A Physically-Based Preconditioner for Quasi-Planar Scattering Problems

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Abstract—A physically-based preconditioner for iterative method of moments solutions of quasi-planar penetrable surface scattering is presented. The required preconditioner inverse operation is computed in order $N \log N$ operations by first representing the right-hand side vector to be multiplied by the preconditioner inverse in terms of its plane wave spectrum. Individual plane wave amplitudes are then combined with appropriate Fresnel reflection coefficients to generate the solution for induced currents, and transformed back to the spatial domain. The proposed technique is applied in simulations of scattering from rough surfaces, and the number of iterations required to obtain a convergent solution is compared to that achieved by a banded-matrix preconditioning method. The results show that the physically-based preconditioner produces rapid convergence for surfaces with moderate heights and slopes.

Index Terms—Numerical methods, rough surface scattering.

I. INTRODUCTION

SCATTERING from quasi-planar surfaces, including rough surfaces, planar devices, etc., is of interest in many fields of applied optics and electromagnetics. Numerical methods have been applied extensively to solve these problems (see for example [1]-[9]), primarily based on a method of moments (MOM) formulation. The use of the method of moments results in a matrix equation which can be solved to obtain induced currents on the surface. However, direct solution of this matrix equation by LU decomposition requires an $O(N^3)$ operation, where N is the number of unknowns, making the complexity impractical for large-scale problems. As a result, numerous iterative schemes have been developed and applied to reduce the complexity to $O(N^2)$, with further reductions in operation count possible through the incorporation of fast algorithms for computing the matrix-vector multiply (e.g. [8]).

Even for such fast algorithms, the CPU time needed for solving a quasi-planar surface scattering problem remains directly proportional to the number of iterations required by the iterative solver. A recent work [9] has documented convergence problems for penetrable (i.e. dielectric) surfaces in some cases, especially those involving weak dielectric contrast, when using commonly applied iterative algorithms. Iterative solver convergence can be improved by preconditioning the system with a matrix that approximates the original MOM matrix while remaining easily invertible. Improvements in iterative algorithm convergence can also be achieved in some situations by varying the choice of integral equations that is

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solved [10]; however even in such cases preconditioners may remain useful to achieve further reductions in CPU time.

It has been shown previously [4] that iterative methods such as the forward-backward method (FBM)/method of ordered multiple interactions (MOMI) [2], [3] are equivalent to the use of a block Jacobi preconditioner, while the banded matrix iterative approach (BMIA) [5] is equivalent to banded matrix preconditioning [6]. The former case in each iteration successively neglects coupling to a specific observation point on the surface from points on either the "forward" or "backward" portions of the surface, while the latter in each iteration neglects contributions to an observation point from source points that are outside a specified "strong" distance from the observation point. Both cases use a preconditioner that retains some portions of the original MOM matrix while replacing many of the original MOM matrix elements with zero (resulting in an upper- or lower- triangular preconditioner matrix for the FBM/MOMI or in a banded matrix for the BMIA.) Both are advantageous because the preconditioner can be inverted easily using backsubstitution. However the computational costs of the triangular matrix solution remain $O(N^2)$, and the banded matrix may neglect important coupling among surface points unless the "bandwidth" of the banded matrix is increased, which increases the cost of the banded matrix solution. It is also to be expected that the required bandwidth of a banded matrix solution will increase at least slowly as the surface length (i.e. N) is increased, since the sum of individual small contributions among widely separated points increases as the number of widely separated points is increased.

Instead of structurally approximating the original matrix, this paper presents a physically-based preconditioner through the use of a quasi-planar assumption. The use of this solution is motivated by the fact that preconditioner matrix inversion is essentially a mathematical representation of an approximate solution of the electromagnetic scattering problem. The proposed physically-based preconditioner inverse is implemented on a vector that represents a modified incident field, and uses a fast Fourier transform to decompose this vector in terms of a plane wave spectrum. Individual plane waves are then combined with appropriate Fresnel reflection coefficients to generate the corresponding induced currents, with these currents then transformed back to the spatial domain to produce the result of the inverse preconditioner multiply operation. The approach is similar to that used in the Operator Expansion Method (OEM, [11]) approximate theory of perfectly conducting rough surface scattering, but generalized for penetrable surfaces. Although the use of Fresnel reflection coefficients implies that

the surface is approximated as being flat, results using the OEM have shown that a small slope, rather than small height, approximate theory results. This is because the technique is applied for computing the coupling between pairs of points, which is well approximated even for large height surfaces of moderate slope as that of a flat surface for widely separated pairs of points.

The next section presents a representative integral equation formulation and the resulting matrix equation, and the proposed physically-based preconditioner is then discussed in Section III. In Section IV, the performance of the technique is presented through comparison with the banded-matrix preconditioning method for randomly rough surfaces with various heights, slopes, and lengths. While the discussion to follow focuses on the one dimensional surface case for simplicity (a two dimensional scattering problem), the method is easily extended to three dimensional scattering problems and has already been applied (but not described in detail) in three dimensional scattering results presented in the literature [12].

II. FORMULATION

Consider a plane wave impinging upon a one-dimensional penetrable rough surface of length L with surface profile $z = f(x)$. For incidence in the $x - z$ plane, Maxwell's equations decouple into dual equations for TE and TM waves, and a scalar Kirchhoff diffraction integral in terms of E_y for the TE case and H_y for the TM case can be applied using the Hankel function form of the Green's function. Equations for a dielectric surface medium are

$$
\begin{aligned}
\frac{\psi(\bar{r})}{2} &= \psi_{\text{inc}}^{(1)} + \int dS' \\
&\quad \cdot \left\{ \psi(\bar{r}') \frac{\partial g_1(\bar{r}, \bar{r}')}{\partial n} - g_1(\bar{r}, \bar{r}') \left[\frac{\partial \psi(\bar{r}')}{\partial n} \right]_1 \right\}
\end{aligned} \quad (1)
$$

$$
\begin{aligned}
\frac{\psi(\bar{r})}{2} &= \psi_{\text{inc}}^{(2)} - \int dS' \\
&\quad \cdot \left\{ \psi(\bar{r}') \frac{\partial g_2(\bar{r}, \bar{r}')}{\partial n} - g_2(\bar{r}, \bar{r}') \left[\frac{\partial \psi(\bar{r}')}{\partial n} \right]_2 \right\}
\end{aligned} \quad (2)
$$

where ψ is E_y for the TE case with incident field $E_{y,\text{inc}}$, and H_y for the TM case with incident field $H_{y,\text{inc}}$. The 2-D Green's function is given by

$$
g_j(\bar{r}, \bar{r}') = \frac{i}{4} H_0^{(1)}(k_j |\bar{r} - \bar{r}'|) \quad (3)
$$

where $k_j = \omega \sqrt{\mu_0 \epsilon_j}$ is the propagation constant in medium j. The domain of integration in equations (1) and (2) is the surface profile S' and a principal value integration is implied. Continuity of tangential field components E_y and H_y is implicit in the above formulations with continuity of the along profile field component yielding

$$
\left[\frac{\partial E_y(\bar{r}')}{\partial n} \right]_1 = \left[\frac{\partial E_y(\bar{r}')}{\partial n} \right]_2 \quad (4)
$$

for the TE case (nonmagnetic medium) and

$$
\left[\frac{\partial H_y(\bar{r}')}{\partial n} \right]_1 = \frac{\epsilon_1}{\epsilon_2} \left[\frac{\partial H_y(\bar{r}')}{\partial n} \right]_2 \quad (5)
$$

for the TM case. Note that the integral equations (1) and (2) include incident fields impinging both from above and below the surface ($\psi_{\text{inc}}^{(1)}$ and $\psi_{\text{inc}}^{(2)}$). In many applications, the field incident from below the surface is assumed to be zero. However the inclusion of fields incident from below the boundary will be important when the preconditioning algorithm is developed in the next Section.

The above formulation gives two integral equations in two unknowns (ψ and $\partial \psi / \partial n$) on the surface profile. Applying a point matching method of moments technique results in a matrix equation in terms of the unknown pulse-basis function expansion coefficients of these fields. For the TE case, the matrix equation can be written as

$$
\overline{\mathbf{Z}} \mathbf{I} = \mathbf{V} \quad (6)
$$

Here the size of the impedance matrix $\overline{\mathbf{Z}}$ is $2N \times 2N$, where N is the number of surface points. The impedance matrix has the form

$$
\overline{\mathbf{Z}} = \begin{bmatrix} \overline{\mathbf{Z}}_{11} & \overline{\mathbf{Z}}_{12} \\ \overline{\mathbf{Z}}_{21} & \overline{\mathbf{Z}}_{22} \end{bmatrix} \quad (7)
$$

The first N rows ($\overline{\mathbf{Z}}_{11}$ and $\overline{\mathbf{Z}}_{12}$) correspond to the integral equation above the surface (1), while the last N rows ($\overline{\mathbf{Z}}_{21}$ and $\overline{\mathbf{Z}}_{22}$) represent the integral equation below the surface (2). Elements of the impedance matrix are proportional to Hankel functions of order zero or order one evaluated at arguments corresponding to distances between individual points on the surface profile. Appropriate self terms for these integrals equations are described in the literature [1]-[9]. The vectors $\mathbf{I}$ and $\mathbf{V}$ are given by

$$
\mathbf{I} = \begin{bmatrix} \mathbf{M} \\ \mathbf{J} \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} \mathbf{E}_{\text{inc}}^{(1)} \\ \mathbf{E}_{\text{inc}}^{(2)} \end{bmatrix} \quad (8)
$$

where the vectors $\mathbf{M}$ and $\mathbf{J}$ contain the expansion coefficients of the unknown magnetic surface current density (proportional to ψ in the TE case) and the electric surface current density (proportional to $\frac{\partial E_y(\bar{r}')}{\partial n}$ in the TE case) respectively. The right-hand side vectors, $\mathbf{E}_{\text{inc}}^{(1)}$ and $\mathbf{E}_{\text{inc}}^{(2)}$, contain the incident electric fields from above and below the surface evaluated at points on the surface profile. Note that a matrix equation for TM case can be obtained similarly with incident magnetic fields on the right hand side.

Numerous stationary or non-stationary techniques have been utilized to solve the MOM matrix equation iteratively. Examples of the non-stationary techniques include quasi-minimal residual (QMR), bi-conjugate gradient stabilized (Bi-CGSTAB) [13], and generalized conjugate residual (GCR) methods [6]. Comparison of iterative solutions of scattering from rough surfaces have been discussed in [7], [9]. Iterative methods can be applied to (6) directly or to the preconditioned system

$$
\overline{\mathbf{C}}^{-1} \overline{\mathbf{Z}} \mathbf{I} = \overline{\mathbf{C}}^{-1} \mathbf{V} \quad (9)
$$

where the preconditioning matrix $\overline{\mathbf{C}}$ is chosen to be invertible easily while approximating the original impedance matrix $\overline{\mathbf{Z}}$. As described in [13], solution of equation (9) can be accomplished by modifications to the standard iterative algorithm so that only an additional routine for computing $\overline{\mathbf{C}}^{-1} \tilde{\mathbf{V}}$, where $\tilde{\mathbf{V}}$ is an arbitrary vector, is required.

III. PHYSICALLY-BASED PRECONDITIONER

Because the preconditioner is designed to approximate the original MOM matrix, it is in fact a mathematical representation of an approximate solution of the electromagnetic scattering problem. Therefore any approximate electromagnetic method can be applied, so long as it can be computed efficiently. Here a quasi-planar approximation is described for the TE case; the same procedure is easily modified for the TM case or three dimensional scattering problems.

During the iterative solution, the algorithm provides an arbitrary vector $\tilde{\mathbf{V}}$ to be multiplied by the preconditioner inverse. Even when the original right hand side vector $\mathbf{V}$ contained only fields incident from above the surface profile, the new "right hand side" vector $\tilde{\mathbf{V}}$ produced by the iterative algorithm in general will contain electric fields incident both from above and below the boundary. Separating this vector into the first N and second N rows yields "modified" incident fields evaluated at points $x_n = n\Delta x$ on the surface profile:

$$\tilde{\mathbf{E}}_{\text{inc}}^{(1)} = \left[\tilde{E}_n^{(1)}\right] \qquad \tilde{\mathbf{E}}_{\text{inc}}^{(2)} = \left[\tilde{E}_n^{(2)}\right] \qquad (10)$$

where $n = 0, 1, \ldots, N-1$. These modified incident fields can be expressed in terms of a spectral representation as

$$\tilde{E}_{\text{inc}}^{(1)}(x) = \int_{-\infty}^{\infty} \tilde{e}_{\text{inc}}^{(1)}(k_x) e^{ik_x x} e^{-ik_z z} dk_x \qquad (11)$$

$$\tilde{E}_{\text{inc}}^{(2)}(x) = \int_{-\infty}^{\infty} \tilde{e}_{\text{inc}}^{(2)}(k_x) e^{ik_x x} e^{ik_z z} dk_x \qquad (12)$$

The Rayleigh hypothesis is invoked in the above equations because it is assumed that fields incident from above or below the surface are composed only of plane waves traveling in the $-z$ direction or $+z$ directions, respectively.

For a quasi-planar surface, the exponent $e^{ik_z z}$ is approximately equal to 1, so that equations (11) and (12) can be approximated as Fourier transform relationships and inverted as

$$\tilde{e}_{\text{inc}}^{(1,2)}(k_x) = \frac{1}{2\pi} \int_0^L \tilde{E}_{\text{inc}}^{(1,2)}(x) e^{-ik_x x} dx \qquad (13)$$

Given a set of incident plane waves, induced currents can now be obtained by solving the planar surface reflection problem for each plane wave. The resulting total y component of the electric field in the spectral domain is given by

$$\tilde{m}(k_x) = (1+\Gamma)\tilde{e}_{\text{inc}}^{(1)}(k_x) - (1-\Gamma)\tilde{e}_{\text{inc}}^{(2)}(k_x) \qquad (14)$$

while the electric current is given by

$$\tilde{j}(k_x) = \frac{k_z^{(1)}}{\omega\mu_0}(1-\Gamma)\,\tilde{e}_{\text{inc}}^{(1)}(k_x) + \frac{k_z^{(2)}}{\omega\mu_0}(1+\Gamma)\,\tilde{e}_{\text{inc}}^{(2)}(k_x) \qquad (15)$$

Here the Fresnel reflection coefficient for the TE case is given by

$$\Gamma = \frac{k_z^{(2)} - k_z^{(1)}}{k_z^{(2)} + k_z^{(1)}} \qquad (16)$$

with

$$k_z^{(1,2)} = \sqrt{k_{1,2}^2 - k_x^2} \qquad (17)$$

These currents in the spectral domain can then be transformed back to yield the solution to the preconditioned system

$$\tilde{M}(x) = \int_{-\infty}^{\infty} \tilde{m}(k_x) e^{ik_x x} dk_x \qquad (18)$$

$$\tilde{J}(x) = \int_{-\infty}^{\infty} \tilde{j}(k_x) e^{ik_x x} dk_x \qquad (19)$$

This physically-based preconditioner can also be expressed in matrix form. When discretized, the spectral representation equation becomes a fast Fourier transform of the incident fields on the surface, and the inverse preconditioner matrix can be written as

$$\overline{\mathbf{C}}^{-1} = \begin{bmatrix} \overline{\mathbf{F}}^{-1} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{F}}^{-1} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{T}}_{11} & \overline{\mathbf{T}}_{12} \\ \overline{\mathbf{T}}_{21} & \overline{\mathbf{T}}_{22} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{F}} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{F}} \end{bmatrix} \qquad (20)$$

where $\overline{\mathbf{F}}$ is the fast Fourier transform matrix and the four submatrices $\overline{\mathbf{T}}$ are diagonal with elements corresponding to (14) and (15). As can be seen, this preconditioner is easy to implement by using an FFT routine and scalar multiplication, and requires an operations count of $O(N \log N)$ due to the FFT algorithm. The preconditioner requires storage only of an additional array of size N. Because it avoids structural approximations (i.e. setting matrix elements equal to zero) the proposed preconditioner attempts to approximate coupling among all pairs of surface points. However, due to the Rayleigh hypothesis and quasi-planar assumption, the degree to which it approximates the original MOM matrix should degrade as surface heights and slopes are increased.

IV. NUMERICAL RESULTS

The performance of the proposed physically-based preconditioner is presented in this section. Although preconditioners can be applied to any iterative method, the bi-conjugate gradient stabilized (Bi-CGSTAB) [7],[9], [13] algorithm is used in this study. The convergence rate of Bi-CGSTAB with the physically-based preconditioner is compared to those achieved by the banded matrix preconditioner and without preconditioning.

The problem considered involves scattering from dielectric random rough surfaces described as Gaussian random processes and with either a Gaussian correlation function or a Pierson-Moskowitz (ocean-like) power spectral density. The upper and lower media have relative permittivities 1 and $\epsilon_r = 14 + 14i$, respectively. The surface is illuminated by a TE-polarized tapered plane wave with tapering parameter $L/6$ [2] impinging at $55°$ from normal incidence. The default length of the surface is 2000λ (where λ is the electromagnetic wavelength in free space), and 16384 surface points are used so that each pulse-basis function has a spatial extent of 0.1222λ. Bandwidths of 32 points and 128 points (i.e. coupling from points within horizontal distances $\pm 3.9\lambda$ and $\pm 15.6\lambda$ of an observation point are included respectively) are used for the banded matrix preconditioner.

Convergence of the iterative solver for surfaces with various rms height h and correlation length l parameters is examined through the normalized residual of the solution:

$$R_N = \frac{\|\mathbf{V} - \overline{\mathbf{Z}}\mathbf{I}^{(j)}\|}{\|\mathbf{V}\|} \qquad (21)$$

(a)

(b)
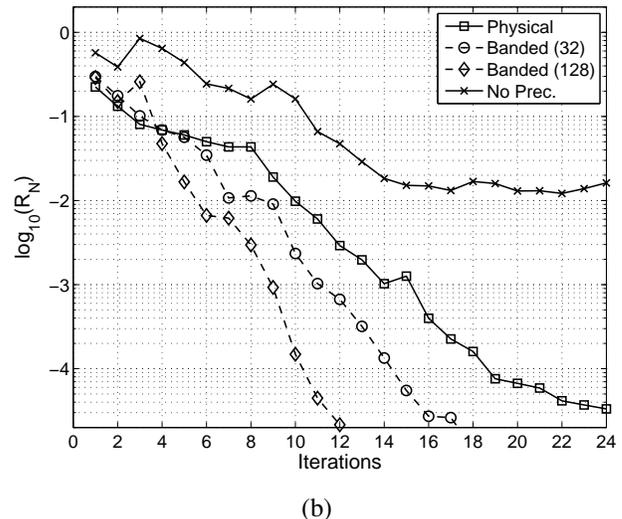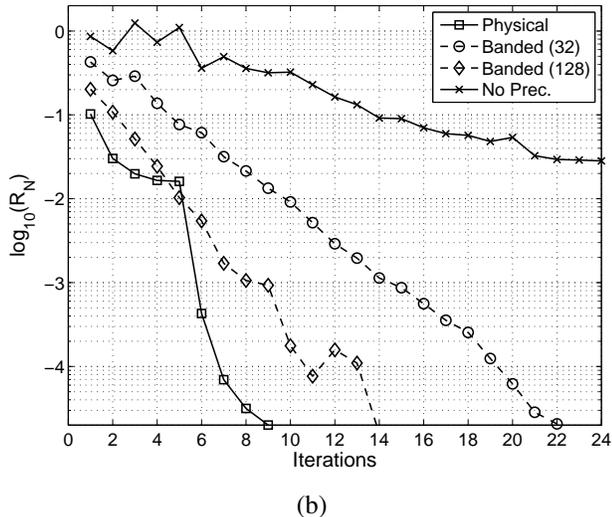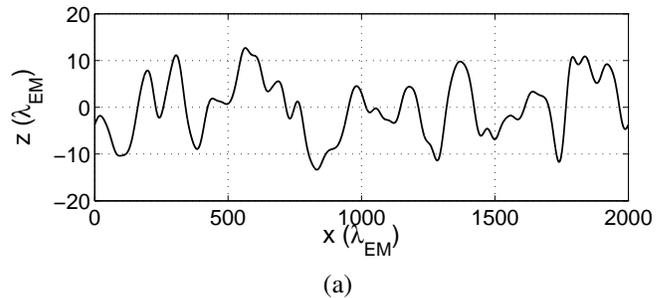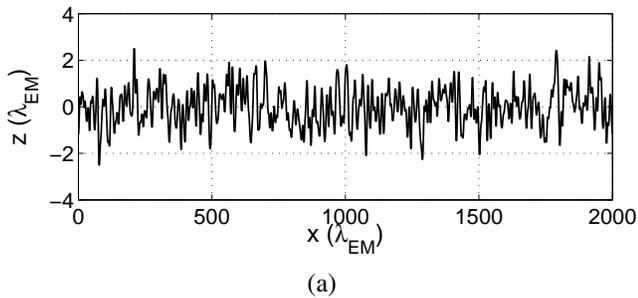
Fig. 1. (a) Surface profile of a Gaussian random surface with rms height 0.77λ and correlation length 5.13λ. (b) Convergence history of the iterative solver with varying preconditioners

(a)

(b)

Fig. 2. (a) Surface profile of a Gaussian random surface with rms height 5.97λ and correlation length 39.8λ. (b) Convergence history of the iterative solver with varying preconditioners

where $\mathbf{I}^{(j)}$ is the solution for induced currents on the jth iteration of the iterative solver.

Fig. 1 compares the convergence history of the physically-based preconditioner with those of the banded matrix and no preconditioner for Gaussian correlated surfaces with rms height 0.77λ and correlation length 5.13λ ($h/l = 0.15$). The surface profile is also shown. As expected for surfaces with relatively small height, the physically-based preconditioner converges much faster than the 32 point bandwidth preconditioner, reaching $R_N = 10^{-4}$ in 7 iterations compared to 20 iterations. When the bandwidth is increased to 128 points, the banded matrix preconditioner includes coupling among more points on the surface, and therefore converges faster, reaching the specified residual after 11 iterations. With no preconditioner, the iterative solver converges very slowly as discussed previously [9].

Convergence histories for Gaussian correlated surfaces with an rms height of 5.97λ and correlation length 39.8λ (same $h/l = 0.15$) are shown in Fig. 2. Due to the larger surface height so that the quasi-planar assumption is less valid, the physically-based preconditioner's convergence is less rapid than that of the banded matrix preconditioner. The physically-based preconditioner needs 19 iterations to reach $R_N = 10^{-4}$, while the banded matrix preconditioner needs 15 iterations for the bandwidth of 32 points, and 11 iterations for the bandwidth of 128 points. Both techniques however provide improvements

TABLE I
NUMBER OF ITERATIONS AND CPU TIME REQUIRED TO REACH THE NORMALIZED RESIDUAL OF 10^{-4} ($h/l = 0.15$)

	$h = 0.77\lambda$		$h = 5.97\lambda$	
	iterations	CPU time	iterations	CPU time
Physical	7	98 mins	19	256 mins
Banded (32)	20	268 mins	15	203 mins
Banded (128)	11	158 mins	11	158 mins

over the case with no preconditioning, indicating that the physically based preconditioner remains useful even for surfaces with large heights.

Table I compares the number of iterations and total CPU time requirements for each preconditioner applied to the surfaces shown in Figs. 1 and 2. Codes were run on a 3.0 GHz Pentium 4 processor. Unlike the physically-based preconditioner, the time for the banded matrix preconditioner also includes the setup time for LU decomposition of the banded matrix which scales linearly with the bandwidth; storage requirements for the banded matrix preconditioner are on the order of bN where b is the bandwidth used. A larger bandwidth is needed in order to improve the convergence which in turn increases the number of operations for both LU decomposition and the preconditioner inverse multiply. The CPU time per iteration of the physically-based preconditioner
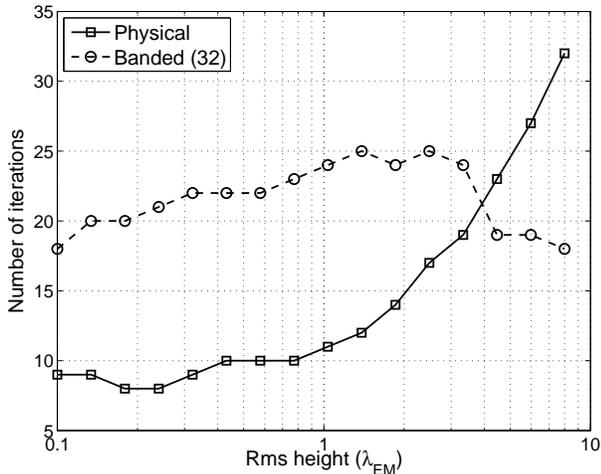
Fig. 3. Number of iterations required to reach the normalized residual of 2×10^{-5} for surfaces with rms height to correlation length ratio (h/l) of 0.15

Fig. 4. Number of iterations required to reach the normalized residual of 2×10^{-5} for surfaces with rms height of 0.35λ

and the banded matrix preconditioner with bandwidth 32 points are comparable, so that the difference in total times between the two is due to the setup time required for the banded matrix preconditioner and the number of iterations needed to reach convergence.

Convergence is next examined for surface profiles as the rms height is increased for a fixed h/l of 0.15. The number of iterations required to reach $R_N = 2 \times 10^{-5}$ for both preconditioners is plotted in Fig. 3. For the banded matrix preconditioner, the bandwidth of 32 points is selected for comparison. Since the computational time per iteration is comparable to the physically-based preconditioner, the required number of iterations will approximately reflect the total time used by the preconditioners. Results for the no preconditioner case are not given here due to the very slow convergence rate observed which made computations of the necessary results difficult. The physically-based preconditioner takes advantage of surface properties in the smaller height cases and therefore converges much faster when the rms height is small to moderate. The number of iterations for the banded matrix preconditioner is less affected by the surface profile. For the parameters chosen here, the physically based approach loses its advantages over the banded matrix technique at a surface rms height of around 4 wavelengths.

Fig. 4 plots the number of iterations required to reach $R_N = 2 \times 10^{-5}$ as h/l increases (the correlation length decreases) at a fixed rms height of 0.35λ . The result shows that the physically-based preconditioner remains advantageous compared to the banded matrix method up to h/l values of 0.6 (extremely large slopes) for these parameters. Note that the small correlation lengths for the larger h/l values in Figure 4 include very fine scale roughness features in the surface profile. The number of iterations needed by the banded matrix preconditioner shows only slight increases with h/l.

Fig. 5 examines the number of iterations to reach $R_N = 2 \times 10^{-5}$ as a function of surface length for a fixed rms height of 0.35λ and a fixed correlation length of 2.33λ ($h/l = 0.15$).
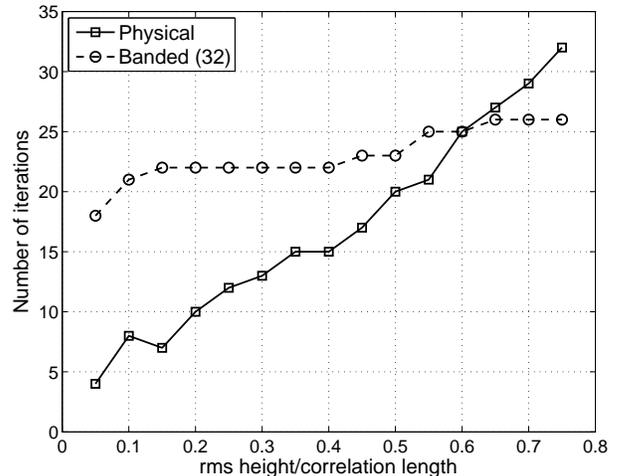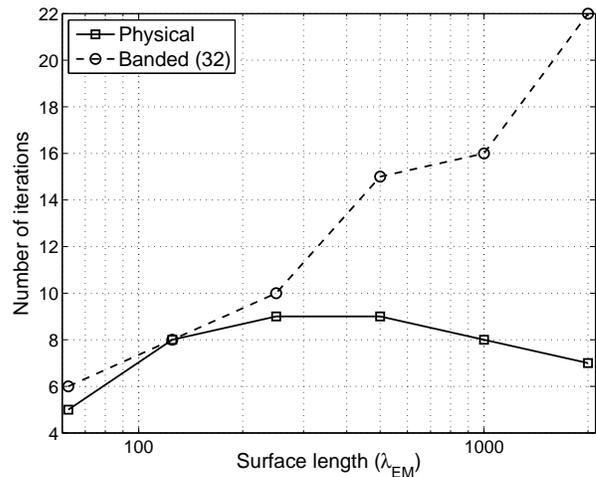

Fig. 5. Number of iterations required to reach normalized residual of 2×10^{-5} as a function of surface length (rms height 0.35λ, correlation length of 2.33λ)

The discretization 0.122λ is used for all surface lengths so that changes in length correspond to changes in N. The number of iterations required by the banded matrix preconditioner increases with the surface length, due to an increased importance of "weak" coupling as the surface becomes longer. A larger bandwidth (and CPU cost) would be needed in order to improve the convergence for longer surfaces. In contrast, the convergence of the physically-based preconditioner remains roughly the same because the approach captures approximate coupling among all pairs of points on the surface regardless of surface length.

Finally, as an example of application to multi-scale surfaces, scattering from ocean-like surfaces generated as realizations of the Pierson-Moskowitz spectrum [14] at various wind speeds is considered. A surface size of 2000λ (200 m) at 3 GHz with 16384 surface points is chosen for this problem. This surface length can resolve all length scales of the Pierson-
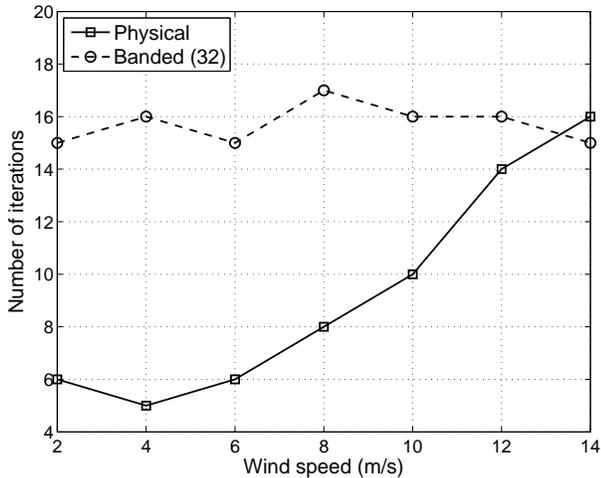
Fig. 6. Number of iterations required to reach normalized residual of 10^{-4} for scattering from a realization of the Pierson-Moskowitz spectrum as a function of wind speed.

Moskowitz spectrum for wind speeds up to approximately 14 m/s. The relative permittivities of the upper and lower media are 1 and $\epsilon_r = 14 + 14i$ respectively. Although $14 + 14i$ is less than the actual permittivity of sea water at 3 GHz ($60.9 + 40.1i$) [15], the difficulties of computing scattering with from media with high permittivities (which are best modeled as impedance surfaces) motivates this choice; this difference in permittivities is not expected to have a major impact on the influence of preconditioning algorithms. The number of iterations required to reach the normalized residual of 10^{-4} is plotted in Fig. 6 for both preconditioners. The result shows that the physically-based preconditioner is advantageous for most of the wind speeds shown in the figure. The convergence of the banded matrix preconditioner remains roughly the same for all wind speeds, and is comparable to that of the physically-based preconditioner at wind speed of 14 m/s (the rms height is approximately 10.58λ). These results show that the physically-based preconditioner remains useful for multi-scale surfaces.

V. CONCLUSION

A physically-based preconditioner for quasi-planar scattering problems is presented in this paper. Unlike the banded matrix approach, this preconditioner takes advantage of surface properties in order to compute approximate coupling among all pairs of surface points. The numerical results confirm that the approach works well for surfaces with small to moderate heights and slopes. The convergence rate also remains roughly the same as the surface length increases. The idea presented here can be extended directly to other integral equation formulations, and has already been applied in three dimensional scattering problems [12].

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