1$) as a function of the truncation wavenumber of the intermediate waves used in the scattering computation. A standard two-scale model would assume that no variations should be observed as the intermediate waves are changed, because only the hydrodynamic modulation of the Bragg waves by long waves (which does not vary with the cutoff wavenumber) has impact here. Analytical approximations of the hydrodynamic modulation [12] show a value of approximately 4 that is confirmed by the numerical hydrodynamic simulations. The increasing results in the numerical values as the cutoff wavenumber is
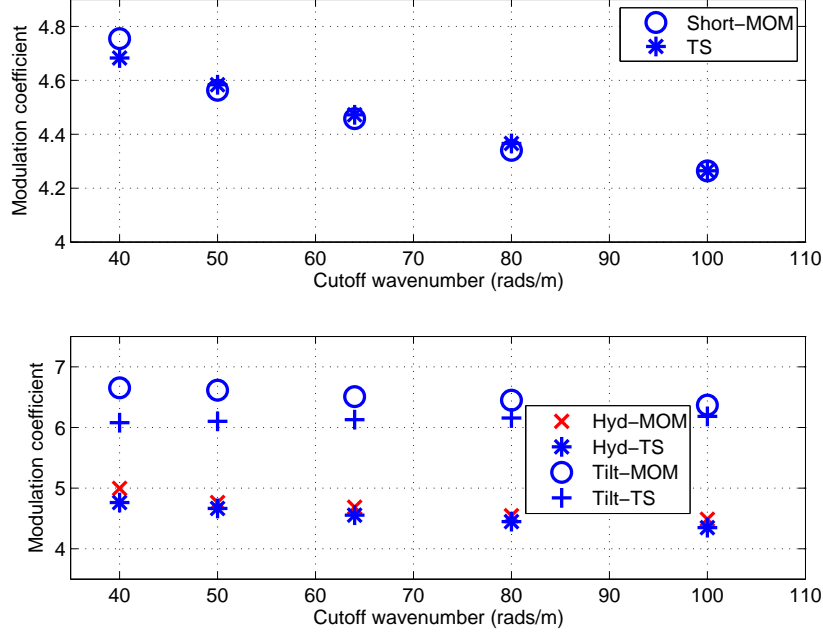
Fig. 3. Normalized first order modulation coefficients as a function of intermediate wave cutoff wavenumber k_c. (a) In phase (i.e. $R_1 \cos \psi_1$) modulation of "short-wave" surfaces (i.e. plot (d) of Figure 1) compared with predictions of a modulated two-scale theory (b) In phase and quadrature modulations for truncated "short wave" surfaces including the sinusoidal long wave, compared with predictions of a modulated two-scale theory

decreased (i.e. more intermediate waves included) show that effects beyond the two-scale theory are occurring.

A three scale model would explain these changes in terms of a locally varying rms slope of surface facets due to hydrodynamic modulations of the intermediate waves by the long wave. An investigation of this hypothesis was performed by computing the slope variance of the surface within the tapered wave footprint and examining time variations of this quantity. The results are presented in Figure 4 for three values of the intermediate wave truncation number. The results clearly show an intermediate wave slope variance that varies with position on the long wave, so that changes in "facet" scattering beyond those associated with Bragg wave hydrodynamic modulations alone can occur. Comparison with the long wave slope in the lower plot shows that the modulations of local slope variances are in phase with the surface height, consistent with hydrodynamic modulations. The fact that these modulations become stronger as more intermediate waves are included explains the increasing trend for smaller cutoff wavenumbers in Figure 3.

B. Two-scale model interpretation

A more quantitative evaluation of Figure 3 was produced by attempting to predict observed changes in the modulation coefficient using a local (not global) two-scale theory. Facet radar cross section values in this model were computed using the Bragg scattering RCS "tilted" (i.e. averaged) over the slope variance of the intermediate waves. Modulations were computed by spatially stepping over the long wave surface height, applying the two-scale model at each facet location using the computed spatially varying intermediate wave slope variance, and numerically extracting the resulting first order
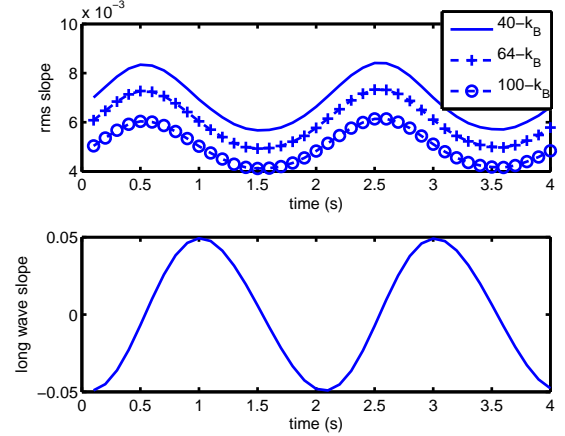

Fig. 4. Local slope variance of intermediate waves as a function of time for three truncation wavenumbers described as a lower cutoff to k_B=210 rads/m.

modulation coefficient. Results from this approach are plotted in the upper plot of Figure 3, and show a reasonable match to the numerical data. In this process, the numerically obtained modulation coefficient of the intermediate wave slope variance was increased by 20 percent in order to produce the results shown; the reason for this requirement is currently under investigation. Nevertheless, the results provide credence to a three-scale model interpretation of modulation mechanisms in sea backscattering.

C. Results including tilt effects

The lower plot of Figure 3 is identical to the upper plot, except that the results are obtained from scattering simulations including the sinusoidal long waves. In this case, tilt effects

in phase with the long wave slope occur, so that both the in-phase ($R_1 \cos \psi_1$, "hydrodynamic") and quadrature phase ($R_1 \sin \psi_1$, "tilt") components are plotted. Observed hydrodynamic modulations are similar to those for the short wave only surfaces, and again are reasonably matched by the modulated two-scale theory as in the upper plot. Tilt modulations are also reasonably matched by the modulated two-scale theory (in this case taking variations in the local incidence angle into account), although the variations observed in the numerical data are somewhat larger than those predicted by the theory. Note a small variation in the tilt modulation is consistent with the three scale theory, as increasing intermediate wave slopes decrease the sensitivity of the facet cross section to tilt effects. Continued analyses are in process to assess the observed differences between the numerical data and three scale model predictions.

IV. Concluding remarks

A numerical approach for performing detailed investigations of modulation mechanisms in sea backscattering was described. While combined numerical electromagnetic/hydrodynamic studies for sea scattering have been presented in the past, the new aspect of this work involves repeated simulations as the spectral content of the surface is varied in order to separate long wave tilting, Bragg wave hydrodynamic, and intermediate wave slope variance modulations. The results obtain support a three-scale model interpretation of this process, although moderate differences with a three-scale model prediction were observed that are currently under investigation.

References

[1] W. C. Keller and J. W. Wright, "Microwave scattering and the straining of wind-generated waves," *Radio Sci.*, vol. 10, pp. 139–147, 1975.

[2] W. Alpers and K. Hasselmann, "The two-frequency microwave technique for measuring ocean-wave spectra from an airplane or statellite," *Boundary Layer Meteorol.*, vol. 13, pp. 215–230, 1978.

[3] W. J. Plant, "The modulation transfer function: concept and applications," in *Radar Scattering from Modulated Wind Waves* (G. J. Komen and W. A. Oost, eds.), pp. 155–172, Norwell, Mass.: Kluwer Academic, 1989.

[4] T. Hara and W. J. Plant, "Hydrodynamic modulation of short wind-wave spectra by long waves and its measurement using microwave backscatter," *J. Geophys. Res.*, vol. 99, no. C5, pp. 9767–9784, 1994.

[5] R. Romeiser, A. Schmidt, and W. Alpers, "A three-scale composite surface model for the ocean wave-radar modulation transfer function," *J. Geophys. Res.*, vol. 99, no. C5, pp. 9785–9801, 1994.

[6] A. Schmidt, V. Wismann, R. Romesier, and W. Alpers, "Simultaneous measurements of the ocean wave-radar modulation transfer function at L, C, and X bands from the research platform Nordsee," *J. Geophys. Res.*, vol. 100, no. C5, pp. 8815–8827, 1995.

[7] C. L. Rino, T. L. Crystal, A. K. Koide, H. D. Ngo, and H. Guthart, "Numerical simulation of backscatter from linear and nonlinear ocean surface realizations," *Radio Sci.*, vol. 26, no. 1, pp. 51–71, 1991.

[8] J. V. Toporkov and G. S. Brown, "Numerical simulations of scattering from time-varying, randomly rough surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 4, pp. 1616–1625, 2000.

[9] J. T. Johnson, J. V. Toporkov, and G. S. Brown, "A numerical study of backscattering from time-evolving sea surfaces: comparison of hydrodynamic models," *IEEE Trans. Geosci. Remote Sens.*, vol. 39, no. 11, pp. 2411–2420, 2001.

[10] A. R. Hayslip, J. T. Johnson, and G. R. Baker, "Further numerical studies of backscattering from time evolving non-linear sea surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 41, no. 10, pp. 2287–2293, 2003.

[11] B. J. West, K. Brueckner, and R. S. Janda, "A numerical method for sea surface hydrodynamics," *J. Geophys. Res.*, vol. 92, pp. 11803–11824, 1987.

[12] G. D. Pan and J. T. Johnson, "A numerical study of the modulation of short sea waves by longer waves," *IEEE Trans. Geosci. Remote Sens.*, vol. 44, no. 10, pp. 2880–2889, 2006.

[13] D. A. Kapp and G. S. Brown, "A new numerical method for rough-surface scattering calculations," *IEEE Trans. Antennas Propagat.*, vol. 44, no. 5, pp. 711–721, 1996.

[14] H. T. Chou and J. T. Johnson, "A novel acceleration algorithm for the computing of scattering from rough surfaces with the forward-backward method," *Radio Sci.*, vol. 33, no. 5, pp. 1277–1287, 1998.

[15] H. T. Chou and J. T. Johnson, "Formulation of forward-backward method using novel spectral accleration for the modeling of scattering from impedance rough surfaces," *IEEE Trans. Geosci. Remote Sens.*, vol. 38, no. 1, pp. 605–607, 2000.

[16] J. V. Toporkov and M. A. Sletten, "Statistical properties of low-grazing range-resolved sea surface backscatter generated through two-dimensional direct numerical simulations," *IEEE Trans. Geosci. Remote Sens.*, vol. 45, no. 5, pp. 1181–1197, 2007.

[17] E. I. Thorsos and D. R. Jackson, "The validity of the perturbation approximation for rough surface scattering using a Gaussian roughness spectrum," *J. Acoust. Soc. Am.*, vol. 66, no. 1, pp. 261–277, 1989.