Fourth- and higher-order small-perturbation solution for scattering from dielectric rough surfaces

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A recursive solution of the small-perturbation method for rough surface scattering is presented. These results permit fourth- and higher-order corrections to rough surface scattering coefficients to be determined in a form that explicitly separates surface and electromagnetic properties. Sample results are presented for the fourth-order correction to the specular reflection coefficient of a rough surface and the sixth-order correction to incoherent scattering cross sections. © 2003 Optical Society of America

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1. INTRODUCTION

The small-perturbation method[1] (SPM) for scattering from a rough surface involves a perturbation series in surface height for scattered fields. Explicit expressions up to second order in surface height were provided in Ref. 1; recently explicit expressions up to third order have also been presented.[2] These explicit expressions are advantageous in that surface and electromagnetic properties are decoupled into a product of surface spectral components with electromagnetic "kernel functions" that are independent of surface properties. The computation of statistical averages required for stochastic surfaces is greatly simplified in this case. However, because the explicit expressions for the third-order kernel functions are already quite complex, the complexity of such expressions for fourth- and higher-order terms is expected to make them cumbersome, if not intractable.

Reference 2 also presents a systematic procedure for determining fourth- and higher-order solutions, but the simplified results of this procedure were not provided. Although the systematic procedure described can be applied in a numerical computation of fourth- and higher-order SPM fields for a deterministic surface, the results obtained couple surface and electromagnetic effects, so that a Monte Carlo procedure is needed to compute statistical averages for a stochastic surface.

For scattering from a Gaussian random process surface, average incoherent cross sections are composed of even-order terms in surface height, beginning at second order. The fourth-order term presented in Ref. 2 required knowledge of the third-order SPM field solution. Knowledge of fourth- and higher-order field solutions is needed to compute the sixth-order correction to incoherent cross sections. Although the utility of this correction is debatable for computing scattering from a surface with small to moderate roughness, the kernel functions involved also appear in the small-slope theory of scattering from a rough surface[3] as well as in the small-slope theory of emission from a rough surface.[4,5] Furthermore, the second-order contribution to incoherent cross sections vanishes for cross-polarized backscattering, so that the sixth-order term provides the first correction to the dominant fourth-order result. Information on this correction is important for obtaining knowledge of the convergence of the SPM solution for cross-polarized backscattering.

In this paper the systematic procedure described in Ref. 2 is applied to construct a complete, recursive, and arbitrary-order solution for scattered fields in the SPM method. The forms obtained explicitly separate surface and electromagnetic properties, and a recursive algorithm for the SPM kernel functions is provided to simplify computation of higher-order terms. Sample results from the fourth-order theory are presented in terms of the fourth-order reflection-coefficient correction and the sixth-order correction to incoherent scattering cross sections. The method can be applied in studies of periodic or nonperiodic surfaces that are either deterministic or stochastic.

2. FORMULATION

The basic formulation and notation introduced in Ref. 2 is used below unless otherwise noted. Consider a periodic rough interface $z = f(x, y)$ separating free space from a dielectric region with relative permittivity ϵ. A plane wave is incident on this interface from free space; the resulting scattered and transmitted fields can be completely described in terms of the polarization amplitudes of a set of Floquet modes. Horizontally and vertically polarized scattered mode amplitudes are denoted by $\alpha_{\bar{n}'}$ and $\beta_{\bar{n}'}$, respectively, and $\gamma_{\bar{n}'}$ and $\delta_{\bar{n}'}$ refer to transmitted horizontally and vertically polarized amplitudes, respectively. Here $\bar{n}' = (n', m')$ denotes the mode index of a particular Floquet mode, thus describing the direction of propagation of the corresponding scattered or transmitted field. Note that one can remove the requirement for a periodic interface after the solution is completed by allowing the periods to approach infinity, as in Refs. 1 and 2.

Following the process described in Eqs. (83)–(86) in Ref. 2 but shifting some of the indices appropriately allows the multiple terms in these equations to be combined. Making use of a vector notation, a scattering coefficient $\bar{\zeta} = [\alpha,\ \beta,\ \gamma,\ \delta]^{\mathrm{T}}$ at Nth order ($N \geqslant 1$) can then be expressed as

$$\bar{\zeta}_{\bar{n}'}^{(N)} = \sum_{\bar{n}_1} \sum_{\bar{n}_2} \cdots \sum_{\bar{n}_{N-1}} h_{\bar{n}_1} h_{\bar{n}_2} \ldots h_{\bar{n}_{N-1}} h_{\bar{n}'-\bar{n}_1-\ldots\bar{n}_{N-1}} \times \bar{g}^{(N)}(\bar{n}',\ \bar{n}_1,\ldots,\bar{n}_{N-1}), \tag{1}$$

where $h_{\bar{n}}$ refers to the Fourier coefficients of the surface; note that N of these are included so that the overall term is Nth order in surface height. For $N = 1$ the sums vanish and only a single Fourier coefficient $h_{\bar{n}'}$ multiplies $\bar{g}^{(1)}(\bar{n}')$.

The Nth-order SPM kernel is expressed in terms of lower-order kernels as follows:

$$\bar{g}^{(N)}(\bar{n}',\ \bar{n}_1,\ldots,\bar{n}_{N-1}) = \bar{g}^{(N,0)}(\bar{n}',\bar{n}_1,\ldots,\bar{n}_{N-1}) + \sum_{l=1}^{N-1} [\bar{\bar{\nu}}^{(N-l)}(\bar{n}',\ \bar{n}_s^{(l)},\ \bar{n}_{l+1},\ldots,\bar{n}_{N-1}) \cdot \bar{g}^{(l)}(\bar{n}_s^{(l)},\ \bar{n}_2,\ldots,\bar{n}_l)], \tag{2}$$

where $\bar{n}_s^{(l)}$ is

$$\bar{n}_s^{(l)} = \sum_{i=1}^{l} \bar{n}_i\,. \tag{3}$$

The $\bar{\bar{\nu}}^{(N-l)}(\bar{n}',\ \bar{n},\ \bar{n}_1,\ldots,\bar{n}_{N-l-1})$ quantity above is a four-by-four tensor with elements ν_{ij} at row i and column j, and the kernel-function vector $\bar{g}^{(l)}$ for $l = 1$ to N is a four-element column vector defined analogously to $\bar{\zeta}$. From Ref. 2, elements of the $\bar{\bar{\nu}}^{N-l}$ tensor are

$$\begin{bmatrix} \nu_{11} \\ \nu_{13} \\ \nu_{31} \\ \nu_{33} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_0 \\ A_1 \end{bmatrix} \frac{1}{\kappa_H'} \left(-c_{\bar{n},\bar{n}'} \begin{bmatrix} k_{z1\bar{n}'} + k_{z\bar{n}} \\ -k_{z1\bar{n}'} + k_{z1\bar{n}} \\ -k_{z\bar{n}'} + k_{z\bar{n}} \\ k_{z\bar{n}'} + k_{z1\bar{n}} \end{bmatrix} + \begin{bmatrix} \kappa_{c0} \\ \kappa_{c1} \\ \kappa_{c0} \\ \kappa_{c1} \end{bmatrix} \right),$$

$$\begin{bmatrix} \nu_{22} \\ \nu_{24} \\ \nu_{42} \\ \nu_{44} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_0 \\ A_1 \end{bmatrix} \frac{1}{\kappa_V'} \left(-c_{\bar{n},\bar{n}'} \begin{bmatrix} k_{z1\bar{n}'} + \epsilon k_{z\bar{n}} \\ (-k_{z1\bar{n}'} + k_{z1\bar{n}})\sqrt{\epsilon} \\ (-k_{z\bar{n}'} + k_{z\bar{n}})\sqrt{\epsilon} \\ \epsilon k_{z\bar{n}'} + k_{z1\bar{n}} \end{bmatrix} + \begin{bmatrix} \epsilon\kappa_{c0} \\ \sqrt{\epsilon}\kappa_{c1} \\ \sqrt{\epsilon}\kappa_{c0} \\ \kappa_{c1} \end{bmatrix} \right),$$

$$\begin{bmatrix} \nu_{12} \\ \nu_{14} \\ \nu_{32} \\ \nu_{34} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_0 \\ A_1 \end{bmatrix} \frac{1}{\kappa_H'} \left(-\frac{s_{\bar{n},\bar{n}'}}{k_0} \begin{bmatrix} (-k_0^2 - k_{z1\bar{n}'}k_{z\bar{n}}) \\ (k_1^2 - k_{z1\bar{n}'}k_{z1\bar{n}})/(\sqrt{\epsilon}) \\ (-k_0^2 + k_{z\bar{n}'}k_{z\bar{n}}) \\ (k_1^2 + k_{z\bar{n}'}k_{z1\bar{n}})/(\sqrt{\epsilon}) \end{bmatrix} + \begin{bmatrix} -\kappa_{s0}\dfrac{k_{z1\bar{n}'}}{k_0} \\ -\kappa_{s1}\dfrac{k_{z1\bar{n}'}}{k_1} \\ \kappa_{s0}\dfrac{k_{z\bar{n}'}}{k_0} \\ \kappa_{s1}\dfrac{k_{z\bar{n}'}}{k_1} \end{bmatrix} \right),$$

$$\begin{bmatrix} \nu_{21} \\ \nu_{23} \\ \nu_{41} \\ \nu_{43} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_0 \\ A_1 \end{bmatrix} \frac{1}{\kappa_V'} \left(-\frac{s_{\bar{n},\bar{n}'}}{k_0} \begin{bmatrix} (k_1^2 + k_{z1\bar{n}'}k_{z\bar{n}}) \\ (-k_1^2 + k_{z1\bar{n}'}k_{z1\bar{n}}) \\ (k_0^2 - k_{z\bar{n}'}k_{z\bar{n}})\sqrt{\epsilon} \\ (-k_0^2 - k_{z\bar{n}'}k_{z1\bar{n}})\sqrt{\epsilon} \end{bmatrix} + \begin{bmatrix} \kappa_{s0}\dfrac{k_{z1\bar{n}'}}{k_0} \\ \kappa_{s1}\dfrac{k_{z1\bar{n}'}}{k_0} \\ -\kappa_{s0}\dfrac{k_{z\bar{n}'}}{k_0}\sqrt{\epsilon} \\ -\kappa_{s1}\dfrac{k_{z\bar{n}'}}{k_0}\sqrt{\epsilon} \end{bmatrix} \right), \tag{4}$$

where

$$A_0 = \frac{i^{(N-l)}}{(N-l)!}(k_{z\bar{n}})^{(N-l)},$$

$$A_1 = \frac{i^{(N-l)}}{(N-l)!}(-k_{z1\bar{n}})^{(N-l)},$$

$$\frac{1}{\kappa_H'} = \frac{1}{k_{z\bar{n}'} + k_{kz1\bar{n}'}},$$

$$\frac{1}{\kappa_V'} = \frac{1}{\epsilon k_{z\bar{n}'} + k_{kz1\bar{n}'}}, \tag{5}$$

$$\bar{n}'' = \bar{n} + \bar{n}_1 + \bar{n}_2 \cdots + \bar{n}_{N-l-1}, \tag{6}$$

$$\kappa_{c0} = (N - l)\frac{k_{\rho\bar{n}}}{k_{z\bar{n}}}(k_{\rho\bar{n}'} - c_{\bar{n}',\bar{n}''}k_{\rho\bar{n}''}),$$

$$\kappa_{s0} = (N - l)\frac{k_{\rho\bar{n}}}{k_{z\bar{n}}}k_{\rho\bar{n}''}s_{\bar{n}',\bar{n}''},$$

$$\kappa_{c1} = (N - l)\frac{k_{\rho\bar{n}}}{k_{z1\bar{n}}}(k_{\rho\bar{n}'} - c_{\bar{n}',\bar{n}''}k_{\rho\bar{n}''}),$$

$$\kappa_{s1} = (N - l)\frac{k_{\rho\bar{n}}}{k_{z1\bar{n}}}k_{\rho\bar{n}''}s_{\bar{n}',\bar{n}''}. \tag{7}$$

The $k_{z\bar{n}}$ and $k_{z1\bar{n}}$ terms are the z components of the $\bar{n}$th Floquet mode propagation directions above and below the surface (positive values), respectively, and $k_{\rho\bar{n}}$ is the corresponding transverse component. The $c_{\bar{n},\bar{n}'}$ and $s_{\bar{n},\bar{n}'}$ terms are the cosine and sine functions, respectively, defined in Ref. 2, and $k_1 = \sqrt{\epsilon}k_0$ and $k_0 = 2\pi/\lambda$ is the wavenumber for the incident plane wave of wavelength λ.

Derivations up to this point are valid for both horizontal and vertical incident polarizations. However, the zeroth-order contribution $\bar{g}^{(N,0)}(\bar{n}', \bar{n}_1,\ldots,\bar{n}_{N-1})$ is distinct for the two polarizations. For horizontal incidence,

$$g_\alpha^{(N,0)} = \frac{i^N}{N!}\left(\frac{-k_0}{\kappa_H'}\right)\left[-c_{i,n'}\left(\frac{-k_{z1\bar{n}'}}{k_0}R_{1h}^{(N)} - \frac{k_{zi}}{k_0}R_{2h}^{(N)}\right)\right.$$
$$\left. + NR_{3h}^{(N)}L^c\right],$$

$$g_\beta^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_0}{\kappa_V'}\right)\left[-s_{i,n'}\left(\epsilon R_{1h}^{(N)} + \frac{k_{zi}k_{z1\bar{n}'}}{k_0^2}R_{2h}^{(N)}\right)\right.$$
$$\left. - N\frac{k_{z1\bar{n}'}}{k_0}R_{3h}^{(N)}L^s\right],$$

$$g_\gamma^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_0}{\kappa_H'}\right)\left[-c_{i,n'}\left(\frac{-k_{z\bar{n}'}}{k_0}R_{1h}^{(N)} + \frac{k_{zi}}{k_0}R_{2h}^{(N)}\right)\right.$$
$$\left. - NR_{3h}^{(N)}L^c\right],$$

$$g_\delta^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_1}{\kappa_V'}\right)\left[-s_{i,n'}\left(R_{1h}^{(N)} - \frac{k_{zi}k_{z\bar{n}'}}{k_0^2}R_{2h}^{(N)}\right)\right.$$
$$\left. + N\frac{k_{z\bar{n}'}}{k_0}R_{3h}^{(N)}L^s\right], \tag{8}$$

where

$$\bar{n}^* = \bar{n}_1 + \bar{n}_2 \cdots + \bar{n}_{N-1}, \tag{9}$$

$$L^c = (k_{\rho\bar{n}'} - c_{\bar{n}',\bar{n}^*}k_{\rho\bar{n}^*}), \qquad L^s = (s_{\bar{n}',\bar{n}^*}k_{\rho\bar{n}^*}), \tag{10}$$

and

$$R_{1h}^{(N)} = \Gamma_H(k_{zi})^N + (-k_{zi})^N - (1 + \Gamma_H)(-k_{z1i})^N,$$

$$R_{2h}^{(N)} = \Gamma_H(k_{zi})^N - (-k_{zi})^N + \frac{k_{z1i}}{k_{zi}}(1 + \Gamma_H)$$
$$\times (-k_{z1i})^N,$$

$$R_{3h}^{(N)} = -\frac{k_{\rho i}}{k_0}[(-k_{zi})^{N-1} + \Gamma_H(k_{zi})^{N-1} - (1 + \Gamma_H)$$
$$\times (-k_{z1i})^{N-1}]. \tag{11}$$

For vertical incidence,

$$g_\alpha^{(N,0)} = \frac{i^N}{N!}\left(\frac{-k_0}{\kappa_H'}\right)\left[-s_{i,n'}\left(\frac{k_{zi}k_{z1\bar{n}'}}{k_0^2}R_{1v}^{(N)} + R_{2v}^{(N)}\right)\right.$$
$$\left. + N\frac{k_{z1\bar{n}'}}{k_0}R_{3v}^{(N)}L^s\right],$$

$$g_\beta^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_0}{\kappa_V'}\right)\left[-c_{i,n'}\left(\frac{\epsilon k_{zi}}{k_0}R_{1v}^{(N)} + \frac{k_{z1\bar{n}'}}{k_0}R_{2v}^{(N)}\right)\right.$$
$$\left. + N\epsilon R_{3v}^{(N)}L^c\right],$$

$$g_\gamma^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_0}{\kappa_H'}\right)\left[-s_{i,n'}\left(\frac{k_{zi}k_{z\bar{n}'}}{k_0^2}R_{1v}^{(N)} - R_{2v}^{(N)}\right)\right.$$
$$\left. + N\frac{k_{z\bar{n}'}}{k_0}R_{3v}^{(N)}L^s\right],$$

$$g_\delta^{(N,0)} = \frac{i^N}{N!}\left(\frac{k_1}{\kappa_V'}\right)\left[-c_{i,n'}\left(\frac{k_{zi}}{k_0}R_{1v}^{(N)} - \frac{k_{z\bar{n}'}}{k_0}R_{2v}^{(N)}\right)\right.$$
$$\left. + NR_{3v}^{(N)}L^c\right], \tag{12}$$

where

$$R_{1v}^{(N)} = \Gamma_V(k_{zi})^N - (-k_{zi})^N + \frac{k_{z1i}}{\epsilon k_{zi}}(1 + \Gamma_V)(-k_{z1i})^N,$$

$$R_{2v}^{(N)} = \Gamma_V(k_{zi})^N + (-k_{zi})^N - (1 + \Gamma_V)(-k_{z1i})^N,$$

$$R_{3v}^{(N)} = \frac{k_{\rho i}}{k_0}\left[(-k_{zi})^{N-1} + \Gamma_V(k_{zi})^{N-1}\right.$$
$$\left. - \frac{1 + \Gamma_V}{\epsilon}(-k_{z1i})^{N-1}\right]. \tag{13}$$

In the preceding equations, Γ_H and Γ_V are the Fresnel reflection coefficients for the incident plane wave, and k_{zi}, k_{1zi}, and $k_{\rho i}$ refer to components of the incident plane wave propagation vectors above and below the interface.

To illustrate the above procedure, consider Eq. (2) for $N = 1, 2$, and 3. For $N = 1$, the sum over l vanishes and the solution is

$$\bar{g}^{(1)}(\bar{n}') = \bar{g}^{(1,0)}(\bar{n}'). \tag{14}$$

For $N = 2$,

$$\bar{g}^{(2)}(\bar{n}', \bar{n}_1) = \bar{g}^{(2,0)}(\bar{n}', \bar{n}_1) + \bar{\bar{\nu}}^{(1)}(\bar{n}', \bar{n}_1) \cdot \bar{g}^{(1)}(\bar{n}_1) \tag{15}$$

$$= \bar{g}^{(2,0)}(\bar{n}', \bar{n}_1) + \bar{\bar{\nu}}^{(1)}(\bar{n}', \bar{n}_1) \cdot \bar{g}^{(1,0)}(\bar{n}_1). \tag{16}$$

Note that the second-order kernel is expressed in terms of $\bar{g}^{(l,0)}$ functions only in the second equation. For $N = 3$,

$$\bar{g}^{(3)}(\bar{n}', \bar{n}_1, \bar{n}_2) = \bar{g}^{(3,0)}(\bar{n}', \bar{n}_1, \bar{n}_2) + \bar{\bar{\nu}}^{(1)}(\bar{n}', \bar{n}_1 + \bar{n}_2) \cdot \bar{g}^{(2)}(\bar{n}_1 + \bar{n}_2, \bar{n}_2) + \bar{\bar{\nu}}^{(2)} \times (\bar{n}', \bar{n}_1, \bar{n}_2) \cdot \bar{g}^{(1)}(\bar{n}_1). \tag{17}$$

Moreover,

$$\bar{g}^{(3)}(\bar{n}', \bar{n}_1, \bar{n}_2) = \bar{g}^{(3,0)}(\bar{n}', \bar{n}_1, \bar{n}_2) + \bar{\bar{\nu}}^{(1)}(\bar{n}', \bar{n}_1 + \bar{n}_2) \cdot \bar{g}^{(2,0)}(\bar{n}_1 + \bar{n}_2, \bar{n}_2) + \bar{\bar{\nu}}^{(1)}(\bar{n}', \bar{n}_1 + \bar{n}_2) \cdot \bar{\bar{\nu}}^{(1)}(\bar{n}_1 + \bar{n}_2, \bar{n}_2) \cdot \bar{g}^{(1,0)}(\bar{n}_2) + \bar{\bar{\nu}}^{(2)}(\bar{n}', \bar{n}_1, \bar{n}_2) \cdot \bar{g}^{(1,0)}(\bar{n}_1) \tag{18}$$

In general, it is possible to express all the higher-order kernels in terms of the same- and lower-order $\bar{g}^{(l,0)}$ functions only. The formulation thus provides a recursive, analytical solution of the SPM equations and is easily programmed with a simple, recursive algorithm to implement Eq. (2).

The SPM kernels derived are to be used in evaluating field amplitudes through Eq. (1). Note however that the "dummy" variables $\bar{n}_1$ through $\bar{n}_{N-1}$ as well as $\bar{n}' - \bar{n}_1 - \cdots - \bar{n}_{N-1}$ in this sum are interchangeable, owing to the simple product of surface Fourier coefficients involved. Thus results for field amplitudes are unchanged if the SPM kernels are symmetrized under an interchange of these arguments. This is equivalent to stating that the SPM kernel function is not unique unless it has been symmetrized under this interchange. For example, with $N = 2$ Eq. (1) is

$$\bar{\zeta}_{\bar{n}'}^{(2)} = \sum_{\bar{n}_1} h_{\bar{n}_1} h_{\bar{n}'-\bar{n}_1} \cdot \bar{g}^{(2)}(\bar{n}', \bar{n}_1). \tag{19}$$

Here $\bar{n}_1$ is a dummy variable, and an interchange of $\bar{n}_1$ with $\bar{n}' - \bar{n}_1$ retains the same form. An appropriate symmetrized second-order kernel is then

$$\bar{g}_S^{(2)}(\bar{n}', \bar{n}_1) = \tfrac{1}{2}[\bar{g}^{(2)}(\bar{n}', \bar{n}_1) + \bar{g}^{(2)}(\bar{n}', \bar{n}' - \bar{n}_1)]. \tag{20}$$

The symmetrized kernel for $N = 3$ is

$$\bar{g}_S^{(3)}(\bar{n}', \bar{n}_1, \bar{n}_2) = \tfrac{1}{6}[\bar{g}^{(3)}(\bar{n}', \bar{n}_1, \bar{n}_2) + \bar{g}^{(3)}(\bar{n}', \bar{n}' - \bar{n}_1 - \bar{n}_2, \bar{n}_2) + \bar{g}^{(3)}(\bar{n}', \bar{n}_1, \bar{n}' - \bar{n}_1 - \bar{n}_2) + \bar{g}^{(3)}(\bar{n}', \bar{n}_2, \bar{n}_1) + \bar{g}^{(3)}(\bar{n}', \bar{n}' - \bar{n}_1 - \bar{n}_2, \bar{n}_1) + \bar{g}^{(3)}(\bar{n}', \bar{n}_2, \bar{n}' - \bar{n}_1 - \bar{n}_2)]. \tag{21}$$

Use of symmetrized kernels in computing Eq. (1) is not required, but comparisons with other SPM formulations should be made in terms of symmetrized kernels only so that unique definitions are considered. When the symmetrized kernels are being used, the domain of summation in Eq. (1) can be reduced appropriately to avoid unnecessary computations.

3. SAMPLE RESULTS

A. Fourth-Order Reflection-Coefficient Correction

For a stochastic rough surface defined to have a zero-mean value and zero-mean Fourier coefficients, the average specular reflection coefficient is

$$\langle \bar{\Gamma}^{\mathrm{eff}} \rangle = \bar{\Gamma}^{(0)} + \bar{\Gamma}^{(2)} + \bar{\Gamma}^{(3)} + \bar{\Gamma}^{(4)} + \cdots \tag{22}$$

$$= \bar{\Gamma} + \sum_{\bar{n}_1} \langle |h_{\bar{n}_1}|^2 \rangle \bar{g}^{(2)}(0, \bar{n}_1) + \sum_{\bar{n}_1} \sum_{\bar{n}_2} \langle h_{\bar{n}_1} h_{\bar{n}_2} h_{-\bar{n}_1-\bar{n}_2} \rangle \times \bar{g}^{(3)}(0, \bar{n}_1, \bar{n}_2) + \sum_{\bar{n}_1} \sum_{\bar{n}_2} \sum_{\bar{n}_3} \langle h_{\bar{n}_1} h_{\bar{n}_2} h_{\bar{n}_3} h_{-\bar{n}_1-\bar{n}_2-\bar{n}_3} \rangle \times \bar{g}^{(4)}(0, \bar{n}_1, \bar{n}_2, \bar{n}_3) \tag{23}$$

to fourth order, where the $\langle \cdot \rangle$ notation indicates an ensemble average. A similar expression valid to third order was provided previously in Ref. 2. Note that $\bar{\Gamma}$ above is a vector form for the Fresnel reflection and transmission coefficients, defined analogously to the SPM kernel functions. For a discrete analog of a Gaussian random process, the third-order term (proportional to the surface bispectrum) vanishes, and the fourth-order Fourier coefficient correlation can be expressed as[6]

$$\langle h_{\bar{n}_1} h_{\bar{n}_2} h_{\bar{n}_3} h_{-\bar{n}_1-\bar{n}_2-\bar{n}_3} \rangle = \delta(\bar{n}_1 + \bar{n}_2) |h_{\bar{n}_1}|^2 |h_{\bar{n}_3}|^2 + \delta(\bar{n}_1 + \bar{n}_3) |h_{\bar{n}_1}|^2 |h_{\bar{n}_2}|^2 + \delta(\bar{n}_2 + \bar{n}_3) |h_{\bar{n}_1}|^2 |h_{\bar{n}_2}|^2. \tag{24}$$

Making use of this expansion and renaming variables permits the fourth-order term in Eq. (23) to be rewritten as

$$\bar{\Gamma}^{(4)} = \sum_{\bar{n}_1} \sum_{\bar{n}_2} \langle |h_{\bar{n}_1}|^2 \rangle \langle |h_{\bar{n}_2}|^2 \rangle [\bar{g}^{(4)}(0, \bar{n}_1, -\bar{n}_1, \bar{n}_2) + \bar{g}^{(4)}(0, \bar{n}_1, \bar{n}_2, -\bar{n}_1) + \bar{g}^{(4)}(0, \bar{n}_1, \bar{n}_2, -\bar{n}_2)]. \tag{25}$$

In the limit that the surface periods approach infinity, as described in Ref. 2, Eq. (25) becomes

$$\bar{\Gamma}^{(4)} = \int \mathrm{d}\bar{k}_1 \int \mathrm{d}\bar{k}_2 W(\bar{k}_1) W(\bar{k}_2) [\bar{g}^{(4)}(0, \bar{k}_1, -\bar{k}_1, \bar{k}_2) + \bar{g}^{(4)}(0, \bar{k}_1, \bar{k}_2, -\bar{k}_1) + \bar{g}^{(4)}(0, \bar{k}_1, \bar{k}_2, -\bar{k}_2)]. \tag{26}$$

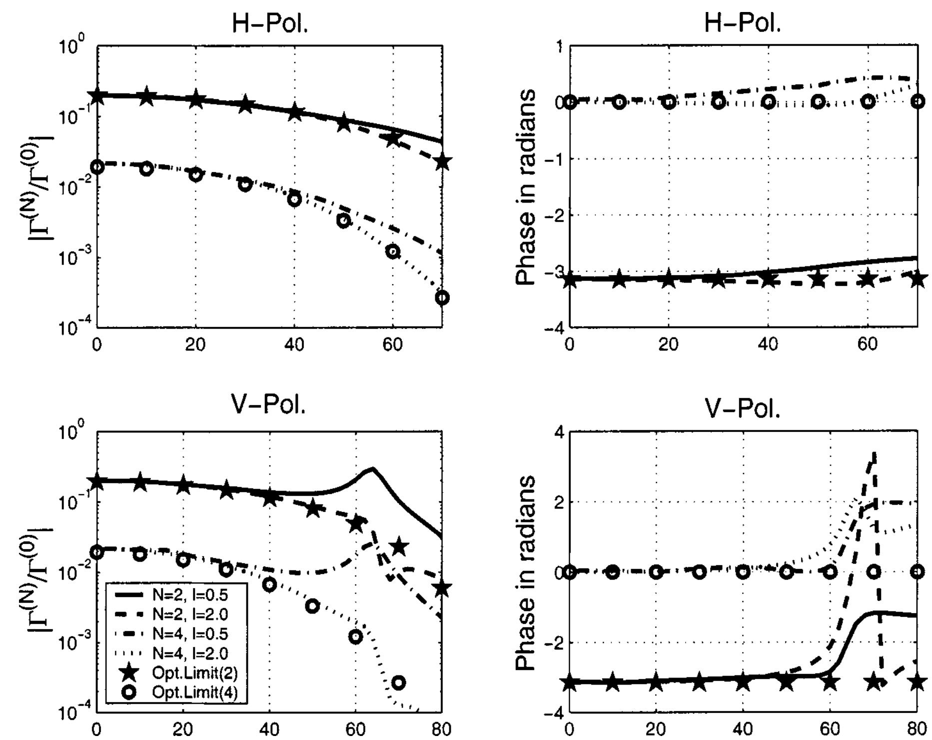

Fig. 1. Specular reflection coefficient versus incidence angle; $\sigma = 0.05\lambda$, $\epsilon = 4 + i$. Magnitudes and phases (in radians) are normalized by Fresnel reflection coefficient for horizontal and vertical incidence.

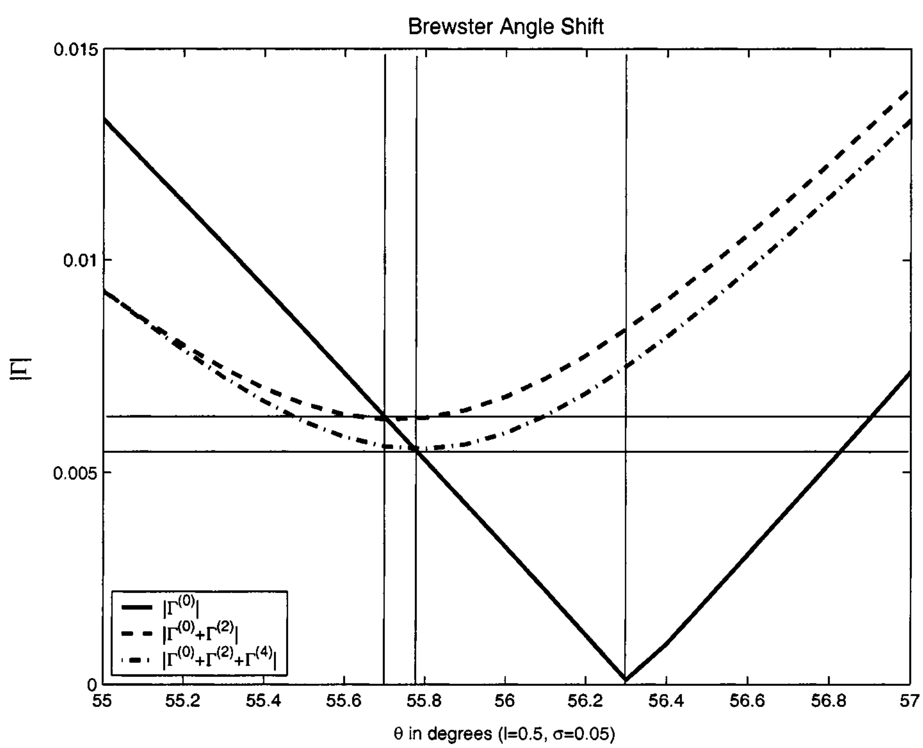

Fig. 2. Brewster angle shift; $\sigma = 0.05\lambda$, $l = 0.5\lambda$, $\epsilon = 2.25$.

Figure 1 illustrates the amplitude and phase of both the second- and the fourth-order average reflection-coefficient corrections (i.e., $N = 2$ and $N = 4$) relative to $\Gamma^{(0)}$ versus incidence angle, for both horizontal and vertical incidence. Results are shown for surfaces described by an isotropic Gaussian spectrum:

$$W(k_\rho) = \frac{h^2 l^2}{4\pi} \exp[-(k_\rho l)^2/4]. \tag{27}$$

Here h refers to the surface rms height, and l is the surface correlation length. A surface rms height of 0.05

wavelength and correlation lengths of 0.5 and 2 wavelengths were used in the computations, along with a surface relative permittivity of $\epsilon = 4 + i$. The fourfold integration of Eq. (26) was evaluated with Gauss–Legendre quadrature and was found to converge well as the number of integration points was increased. Second- and fourth-order terms from a power series expansion of the reflection coefficient from the physical optics approximation,[7]

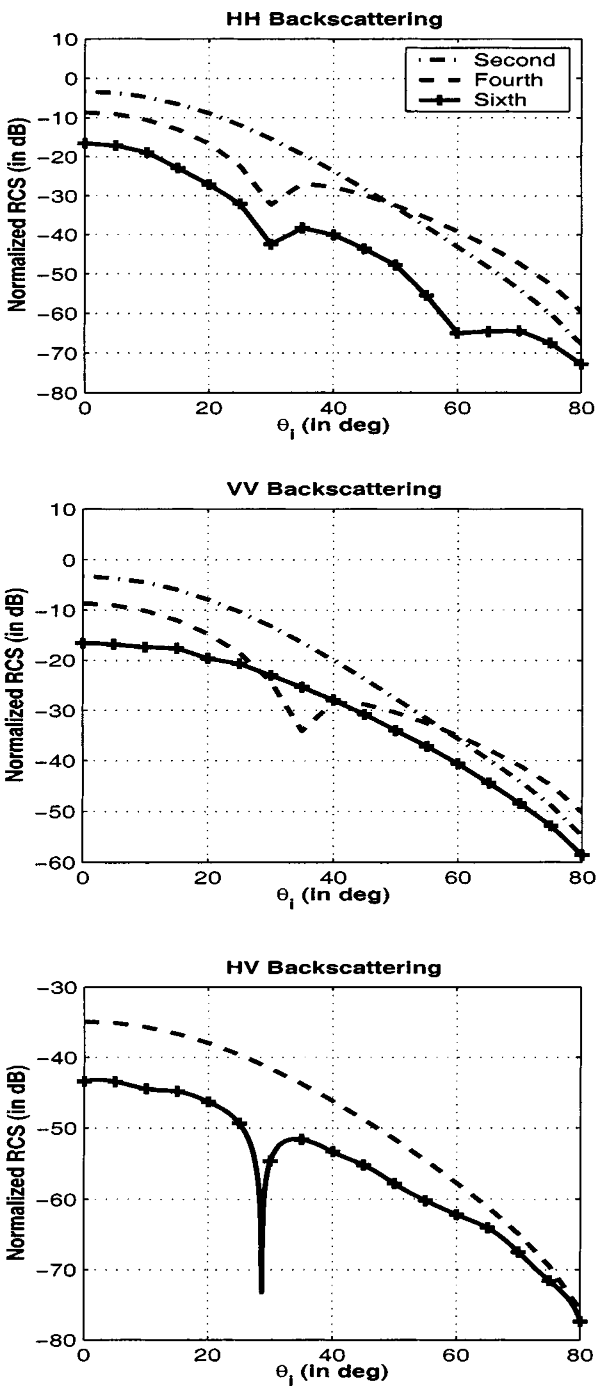

Fig. 3. Second-, fourth-, and sixth-order backscattering cross sections versus observation angle; $\sigma = 0.05\lambda$, $l = 0.5\lambda$, $\epsilon = 4 + i$ for HH, VV, and HV. RCS, radar cross section.

$$\Gamma_{\text{optical}} = \Gamma^{(0)} \exp(-2k^2h^2 \cos^2 \theta_i) \approx \Gamma^{(0)}(1 - 2k^2h^2 \cos^2 \theta_i + 2k^4h^4 \cos^4 \theta_i - \ldots), \tag{28}$$

are also included for comparison.

Results show a generally decreasing importance of both second- and fourth-order corrections as the observation angle is increased, particularly for horizontal polarization. Good agreement between the optical expansion and the SPM computations for the longer correlation length is observed, but significant deviations are found for the shorter correlation length. This is not surprising given the large-scale assumption implicit in the optical expansion. A previous study[8] of higher-order coherent reflection of scalar waves from impenetrable rough surfaces showed similar conclusions with regard to convergence to the optical limit. Deviations from the optical limit are largest in the vertical polarization case near the pseudo-Brewster angle at approximately 63.7 deg, where the Fresnel reflection coefficient magnitude is approximately 0.05. Here the small corrections are more important owing to the small zeroth-order term obtained. Results also show that the fourth-order correction is relatively insignificant for these surface statistics; however the fourth-order term will scale as h^4, compared with h^2 for second order. An increase of the surface rms height to 0.15 wavelength would result in approximately equal amplitudes for the second- and fourth-order corrections, so that higher-order terms in the expansion would likely be required for accuracy.

B. Brewster Angle Shift

To further study the influence of higher-order reflection-coefficient corrections on the Brewster angle effect, we performed an additional calculation using $\epsilon = 2.25$. Figure 2 plots the total reflection coefficient near the flat-surface Brewster angle of 56.31 deg for a surface with rms height $h = 0.05\lambda$ and correlation length $l = 0.5\lambda$. Results show a slight modification of the reflection-coefficient minimum location, because second- and fourth-order terms are individually included. For these surface statistics, the second-order correction shifts the Brewster angle approximately -0.5 deg, and the fourth-order correction causes a positive shift of less than 0.1 deg. Results similar to these with use of the second-order term only have been studied.[9,10] Again the importance of the fourth-order correction would become more pronounced as surface heights increased.

C. Incoherent Average Scattering Cross Sections for a Gaussian Random Process

As discussed in Ref. 2, average incoherent scattering cross sections are composed of even-order contributions for Gaussian random-process surfaces, beginning at second order. Reference 2 presented the fourth-order correction based on the third-order field solution; the current formulation allows computation of the sixth-order correction through use of the field solution up to fifth order. Although this term is not expected to be an important con-

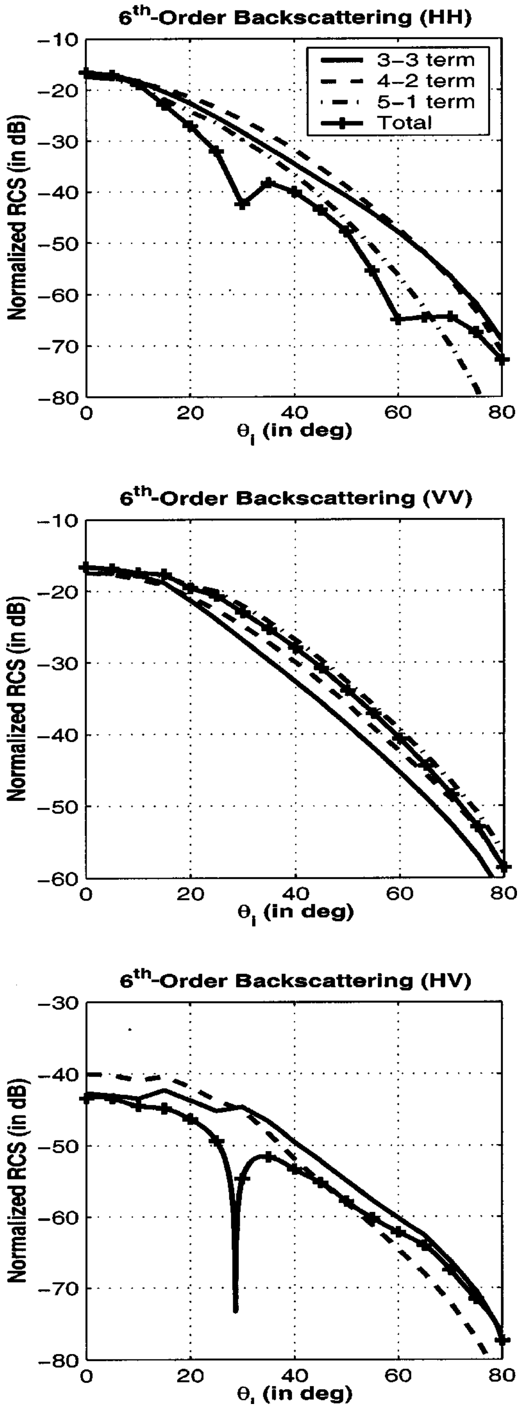

Fig. 4. Terms contributing to the sixth-order backscattering cross section versus observation angle, $\sigma = 0.05\lambda$, $l = 0.5\lambda$, $\epsilon = 4 + i$ for HH, VV, and HV. RCS, radar cross section.

tribution for small height surfaces in copolarization, the fact that the second-order term vanishes for cross-polarized backscattering increases the importance of the sixth-order contribution. This sixth-order term is thus the first correction to the previous fourth-order solution and therefore provides information on the expected accuracy of this solution.

Following Ref. 2, the sixth-order contribution to average incoherent scattering cross sections for a periodic Gaussian process surface is

$$\begin{aligned}\sigma_{\bar{n}'} = 4\pi k_{z\bar{n}'}^2 \operatorname{Re}\Bigg\{ \sum_{\bar{n}_1}\sum_{\bar{n}_2}\sum_{\bar{n}_3}\sum_{\bar{n}_4} & [\langle h_{\bar{n}_1} h_{\bar{n}_2} h_{-\bar{n}_3} h_{-\bar{n}_4} h_{\bar{n}'-\bar{n}_1-\bar{n}_2} h_{\bar{n}_3+\bar{n}_4-\bar{n}'}\rangle \\ & \times g^{(3)}(\bar{n}', \bar{n}_1, \bar{n}_2) g^{(3)*}(\bar{n}', \bar{n}_3, \bar{n}_4) \\ & + \langle h_{-\bar{n}_1} h_{\bar{n}_2} h_{\bar{n}_3} h_{\bar{n}_4} h_{\bar{n}_1-\bar{n}'} h_{\bar{n}'-\bar{n}_2-\bar{n}_3-\bar{n}_4}\rangle \\ & \times 2g^{(2)*}(\bar{n}', \bar{n}_1) g^{(4)}(\bar{n}', \bar{n}_2, \bar{n}_3, \bar{n}_4) \\ & + \langle h_{\bar{n}_1} h_{\bar{n}_2} h_{\bar{n}_3} h_{\bar{n}_4} h_{-\bar{n}'} h_{\bar{n}'-\bar{n}_1-\bar{n}_2-\bar{n}_3-\bar{n}_4}\rangle \\ & \times 2g^{(1)*}(\bar{n}') g^{(5)}(\bar{n}', \bar{n}_1, \bar{n}_2, \bar{n}_3, \bar{n}_4)]\Bigg\}. \end{aligned} \tag{29}$$

The scalar valued SPM kernels above are for a particular polarization, and $\bar{n}'$ is chosen to represent a particular bistatic scattering angle. Three contributions are observed above: those from the third-order fields squared, a cross product between second- and fourth-order fields, and a cross product between first- and fifth-order fields. In the case of cross-polarized backscattering, the final term vanishes so that a fourth-order field solution is sufficient for computing the sixth-order cross-section correction.

For a periodic Gaussian random process, a correlation of six Fourier coefficients can be split into a sum of 15 terms involving products of three spectra and four delta functions. Using a process similar to that used to obtain Eq. (25) and transforming the result into the continuous limit allows the sixth-order cross-polarized cross section to be expressed as a fourfold integration. Figure 3 illustrates backscattering results at second, fourth, and sixth order for a surface with an isotropic Gaussian correlation function, rms height 0.05λ, and correlation length 0.5λ. The dielectric constant of the surface is $4 + i$, and the three plots correspond to HH, VV, and VH (cross-polarized) cases, respectively. Results show the fourth-order co-polarized correction to be appreciable for these statistics, while the sixth-order term is less important in copolarization but appreciable in cross polarization. The "null" behavior of the sixth-order cross-polarization correction observed is due to a sign change: For smaller angles the sixth-order correction is negative, while for larger angles it is positive. To enhance the figure's resolution near the sign change (results computed in a 5-deg step), a cubic-spline interpolation of the sixth-order term (in linear units) is also plotted. Although neglect of the sixth-order correction appears reasonable for these surface statistics at smaller observation angles, this term will scale as h^6 as surface heights increase, while the second- and fourth-order terms scale only as h^2 and h^4, respectively. Figure 4 plots the $3 - 3$, $4 - 2$, and $5 - 1$ contributions to the sixth-order correction in all polarizations. In most cases these terms exhibit cancellation effects so that the total sixth-order result plotted is less than the maximum of the three terms.

4. CONCLUSIONS

A complete, arbitrary-order solution for SPM kernel functions has been presented in this paper. Higher-order kernel functions were expressed in terms of lower-order kernels in a recursive fashion. Applications of these results were demonstrated; use of the fourth-order theory in a fourth-order study of emission from a rough surface will be reported in the future.

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