On the Geometrical Optics (Hagfors' Law) and Physical Optics Approximations for Scattering from Exponentially Correlated Surfaces

Joel T. Johnson, *Senior Member, IEEE*, Karl F. Warnick, *Senior Member, IEEE*, and Peng Xu

***Abstract*— High frequency approximations to the physical optics (PO) theory of scattering from exponentially correlated rough surfaces are examined and used to interpret the expected accuracy of the PO theory.**

As an introduction, a review of the PO theory for Gaussian correlated surfaces is provided, and in this process an analytical summation of the PO series for specular scattering from Gaussian correlated surfaces is obtained. A similar form is then derived for specular scattering from exponentially correlated surfaces and contrasted to the Gaussian case. These series allow the accuracy of the leading order term (i.e. the geometrical optics (GO) limit) in the high frequency approximation of physical optics scattering for Gaussian or exponentially correlated surfaces to be investigated analytically.

The leading order term in the high frequency expansion for general physical optics scattering from exponentially correlated surfaces (Hagfors' Law) is then reviewed and interpreted in terms of a recently published theory of physical optics for surfaces with infinite rms slopes. The approximate "cutoff" wavenumber from Hagfors' Law at which the high frequency portion of the spectrum of an exponentially correlated surface can be truncated without producing large errors in PO predicted scattering is also discussed.

Using this cutoff wavenumber, an approximate region of validity of the complete physical optics theory for exponentially correlated surfaces is obtained. The validity condition indicates that, for fixed surface statistics, the PO method produces accurate predictions of true surface scattering only up to a specific frequency, and that PO is inaccurate in the high frequency limit. Comparisons of PO predictions with those of a Monte Carlo numerical simulation are used to show that the validity condition derived appears to provide a reasonable indication of PO accuracy. These results have important implications for current investigations of scattering from exponentially correlated surfaces and for the use of Hagfors' Law, as it is traditional to accept PO as the appropriate high frequency limit in most existing approximate models of surface scattering.

***Index Terms*— Rough Surface Scattering**

I. Introduction

THE geometrical optics (GO) approximation [1]-[2] is widely used to model near-specular scattering from random rough surfaces due to its simplicity and the requirement that only the rms slope of the rough surface be known. The GO result is obtained as the leading order term in a high frequency expansion of the standard physical optics (PO) theory. With "single-scale" surfaces for which all roughness scales are reasonably large compared to the electromagnetic wavelength, the rms slope required in the GO can be directly computed from the statistical properties of the roughness; this is the case with the widely used Gaussian correlation function model of a Gaussian random process surface. For "multi-scale" surfaces containing roughness on a wide range of length scales (including length scales comparable to or shorter than the electromagnetic wavelength) use of the geometrical optics theory requires careful consideration. One problem that can occur in such cases involves the fact that the rms slopes of multi-scale surfaces are often infinite. A common method used in practice is to retain the GO theory with an additional "cutoff wavenumber" parameter to be used in determining the rms surface slope. The cutoff wavenumber defines a high-frequency truncation point in the surface spectrum beyond which contributions to the rms slope are neglected. This approach is widely used in modeling scattering from the sea surface; reference [3] shows that reasonable near-specular backscattering responses from ocean-like surfaces can be predicted if the cutoff wavenumber is chosen as twice the electromagnetic wavenumber. Other analytical studies reaching similar conclusions have also been reported [4].

References [5]-[6] have further demonstrated an analytical basis for this approach for Gaussian random process surfaces, and provide a method for determining an appropriate cutoff wavenumber. These references also discuss criteria for predicting the expected accuracy of the PO approximation once the cutoff wavenumber is determined.

This paper provides a detailed examination of the use of both the GO and PO theories for surfaces with exponential correlation functions. This multi-scale description of surface roughness has an infinite rms slope, but is widely used in remote sensing applications, particularly those involving soil surfaces. The high frequency limit for scattering from an exponentially correlated surface (known as Hagfors' Law) has previously been derived [7], and is also widely used in modeling radar returns from planetary surfaces. A recent paper [8] provides a complementary examination of Hagfors' law, and shows that the scattering cross sections obtained fail to satisfy power conservation when integrated over bistatic scattering angles, as is expected for any scattering cross sections based on the PO theory. Reference [8] also explores the impact of a finite surface area on Hagfors' Law, and furthermore compares scattering from surfaces with exponential correlation functions with those described by a more general fractal model. However

Manuscript received Month dd, yyyy; revised Month dd, yyyy. J. T. Johnson is with The Ohio State University, Dept. of Electrical and Computer Engineering and ElectroScience Laboratory. K. F. Warnick is with Brigham Young University, Dept. of Electrical and Computer Engineering. Peng Xu is with the School of Electronic Information, Wuhan University, China.

fractal surface descriptions beyond the exponential correlation function model are not considered further here, as this paper is focused on the expected accuracy of the PO/GO theories for exponentially correlated surfaces and the impact of surface spectral truncation on scattering predictions.

The paper begins by determining a complete analytical high frequency expansion of the physical optics theory for the case of specular scattering in order to examine the potential accuracy of more general high frequency expansions (such as Hagfors' Law) for which only the first series term is determined. As a review of the traditional GO approximation and for contrast with the exponentially correlated case, Section II presents this process for Gaussian correlated surfaces and derives a simple analytical representation for the resulting specular scattering. The process is then applied for exponentially correlated surfaces in Section III, and it is shown that the resulting specularly scattered cross sections remain frequency dependent in contrast to traditional expectations. The expressions obtained in both the Gaussian and exponential correlation function cases show that the first term in the high frequency series (Hagfors' Law in the exponential case) provides an excellent match to specular PO predictions as long as the surface rms height is greater than approximately one half the electromagnetic wavelength.

In Section IV, the methods of [5]-[6] are applied for the exponential case to re-derive Hagfors' Law (i.e. the first term in a high frequency expansion) that is applicable both for specular and non-specular scattering. Hagfors' Law of scattering shows a frequency dependence in the high frequency limit, and includes a term that can be identified as a frequency dependent effective slope variance of the surface.

The extent to which the resulting equations can be interpreted as originating from a surface with a spectrum truncated below a specified cutoff wavenumber is then examined in Section V. Using the methods of [6], the cutoff wavenumber implicit in Hagfors' Law, which increases as the square of the frequency, is re-derived. Because the truncated surface has a finite slope variance, the traditional GO approximation following [6] can be applied, but is shown to produce only moderate accuracy in matching the true leading order PO high frequency limit. However, further computations with a recently proposed band-limited exponential surface model [9] in Section VI confirm that true exponential surface scattering can be modeled with reasonable accuracy as originating from the specified truncated surface.

An approximate validity condition for the use of the physical optics theory for scattering from exponentially correlated surfaces is then derived in Section VII based on the properties of the truncated surface. Comparisons of PO predictions with those of an exact numerical method are also illustrated in Section VII in order to investigate the validity condition proposed. Implications of the validity condition for studies of scattering from exponentially correlated surfaces are then presented in Section VIII.

II. High frequency asymptotic series for specular PO scattering from an isotropic Gaussian correlated surface

To introduce the paper's notation and to review the traditional GO expansion, a complete high frequency expansion of specular scattering from Gaussian correlated surfaces is first examined.

Begin with the Gaussian correlation function

$$C(\rho) = \exp\left(-\left(\frac{\rho}{l}\right)^2\right) \tag{1}$$

Here l represents the surface correlation length and h is used to denote the surface rms height. The ensemble averaged incoherent normalized radar cross section (NRCS) for these surfaces obtained from the physical optics theory is given by [2]

$$\sigma_{\alpha,\beta} = \frac{Q^4\left|\Gamma_{\alpha,\beta}\right|^2}{4Q_z^2} l^2 e^{-Q_z^2h^2}\sum_{n=1}^{\infty}\frac{\left(Q_z^2h^2\right)^n}{n!n}e^{-Q_\rho^2 l^2/(4n)} \tag{2}$$

where

$$\begin{aligned} \overline{Q} &= \left(\overline{K}-\overline{K}_0\right) && (3)\\ Q &= \left|\overline{Q}\right| && (4)\\ Q_z &= \hat{z}\cdot\overline{Q} && (5)\\ Q_\rho &= \left|\overline{Q}-\hat{z}Q_z\right| && (6)\end{aligned}$$

The subscripts α, β refer to the scattered and incident field polarizations, respectively, and are to be chosen from H for horizontal or V for vertical. The function $\Gamma_{\alpha,\beta}$ depends on the Fresnel reflection coefficients of the mean surface evaluated at the stationary phase point as described in [10]-[11]; for in-plane scattering problems Γ is identical to the Fresnel reflection coefficient evaluated at the stationary phase point in the appropriate polarization when $\alpha = \beta$ (co-pol scattering) and vanishes for $\alpha \neq \beta$. Polarization subscripts on Γ and σ will be dropped henceforth for simplicity. In the above it is assumed that $\overline{K}_0$ and $\overline{K}$ are the vector wavenumbers of the incident and scattered plane waves, respectively, and that the direction $\hat{z}$ represents the normal to the mean surface pointing into the region from which the incident field originates. The symbol K will be used to denote the scalar wavenumber in the incident medium (i.e. $\left|\overline{K}_0\right|$) in what follows.

If specular scattering is considered (i.e. $\overline{K}$ and $\overline{K}_0$ are identical except for a change in the sign of their $\hat{z}$ components, so that $Q_\rho = 0$), then the co-polarized NRCS expression simplifies to

$$\sigma = \left|\Gamma\right|^2 k_z^2 l^2 e^{-4k_z^2h^2}\sum_{n=1}^{\infty}\frac{\left(4k_z^2h^2\right)^n}{n!n} \tag{7}$$

where k_z is defined as $\hat{z}\cdot\overline{K} = -\hat{z}\cdot\overline{K}_0$, and Γ represents the Fresnel reflection coefficient evaluated at the incidence angle (the stationary phase point) for the mean surface boundary. In standard practice, the NRCS would be evaluated by numerically summing the above series.

However, the series expansion in equation (7) can be summed analytically following [12] as

$$\sum_{n=1}^{\infty} \frac{\left(4k_z^2h^2\right)^n}{n!n} = \mathrm{Ei}(4k_z^2h^2) - \ln\left(4k_z^2h^2\right) - \gamma \quad (8)$$

where Ei represents the exponential integral function and $\gamma = 0.577215664$ is Euler's constant. This results in a simple analytical form for the specularly scattered NRCS as

$$\sigma = |\Gamma|^2 k_z^2 l^2 e^{-4k_z^2h^2}\left(\mathrm{Ei}(4k_z^2h^2) - \ln\left(4k_z^2h^2\right) - \gamma\right) \quad (9)$$

which is the principal result of this section.

The GO is obtained in the high frequency (i.e. k_z approaches infinity) limit. In this limit, the argument of the exponential integral function becomes large, so that the asymptotic form

$$\mathrm{Ei}\left(4k_z^2h^2\right) \approx \frac{e^{4k_z^2h^2}}{4k_z^2h^2}\left(1 + \frac{1!}{4k_z^2h^2} + \frac{2!}{\left(4k_z^2h^2\right)^2} + \cdots\right) \quad (10)$$

is useful. Substituting this into the NRCS equation yields

$$\begin{aligned}\sigma \approx |\Gamma|^2 k_z^2 l^2 \Bigg\{ & \frac{1}{4k_z^2h^2}\left(1 + \frac{1!}{4k_z^2h^2} + \frac{2!}{\left(4k_z^2h^2\right)^2} + \cdots\right) \\ & - e^{-4k_z^2h^2}\left(\ln\left(4k_z^2h^2\right) + \gamma\right)\Bigg\} \end{aligned} \quad (11)$$

with the leading order term

$$\sigma \approx |\Gamma|^2 \frac{l^2}{4h^2} \quad (12)$$

$$= \frac{|\Gamma|^2}{s^2} \quad (13)$$

where s^2 is the slope variance of the surface $4h^2/l^2$. This leading order term is the traditional GO result for specular scattering, and should provide an accurate approximation of the PO series for $4k_z^2h^2 >> 1$. The traditional GO NRCS is independent of frequency and uses the true slope variance of the surface.

The analytical summation of the PO series in Equation (9) provides a simple method for estimating the degree to which the GO approximation of PO is accurate for Gaussian correlated surfaces. For example, in near normal incidence backscattering the error in neglecting higher order terms in equation (11) is less than 3 percent when $h > \lambda/2$ where λ is the electromagnetic wavelength. Note the question of accuracy of the PO approximation for Gaussian correlated surfaces is not addressed here, but has been studied numerous times in the literature.

A similar process is applied in the next section to determine a high frequency asymptotic series for specular PO scattering from exponentially correlated surfaces.

III. High frequency asymptotic series for specular scattering from an isotropic exponentially correlated surface

In this case, the correlation function is

$$C(\rho) = \exp\left(-\frac{\rho}{l}\right) \quad (14)$$

The ensemble averaged incoherent normalized radar cross section (NRCS) for these surfaces obtained from the physical optics theory is given by [13]

$$\sigma = \frac{Q^4|\Gamma|^2}{4Q_z^2}\frac{2}{l}e^{-Q_z^2h^2}\sum_{n=1}^{\infty}\frac{\left(Q_z^2h^2\right)^n}{(n-1)!}\left[Q_\rho^2 + \frac{n^2}{l^2}\right]^{-3/2} \quad (15)$$

In the specular direction this expression for co-polarization becomes

$$\sigma = 2|\Gamma|^2 k_z^2 l^2 e^{-4k_z^2h^2}\sum_{n=1}^{\infty}\frac{\left(4k_z^2h^2\right)^n}{n!n^2} \quad (16)$$

To proceed, rewrite the sum above as the function

$$f(x) = \sum_{n=1}^{\infty}\frac{x^n}{n!n^2} \quad (17)$$

where $x = 4k_z^2h^2$. Although this sum is not easily approximated analytically, its asymptotic properties can still be examined. Begin with the fact that $f(x)$ satisfies the differential equation

$$xf'(x) + x^2f''(x) = e^x - 1 \quad (18)$$

where the prime indicates differentiation with respect to the argument. An asymptotic analysis of this differential equation suggests that the substitution

$$f(x) \approx \frac{e^x}{x^2}g(x) \quad (19)$$

is appropriate. The resulting differential equation for $g(x)$ can then be solved perturbatively in $1/x$ for x large. This process yields eventually

$$f(x) \approx \frac{e^x}{x^2}\left(1 + \frac{3}{x} + \frac{11}{x^2} + \cdots\right) \quad (20)$$

up to second order in $\frac{1}{x}$ for $g(x)$. Figure 1 is a plot of $f(x)x^2/e^x$ that illustrates convergence toward unity for x large; the approximation including terms up to second order in $1/x$ is also included in the plot.

Using this result, the specular NRCS for large $4k_z^2h^2$ becomes

$$\begin{aligned}\sigma \approx & \frac{|\Gamma|^2}{2}\frac{l^2}{h^2}\frac{1}{4k_z^2h^2} \\ & \left(1 + \frac{3}{4k_z^2h^2} + \frac{11}{16k_z^4h^4} + \cdots\right)\end{aligned} \quad (21)$$

For near normal incidence backscattering, corrections to the leading order term are less than 9 percent for $h > \lambda/2$. The leading order term is

$$\begin{aligned}\sigma &\approx \frac{|\Gamma|^2}{2}\frac{l^2}{h^2}\frac{1}{4k_z^2h^2} \\ &= \frac{|\Gamma|^2}{2\tilde{s}^2} = \frac{|\Gamma|^2 C}{2}\frac{K^2}{k_z^2}\end{aligned} \quad (22)$$

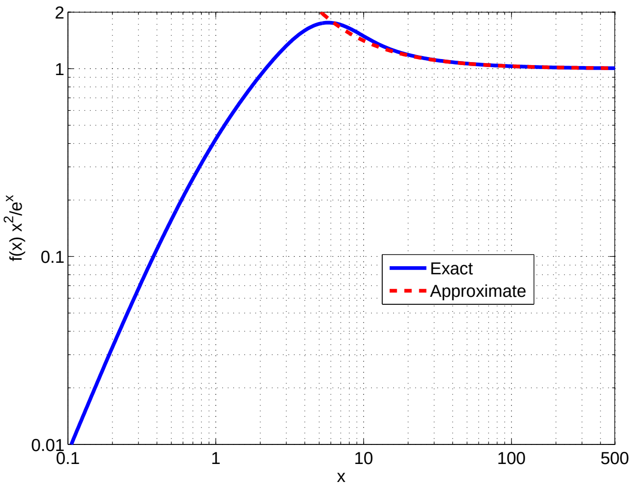

Fig. 1. A plot of $f(x)$ divided by e^x/x^2; the approximate $g(x)$ up to second order in $1/x$ is also included

where in analogy to the Gaussian case (equation (13)), $\tilde{s}^2$ is used to represent an effective slope variance; the reason for including the factor 2 in the denominator above will be made clear in Section V. The quantity $C = (l/[2Kh^2])^2$ introduced in the final statement of equation (22) is the constant traditionally used in Hagfors' Law; equation (22) is Hagfors' Law evaluated for specular scattering.

The effective slope variance is

$$\tilde{s}^2 = \left(\frac{h}{l}\right)^2 \left(4k_z^2h^2\right) \quad (23)$$

$$= \frac{k_z^2}{K^2C} \quad (24)$$

and involves both a slope-like term h/l and the quantity $4k_z^2h^2$. The latter indicates that the leading order high frequency limit (Equation (22)) remains dependent on frequency and the observation angle. This behavior is distinct from the Gaussian case, but could perhaps be justified in terms of a k_zh dependent cutoff wavenumber when computing the rms slope of the exponential surface. This possibility is investigated in Section V. While the quantity $1/\sqrt{C}$ is typically taken as an effective rms slope in the Hagfors model, equation (24) makes clear that the $1/\sqrt{C}$ corresponds to the effective rms slope only when normal incidence backscattering is considered.

IV. General Leading Order High Frequency Limit for Exponential Surfaces: Hagfors' Law

References [5]-[6] present a generalization of the high frequency limit for PO scattering from infinite slope surfaces. For surfaces with finite slope variance, the limit takes on the traditional form that is independent of frequency. For surfaces with infinite slope variance, the high frequency limit of the physical optics integral becomes an "alpha-stable distribution" that is difficult to express analytically in general but can be expanded in an asymptotic series. In this section, the procedures of [5]-[6] are applied to show that the alpha-stable distribution for exponentially correlated surfaces has the simple analytical form of Hagfors' Law. The expansion of this distribution is then examined in Section V.

Begin with the cross section written in terms of the physical optics integral:

$$\sigma = \frac{Q^4|\Gamma|^2}{4Q_z^2}\frac{1}{\pi}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\; e^{iQ_xx}e^{iQ_yy} \left(e^{-Q_z^2h^2(1-C(\rho))} - e^{-Q_z^2h^2}\right) \quad (25)$$

$$= \frac{Q^4|\Gamma|^2}{4Q_z^2}2\int_0^{\infty}d\rho\;\rho\;J_0(Q_\rho\rho) \left(e^{-Q_z^2h^2(1-C(\rho))} - e^{-Q_z^2h^2}\right) \quad (26)$$

with J_0 denoting the Bessel function of zeroth order.

Typically the GO limit is obtained by expanding the correlation function at the origin. However for a surface with infinite rms slopes, a Taylor expansion of $C(\rho)$ at the origin does not exist. Instead, reference [5] describes an expansion of the correlation function at a point near, but not at, the origin, through use of the form

$$C(\rho) \approx 1 - \frac{1}{Q_z^2h^2}\left(\frac{\rho}{\rho_0}\right)^{\alpha} \quad (27)$$

$$= q(\rho) \quad (28)$$

The parameters α and ρ_0 are chosen to satisfy

$$C(\rho_0) = 1 - \frac{1}{Q_z^2h^2} \quad (29)$$

$$C'(\rho_0) = q'(\rho_0) \quad (30)$$

where the prime denotes differentiation. The approximation $q(\rho)$ matches the values of $C(\rho)$ at the origin and the point ρ_0 (where the argument of the exponent in the PO integral is -1), as well as the derivative of the correlation function at ρ_0. For the exponential correlation function, the values of ρ_0 and α can be shown to be

$$\rho_0 = -l\log\left(1 - \frac{1}{Q_z^2h^2}\right) \approx \frac{l}{Q_z^2h^2} \quad (31)$$

$$\alpha = \left(1 - Q_z^2h^2\right)\log\left(1 - \frac{1}{Q_z^2h^2}\right) \approx 1 \quad (32)$$

with the final approximate forms applying as $Q_z^2h^2$ approaches infinity. Note these choices result in the expansion $C(\rho) \approx 1 - \rho/l$ being used, but this is now justified as a limit obtained when points near, but not at, the origin are considered.

Substituting q for C into equation (26) and neglecting the term $e^{-Q_z^2h^2}$ gives

$$\sigma \approx \frac{Q^4|\Gamma|^2}{4Q_z^2}2\int_0^{\infty}d\rho\;\rho\;J_0(Q_\rho\rho)e^{-\frac{\rho}{\rho_0}} \quad (33)$$

in the limit $Q_z^2h^2$ approaches infinity. This integral can be evaluated using an integral table [14] to obtain

$$\sigma \approx \frac{Q^4|\Gamma|^2}{4Q_z^2}2\frac{l^2}{Q_z^4h^4}\left[1 + \left(\frac{Q_\rho}{Q_z}\right)^2\frac{l^2}{Q_z^2h^4}\right]^{-3/2} \quad (34)$$

$$= \frac{|\Gamma|^2}{2\tilde{s}^2}\left[1 + \left(\frac{Q_\rho}{Q_z}\right)^2\frac{1}{\tilde{s}^2}\right]^{-3/2}\left(1 + \frac{Q_\rho^2}{Q_z^2}\right)^2 \quad (35)$$

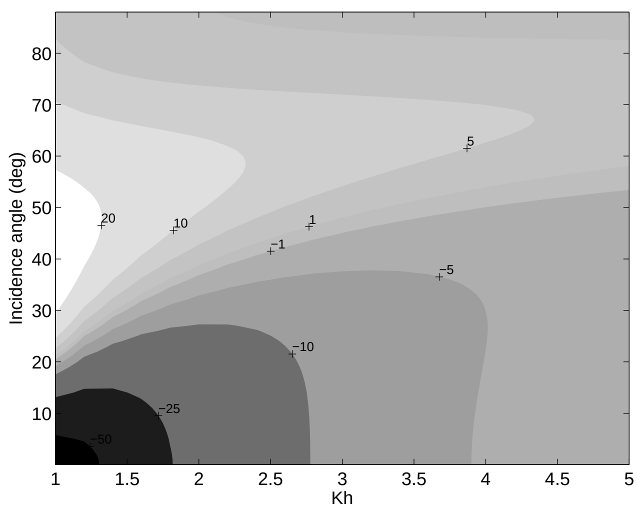

Fig. 2. Filled contour plot of the percent difference between equations (35) and (15), versus Kh and incidence angle, for $l/h = 3$ and at backscattering. Errors are less than 10 percent for $kh > 2.7$, indicating the accuracy of Hagfors' Law in approximating PO predictions

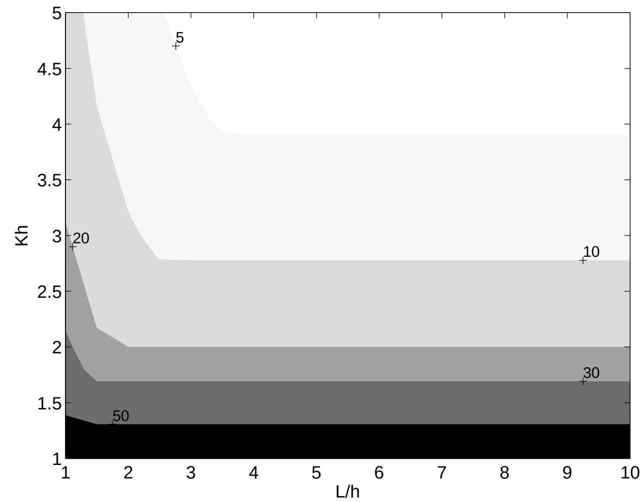

Fig. 3. Filled contour plot of the maximum absolute value of the percent difference between equations (35) and (15), with the maximum taken over backscattering at incidence angles $0, 1, \cdots 89$ degrees. The leading order high frequency limit provides an excellent match to PO predictions for Kh values greater than approximately 2.8.

where

$$\tilde{s}^2 = \left(\frac{h^2}{l^2}\right) Q_z^2 h^2 = \frac{Q_z^2}{4K^2C}, \tag{36}$$

which matches equation (24) for specular scattering. Again Γ above for in-plane scattering problems refers to the Fresnel reflection coefficient evaluated at the stationary phase point (i.e. computed for a surface normal that bisects the incidence and scattering directions). Equation (35) is found to match the leading order result obtained in equation (21) for specular scattering ($Q_\rho = 0$), and remains dependent on frequency. Equation (35) is Hagfors' Law of scattering from an exponential surface, and can be regarded as the leading order high frequency approximation of the PO theory for this case. It also represents the "alpha stable distribution" of references [5]-[6] for exponentially correlated surfaces. Note that it does not have the form of the typical GO approximation, which would involve an exponential function in Q_ρ^2/Q_z^2.

To examine the accuracy of Hagfors' Law (equation (35)) in approximating the original PO result (equation (15)) a set of computations were performed for backscattering at incidence angles from 0 to 89 degrees in steps of one degree, as well as for Kh values $1, 1.2, \cdots, 5$ and l/h values $1, 1.5, \cdots 10$. The percent difference between the predictions of equation (35) and those of equation (15) is illustrated in Figure 2 as a filled contour plot for the choice $l/h = 3$. Note that this plot is independent of the surface dielectric properties. Percent differences are shown for the range -25 to +20 percent. Results show very small errors (within 10 percent) for Kh values larger than 2.7 as previously predicted, with the largest errors somewhat surprisingly occurring for small incidence angles. Unlike the traditional GO approximation, equation (35) appears reproduce PO predictions even at large incidence angles, so long as Kh is not small.

Figure 3 plots versus Kh and l/h the maximum absolute value of the percent error between equations (35) and (15), with the maximum taken over backscattering data at incidence angles $0, 1, \cdots 89$ degrees. The results again show relatively small differences that decrease rapidly as Kh increases. The percent error in the approximation approaches a constant value for large L/h, and in this region the maximum percent error is for normally incident backscattering. A percent error independent of L for normally incident backscattering is consistent with the behavior of the higher order terms for specular scattering in equation (21). Note the accuracy of the physical optics theory in general is questionable for the smaller L/h values in the figure. Overall, these results show that equation (35) is a highly useful approximation for the physical optics theory of scattering from an exponential correlated surface, for moderate to large Kh values.

Figure 4 presents results analogous to those in Figure 2, again with $l/h = 3$, but for in-plane bistatic scattering at incidence angle 20 degrees. The scattering angle is here defined so that a value of -20 degrees corresponds to backscattering while $+20$ degrees is the specular angle. Very small errors (within approximately 10 percent) are again observed at all scattering angles for Kh larger than 2.7. As in Figure 2, the largest errors continue to be located near the specular region.

V. Approximation with a "Truncated" Exponentially Correlated Surface

While equation (35) does not have the form of the traditional geometrical optics theory, its predictions could possibly be approximated by the traditional geometrical optics theory if a method for determining the appropriate rms slope were defined. Reference [6] discusses a method for doing so for general infinite rms slope surfaces; the arguments supporting this method are examined here for exponentially correlated surfaces.

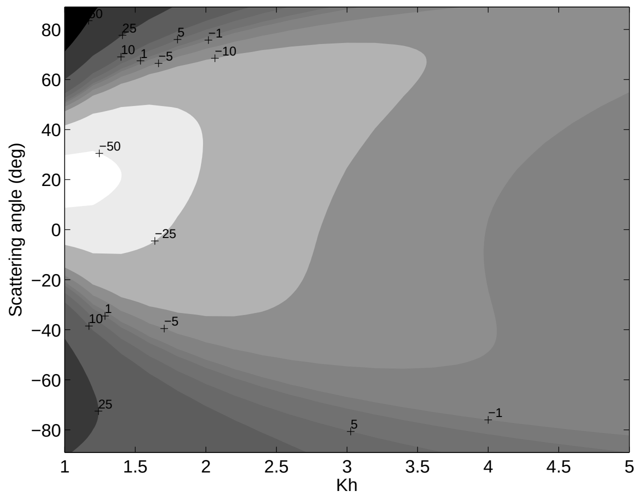

Fig. 4. Filled contour plot of the percent difference between equations (35) and (15), versus Kh and in-plane bistatic angle, for $l/h = 3$ and polar incidence angle $20°$. Hagfors' Law remains a good approximation for PO bistatic scattering so long as Kh is large.

A. PO and alpha-stable distributions

Begin with the form of the high frequency approximation presented in equation (33):

$$\sigma \approx \frac{Q^4 |\Gamma|^2}{4Q_z^2} 2 \int_0^\infty d\rho \; \rho \; J_0(Q_\rho \rho) e^{-\frac{\rho}{\rho_0}} \quad (37)$$

$$= \frac{Q^4 |\Gamma|^2}{4\pi Q_z^2} \int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \; e^{iQ_x x} e^{iQ_y y} \, e^{-\frac{\sqrt{x^2+y^2}}{\rho_0}} \quad (38)$$

The integration above has the form of a Fourier transform evaluated at the point Q_x, Q_y, and is the basis for the interpretation of PO scattering in terms of an alpha-stable distribution. If the function being transformed is regarded as the characteristic function of a random vector, the result of the transform is the probability density function of the random vector evaluated at the point Q_x, Q_y (although scaled by constant multipliers).

Following this process, the characteristic function corresponding to equation (38) is

$$\phi(x,y) = e^{-\frac{\sqrt{x^2+y^2}}{\rho_0}} \quad (39)$$

$$= e^{-\frac{Q_z^2 h^2 \sqrt{x^2+y^2}}{l}} \quad (40)$$

which from reference [15] is known to be the characteristic function of a "symmetric alpha stable" random vector; the particular random vector here has an isotropic Cauchy probability density function [15].

Symmetric alpha stable ($S\alpha S$) random vectors have the property that a random vector defined as a sum of many independent, identically distributed (iid) $S\alpha S$ random vectors will have a probability density function of similar form to that of each of the original iid random vectors. The particular case of interest here involves a random vector defined as

$$\overline{Y} = \frac{1}{b_n} \sum_{r=1}^{n} \overline{X}_r \quad (41)$$

where each $\overline{X}_r$ random vector is independent and has the characteristic function of equation (40). The sum random vector $\overline{Y}$ will also have the characteristic function of equation (40) if the "norming constant" b_n is chosen to be n. This is verified by the fact that the characteristic function of a sum of n iid random vectors is the characteristic function of one of the iid random vectors raised to the nth power, while the characteristic function of $\overline{X}/b_n$ is $\phi(x/b_n, y/b_n)$.

Reference [6] recommends that equation (38) be interpreted as proportional to the pdf (or "distribution") of a sum of $n = Q_z^2 h^2$ random vectors, each with a pdf $p(q_x, q_y)$ that corresponds to the inverse Fourier transformation of $e^{-\sqrt{x^2+y^2}/l}$:

$$p(q_x, q_y) = \frac{l^2}{2\pi} \left[1 + q_\rho^2 l^2\right]^{-3/2} \quad (42)$$

where $q_\rho^2 = q_x^2 + q_y^2$; the above equation is identical to the power spectrum of the exponential surface divided by the surface height variance, called the normalized power spectrum in what follows. Though the number of random vectors clearly can be non-integer in this case, reference [6] states that the results to follow are not affected by this fact. PO NRCS values are then directly proportional to the resulting alpha-stable distribution obtained in this process.

B. Expansion of alpha-stable distribution using truncated surfaces

While for exponential surfaces the alpha-stable distribution can be expressed analytically (equation (35)), it is also possible to express this alpha-stable distribution as an expansion involving quantities that are more similar to traditional GO and composite surface model forms. The expansion uses an alternate means for obtaining the pdf of a sum of random vectors: repeated convolution of the pdfs of each random vector in the sum. Using this repeated convolution form, reference [6] proposes a separation of the individual pdf's into two truncated portions as

$$p_1(q_x, q_y) = p(q_x, q_y) \quad (43)$$

for $q_\rho < k_{max}$, and zero otherwise. The function p_2 is defined as $p(q_x, q_y) - p_1(q_x, q_y)$. The repeated convolutional form can then be written as a series of terms involving varying numbers of p_1 functions convolved with p_2 functions.

Reference [6] argues that the term involving p_1 convolved with itself n times should provide the leading order approximation in the high frequency limit for near-nadiral backscattering. Because the truncated pdf p_1 has a finite rms slope, the leading order term can be approximated through the central limit theorem using a form similar to the traditional geometrical optics method with the rms slope determined using the truncated pdf:

$$\sigma \approx \frac{Q^4 |\Gamma|^2}{Q_z^2} \frac{\hat{m}_0^{Q_z^2 h^2}}{Q_z^2 \tilde{s}^2} e^{-\frac{Q_\rho^2}{Q_z^2 \tilde{s}^2}} \quad (44)$$

where the moments of p_1 of interest are

$$m_0(k_{max}) = h^2 \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \; p_1(q_x, q_y) \tag{45}$$

$$= h^2 \left(1 - \frac{1}{R}\right) \tag{46}$$

$$\hat{m}_0(k_{max}) = \frac{m_0(k_{max})}{h^2} \tag{47}$$

$$m_2(k_{max}) = h^2 \int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \; q_\rho^2 \; p_1(q_x, q_y) \tag{48}$$

$$= \frac{h^2}{l^2} \frac{(R-1)^2}{R} \tag{49}$$

with $R = \sqrt{1 + (k_{max} l)^2}$. The slope quantity $\tilde{s}^2$ in equation (44) is to be taken as $m_2(k_{max})$ at present. The factor $\hat{m}_0^{Q_z^2 h^2}$ is obtained in equation (44) in the central limit theorem due to the fact that p_1 integrated over the (q_x, q_y) plane is $\hat{m}_0$. As is well known, m_2 (the slope variance of the truncated exponential surface) approaches infinity as k_{max} becomes arbitrarily large.

C. Prediction of cutoff wavenumber

The final issue involves determination of the truncation wavenumber k_{max} to utilize. As discussed in [6], the norming constant b_n in (41) can also be determined through the multi-dimensional central limit theorem for infinite variance random vectors. The theorem specifies that the norming constant satisfies an equation involving the rms slope of the truncated surface, as follows:

$$\frac{b_n^2}{l^2} = \frac{n}{h^2} m_2\left(\frac{b_n}{l}\right) \tag{50}$$

which for the known $b_n = n = Q_z^2 h^2$ provides a condition on the effective slope as

$$\tilde{s}^2 = m_2\left(\frac{Q_z^2 h^2}{l}\right) \tag{51}$$

This equation can be shown to hold for the previously defined value of $\tilde{s}^2$ (equation (36)) as $Q_z^2 h^2$ becomes large through the use of equation (49). Because the argument $Q_z^2 h^2 / l$ of m_2 above represents a truncation of the surface spectrum, reference [6] argues that

$$k_{max} = Q_z^2 h^2 / l = Q_z \tilde{s} \tag{52}$$

should be used as the cutoff wavenumber when defining a truncated rms slope for the surface. The original derivation of Hagfors' law [7] obtained an identical cutoff wavenumber based on consideration of the regions of maximum contribution in the physical optics integral.

D. Evaluation of traditional GO approximation for truncated surface

Using the definition of $\hat{m}_0$ for this value of k_{max}, it can be shown that $\hat{m}_0(k_{max})^{Q_z^2 h^2}$ approaches to $\frac{1}{e}$ in the high frequency limit, and the scattering cross section becomes

$$\sigma \approx \left(\frac{Q^4 |\Gamma|^2}{Q_z^4} \frac{1}{\tilde{s}^2} e^{-\frac{Q_\rho^2}{Q_z^2 \tilde{s}^2}}\right) \left(\frac{1}{e}\right) \tag{53}$$

$$= \frac{|\Gamma|^2}{e \tilde{s}^2} e^{-\frac{Q_\rho^2}{Q_z^2 \tilde{s}^2}} \left(1 + \frac{Q_\rho^2}{Q_z^2}\right)^2 \tag{54}$$

While this result is similar to the complete high frequency approximation (equation (35)), it fails to match equation (35) in the high frequency limit for specular scattering by the factor $\frac{2}{e}$, which represents a 1.33 dB error. In addition, equation (54)'s dependence on Q_ρ^2 / Q_z^2 at first order is one half that of equation (35).

Equation (54) is in error for two reasons. First, the slope variance of the truncated surface (i.e. p_1) is large enough that the Gaussian function obtained by applying the central limit theorem to the multiple convolution of p_1 is not sufficiently accurate. The true multiple convolution of p_1 is more peaked and has longer tails than a Gaussian (e.g., it is more like a long-tailed stable distribution than a Gaussian). Physically, this means that the small-scale surface roughness is large enough in amplitude even for the truncated spectrum that the facets of the surface cannot be approximated as smooth. Second, the small scale roughness above the cutoff wavenumber in p_2 is large enough that the second term of the convolution series is non-negligible relative to the first term even for specular scattering. This has essentially the same physical interpretation as the first error contribution. Above-cutoff surface components cause long-wavelength surface facets to scatter diffusely, so facets tilted away by the long-wavelength components scatter appreciably in the specular direction. The second series term is therefore larger than it would be for a smoother surface. Note these errors are more pronounced for the exponential surface (k_ρ^{-3} spectrum) than they would be for an ocean-like surface (k_ρ^{-4}) due to the much greater amplitudes of the short scale roughness in the exponential case.

Although Equation (54) is not asymptotically exact in the high frequency limit, it remains a close approximation to the exact value of the physical optics integral. This suggests that the surface spectrum can be truncated at the cutoff wavenumber k_{max} with only a modest impact on PO predicted scattering cross sections. The next section provides further analysis of the issue of truncation of the spectrum.

VI. Computation of PO scattering from a truncated exponential surface

The preceding section showed that a truncated exponential surface spectrum with the cutoff wavenumber $k_{max} = Q_z^2 h^2 / l$ could produce PO scattering similar to that from a true exponential spectrum in the high frequency limit. However the preceding section also showed that the accuracy of the traditional GO form (54) that results from a single term approximation to the repeated convolution may be limited. In this section, a recently developed model for "band-limited" exponential surfaces [9] is utilized in order to investigate further the effects of surface truncation on PO NRCS predictions.

The band-limited exponential surface model developed in [9] truncates the high frequency portion of the exponential

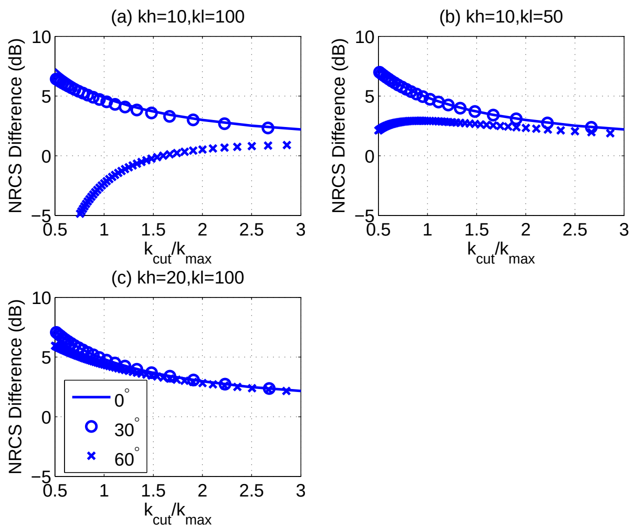

Fig. 5. Convergence of PO NRCS predictions for band-limited exponential surfaces truncated at wavenumber k_{cut} to those obtained from equation (35). Ratio indicated in decibels as a function of k_{cut}/k_{max} for backscattering at incidence angles of 0, 30, and 60 degrees and for three sets of surface Kh, Kl parameters (indicated in figure titles).

spectrum by multiplying it by a Gaussian roll-off at high frequencies. This Gaussian rolloff is defined to take on the value e^{-1} at a wavenumber k_{cut}. Reference [9] discusses the correlation function that results for this surface, as well as a method for evaluating PO NRCS predictions. While this band-limited surface model is not precisely a truncated exponential surface of the type defined in equation (43), an examination of NRCS predictions for the band-limited model as k_{cut} is varied can nevertheless provide information on the effect of spectral truncations.

PO scattering predictions were computed using the band-limited model for k_{cut} ranging from $k_{max}/2$ to $3k_{max}$. Results of these computations are illustrated in Figure 5 in which the ratio of the complete PO NRCS obtained from the bandlimited surfaces to that obtained from the high frequency approximation for true exponential surfaces (equation (35)) is plotted in decibels, versus the ratio k_{cut}/k_{max}. Computations were performed for the cases $Kh = 10$, $Kl = 100$ (plot (a)), $Kh = 10$,$Kl = 50$ (plot (b)), and $Kh = 20$, $Kl = 100$ (plot (c)). The curves shown represent NRCS ratios for backscattering at 0, 30, and 60 degrees incidence. The results show convergence as k_{cut} increases, with the level of error reduced for k_{cut} in the vicinity of k_{max}, though there is spread in the rate of convergence among the curves illustrated. In particular, results for larger Kh or h/L values typically converge more slowly due to the increasing importance of terms neglected in the single term convolutional form. Overall, the results of Figure 5 demonstrate that surface features below the cutoff wavenumber k_{max} determined in Section V appear to play an important role in the total scattered returns, but the contribution of shorter scale features are not completely negligible. Nevertheless, the spectral truncation point k_{max} does provide a reasonable order of magnitude estimate for the length scales producing the dominant scattering contributions.

VII. A VALIDITY CONDITION FOR THE USE OF PHYSICAL OPTICS WITH EXPONENTIALLY CORRELATED SURFACES

Reference [6] proposes a method for assessing the potential accuracy of the physical optics method for a given surface, through a comparison of the average radius of curvature of the truncated surface to the electromagnetic wavelength. The region where the physical optics method is expected to be applicable for backscattering is described by

$$\frac{1}{\sqrt{m_4}}\cos^3\theta \quad >> \quad \frac{2\pi}{K} \tag{55}$$

where θ is the polar observation angle, and m_4 is the fourth moment of the truncated surface spectrum, given by

$$m_4(k_{max}) = h^2\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \; q_\rho^4 \; p_1(q_x,q_y) \tag{56}$$

$$= \frac{h^2}{3l^4}\frac{(R-1)^3\,(R+3)}{R} \tag{57}$$

Substituting equation (57) into equation (55) along with the choice $k_{max} = Q_z^2 h^2/l$ yields the following approximate condition for applicability of physical optics upon simplification in the high frequency limit:

$$(Kh)\left(\frac{h}{l}\right) << 0.2 \tag{58}$$

which can also be written as

$$\tilde{s} << 0.2\frac{Q_z}{K} \tag{59}$$

or

$$C >> 6.25 \tag{60}$$

This first and last conditions are found to be independent of the incidence angle. The results show that the ratio h/l (a "nominal slope" of the surface) must be extremely small as Kh becomes large in order for the physical optics theory to be applicable, and that the PO theory is invalid as the frequency becomes infinite (note that $C = (l/[2Kh^2])^2$ decreases as the frequency increases.)

A comparison of PO predictions with those of a numerically exact method was performed in order to provide a basic assessment of this validity condition. While the capabilities of Monte Carlo methods for computing rough surface 3-D scattering problems continue to increase [16], exponentially correlated surfaces present significant computational challenges due to the large range of roughness scales involved. To reduce computational requirements, numerical simulations were performed for surfaces rough in one direction only (i.e. 2-D scattering problems), using the method of moments (MOM) technique described in [17].

Examination of the PO method in this case requires a re-evaluation of equations (15), (35), (54), and the predicted k_{max} value for one dimensional surfaces. The appendix of reference [17] provides the relevant formulation for equation (15), as well as the definition of the bistatic scattering coefficients to be illustrated. Note however that here the reflection coefficient described in the appendix of reference [17] is replaced with the reflection coefficient evaluated at the stationary phase point.

Expressions analogous to equations (35) and (54) can then be derived as shown in Sections IV and V. Explicit expressions are very similar to those for the 3-D scattering problem, and are omitted here to avoid confusion. The k_{max} value predicted by the multi-dimensional central limit theorem is found to be that of the 3-D case multiplied by $\sqrt{2}$.

One dimensional numerical simulations used penetrable surfaces with length 100 (for the smallest correlation length to be shown) or 200 wavelengths with the relative dielectric constant typically chosen as $15.57 + i3.71$. A tapered wave incident field was utilized to reduce surface edge scattering effects, and a high sampling density retained in order to ensure accurate computations as described in [17]. Results were computed for 150 (smallest correlation length case) or 60 surface realizations with either horizontally (labeled "TE") or vertically (labeled "TM") polarized incident fields. The results to be shown are for in-plane bistatic scattering patterns at incidence angle 40 degrees, with a default Kh value of 2.51. Values of Kl were chosen as 12.6, 41.9, or 83.8, so that the left hand side of the validity condition (equation (58)) is respectively 0.503, 0.1508, or 0.0754 (corresponding C values are 1, 11.1, and 44.3, respectively, while corresponding $\tilde{s}$ values for backscattering are 0.77, 0.23, and 0.12, respectively). Numerical simulations were also performed for $Kh = 5.02$ and $Kl = 83.8$ and 167.4 (using a slightly different dielectric constant of $14.15 + i5.21$) in order to investigate the behavior of the PO approximation as the frequency is increased; these cases represent a doubling of frequency from the $Kh = 2.54$, $Kl = 41.9$ and 83.8 simulations, and have corresponding validity parameters of 0.301 and 0.1508, double those obtained in the lower frequency computation.

Figure 6 illustrates the comparison of true PO (solid curves, analogous to equation (15)), truncated surface GO (dotted curves, analogous to equation (54)), and numerically computed results (dashed curves) for the three lower frequency cases as well as the $Kh = 5.02$, $Kl = 83.8$ case for TE and TM incident fields. Results analogous to equation (35) are not plotted, but were within 0.7 dB of true PO predictions for all cases and all scattering angles. The comparisons in plots (a) through (f) clearly show an improving accuracy of PO predictions compared to MOM as the validity parameter is reduced. Near specular results are typically most accurate in TE polarization, while a tendency of PO to overestimate TM scattering in the near specular region is observed particularly for larger values of the validity parameter. However for non-specular regions, these behaviors are reversed, and PO more closely follows the MOM results in the TM case. The truncated surface GO approximation is observed to provide a reasonable prediction of PO results in the near-specular region, with relatively small errors that in the near specular region are largest at the specular angle.

Plots (g) and (h) correspond to the plots (c) and (d) cases when the frequency is doubled. The results show a decreased accuracy in the PO approximation as the frequency is increased, particularly for TM polarization. Simulations with the frequency doubled from the plots (e) and (f) cases were also performed, but are not illustrated because the results are nearly identical to those at the lower frequency in plots (c) and (d), and again indicate a decrease in the accuracy of the PO approximation as the frequency is increased.

Reference [17] states that the physical optics approximation is "invalid at any frequency" for exponentially correlated surfaces. However this conclusion was reached with regard to the relatively large slope surfaces investigated in reference [17]. The results of Figure 6 clearly show that the PO solution can provide an accurate prediction of exponential surface near-specular scattering, so long as surfaces with small "nominal slopes" (for which the validity condition (58) is satisfied) are considered. Further more detailed evaluations of the accuracy of the PO theory for exponentially correlated surfaces must await future numerical simulations.

VIII. Conclusions

This paper has examined the geometrical and physical optics theories of scattering for exponentially correlated surfaces. As an introduction, an analytical form for PO specular scattering from surfaces with Gaussian correlation functions was presented, and found to provide higher-order corrections to the standard geometrical optics theory in the high frequency limit. A similar expansion was derived for specular scattering from exponentially correlated surfaces, and found to involve a frequency dependent term similar to an effective rms slope of the surface. Both these specular expansions can be useful for assessing the accuracy that the high frequency approximation of physical optics may achieve.

A further detailed examination of the high frequency limit for exponentially correlated surfaces was then performed, following the methods of [5]-[6] which have primarily been applied for pure power-law spectrum (i.e. ocean-like) surfaces in the past. Equation (35) is Hagfors' Law, which provides the leading order high frequency approximation for PO scattering from exponential surfaces in a form that is convenient for use in applications. The approximation has a form that is distinctly different from the traditional GO approximation, and remains dependent on frequency.

Attempts were then made to assess whether the exponential results could be interpreted in terms of scattering from a spectrally truncated surface, as has often been proposed in the past. The approach of [6] applied for exponentially correlated surfaces yielded the Hagfors' Law cutoff wavenumber $k_{max} = Q_z^2 h^2 / l$ for determining surface rms slopes; when used, this cutoff wavenumber was found to produce the slope-like parameters of equation (35) in the high frequency limit. However, the single term approximation of the repeated convolution described in [6] was found to produce a 1.33 dB error for specular scattering even in the high frequency limit, and also to fail to match the correct variations with incidence angle. Nevertheless, computations of PO backscattering from band-limited exponential surfaces of the type proposed in [9] showed that NRCS predictions indeed showed some degree of convergence to the true high frequency limit when surfaces were truncated at wavenumbers near the derived k_{max} value.

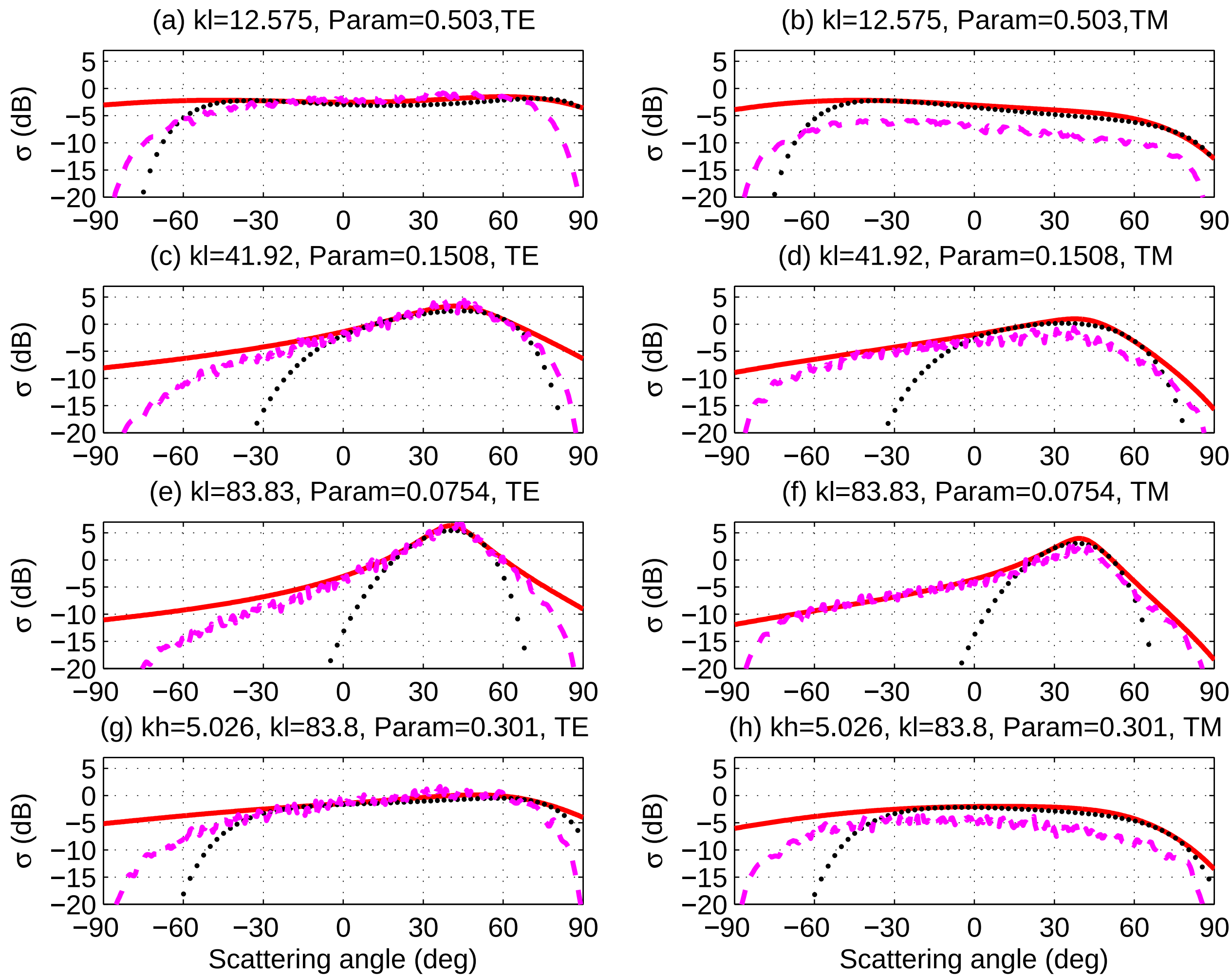

Fig. 6. Comparison of PO (solid curves), truncated GO (dotted curves), and numerically computed (MOM, dashed curves) bistatic scattering coefficients for one dimensional surfaces with $Kh = 2.51$ or $Kh = 5.02$ (plots (g) and (h)), relative permittivity $15.57 + i3.71$ $(14.15 + i5.21$ plots (g) and (h)) and for incidence angle 40 degrees. Kl parameters and incident field polarizations are shown in Figure titles. The value of left hand side of the validity condition (equation (58)) is also shown in the figure titles.

The obtained k_{max} value can be written as

$$k_{max} = Q_z (Q_z h) \left(\frac{h}{l}\right) = Q_z \tilde{s} \tag{61}$$

$$= 4K \cos^2\theta \, (Kh) \left(\frac{h}{l}\right) \tag{62}$$

where the final form applies for backscattering. The inclusion of the factor Kh in the k_{max} definition is particularly unusual for a high frequency theory, since it will approach infinity in the limit, indicating that even surface features much smaller than the electromagnetic wavelength contribute to PO scattered returns. These properties also make clear that the effective rms slope of the surface $\tilde{s}$ increases with frequency. The extreme high frequency content of the exponential surface results in this behavior.

Furthermore, it is interesting to observe that the rms height of the truncated portion of the surface (given by $h_{\mathrm{trun}} = h/\sqrt{R}$ from equation (46)) always satisfies

$$kh_{\mathrm{trun}} = \frac{k}{Q_z} \tag{63}$$

for the specified value of k_{max}; this product is independent of frequency or surface statistics. This choice of cutoff is consistent with rules often put forward in the two-scale theory of rough surface scattering, in that the integrated rms height of the surfaces roughness removed from the "large scale" surface is small compared to the wavelength.

The validity condition derived shows that use of the PO theory for exponential surfaces requires very "smooth" surface statistics, and that for fixed surface statistics the PO theory becomes less applicable as the frequency is increased, in contrast to traditional expectations. Because the validity condition requires that the product of the two final terms in equation (62) is $<< 0.2$ (as well as the Hagfors parameter $C >> 6.25$), the obtained k_{max} value must be less than approximately K in order to produce a reasonably accurate PO prediction of surface scattering. This is not a surprising result, but indicates that PO accuracy is lost in the true high frequency limit due to the increasing importance of contributions from surface roughness length scales that are small compared to the electromagnetic wavelength. A set of numerically computed data was utilized to provide a preliminary verification of the validity condition. Further tests of PO accuracy and the derived validity conditions will require additional studies and simulations with more accurate methods.

The consequences of the derived validity condition can be important for continued studies of scattering from exponentially correlated surfaces, as many of the approximate electromagnetic models currently applied reproduce PO in the high frequency limit. Careful evaluations of other approximate theories [1], such as the integration equation method (IEM) or the small slope approximation (SSA), will be required in order to ensure that reasonable predictions are achieved for exponentially correlated surfaces with rms heights large compared to the electromagnetic wavelength.

REFERENCES

[1] T. M. Elfouhaily and C. A. Guérin, "A critical survey of approximate scattering wave theories from random rough surfaces," *Waves in Random Media,* vol. 14, pp. R1–R40, 2004.
[2] L. Tsang, J. A. Kong, and R. T. Shin, *Theory of Microwave Remote Sensing,* New York: Wiley, 1985.
[3] J. T. Johnson, R. T. Shin, J. A. Kong, L. Tsang, and K. Pak, "A numerical study of the composite surface model for ocean scattering," *IEEE Trans. Geosc. Remote Sens.,* vol. 36, no. 1, pp. 72-83, 1998.
[4] A. Purcell, "Asymptotic expansion of the backscattering strength for a Pierson-Moskowitz sea surface," *J. Computational Acoust.,* vol. 9, pp. 1287–1309, 2001.
[5] K. F. Warnick and D. V. Arnold, "Generalization of the geometrical optics scattering limit for rough conducting surfaces," *J. Opt. Soc. Am. A,* vol. 15, pp. 2355–2361, 1998.
[6] K. F. Warnick, F. W. Millet, and D. V. Arnold, "Physical and geometrical optics for 2-D rough surfaces with power-law height spectra," *IEEE Trans. Ant. Prop.,* vol. 53, pp. 922–932, 2005.
[7] T. Hagfors, "Remote probing of the moon by infrared and microwave emissions and by radar," *Radio Science,* vol. 5, pp. 189–227, 1970.
[8] A. K. Sultan-Salem and G. L. Tyler, "Hagfors' law revisited," *J. Geophys. Res.,* vol. 111, doi:10.1029/2005JE002489, 2006.
[9] A. Darawankul and J. T. Johnson, "Band-limited exponential correlation function for rough surface scattering," *IEEE Trans. Geosc. Rem. Sens.,* vol. 45, pp. 1198–1206, 2007.
[10] T. Elfouhaily and J. T. Johnson, "Extension of the local curvature approximation to third order and full tilt invariance," *Waves in Random and Complex Media,* vol. 16, pp. 97–119, 2006.
[11] T. M. Elfouhaily and J. T. Johnson, "A new model for rough surface scattering," accepted by *IEEE Trans. Geosc. Rem. Sens.,* 2007.
[12] *Handbook of Mathematical Functions,* M. Abramowitz and I. A. Stegun, eds, Ch. 5 "Exponential integral and related functions," Dover: New York, 1970.
[13] A. K. Fung, *Microwave scattering and emission models and their applications,* Artech: Norwood, MA, 1994.
[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products, Fifth Edition,* London: Academic Press Inc., 1994.
[15] G. Samorodnitsky and M. S. Taqqu, *Stable non-Gaussian random processes,* New York: Chapman Hall, 1994.
[16] J. T. Johnson, "Computer simulation studies of rough surface scattering," Chapter in *Nanoscale Light Scattering and Surface Roughness,* A. A. Maradudin, Ed., Springer, 2007.
[17] P. Xu and L. Tsang, "Bistatic scattering and emissivities of lossy dielectric surfaces with exponential correlation functions," *IEEE Trans. Geosc. Rem. Sens.,* vol. 45, pp. 62–72, 2007.