Optical Theorem for Electromagnetic Scattering by a Three-Dimensional Scatterer in the Presence of a Lossless Half Space

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***Abstract*—The classical optical theorem for a scatterer in free space is useful in computing the total extinction cross section when the scattering amplitude in the forward scattering direction is known. In this letter, the classical optical theorem is extended to the case of a scatterer in the presence of a lossless half space. The extended optical theorem is derived based on energy conservation concepts, and the method of stationary phase is employed to obtain the final form. It is found that the total extinction cross section is related to the scattering amplitudes in the specular directions for reflected and transmitted fields. Numerical results are presented to illustrate use of the theorem for evaluating the energy conservation properties of an electromagnetic simulation.**

***Index Terms*—Electromagnetic scattering, electromagnetic theory, subsurface sensing.**

I. Introduction

THE OPTICAL theorem has been known for more than a century and has been frequently applied in many areas of physics [1]–[14]. In electromagnetics (EM), the classical optical theorem is applicable to scattering of a plane wave by objects in homogeneous media (i.e., background media having constant permittivity and permeability throughout all space). The theorem is useful for computing the total extinction cross section when the scattering amplitude in the forward scattering direction is known [2]. This relation facilitates the calculation of the total extinction cross section, since it otherwise must be obtained by integration of the power flux density over a closed surface bounding the scatterer. In addition, the theorem also serves as a energy conservation condition in the verification of analytical and numerical methods in EM scattering theory. Although in the literature the optical theorem is almost exclusively formulated in the frequency domain, the theorem can be formulated in the time domain as well [3], [4] (also known as a time domain energy theorem).

Although the classical optical theorem is very useful, it is limited to the case of scatterers located in homogeneous media. A first extension to this case would involve scatterers in the presence of a lossless isotropic half space; this case is of practical value due to interest in problems involving scattering from an object in the presence of the ground surface. The extended optical theorem also finds application in the computation of the thermally emitted power flux from an object in the presence of a half space [15]. The theorem can be derived by using energy conservation concepts and the method of stationary phase [6] to evaluate associated double integrals asymptotically. Note the use of dyadic Green's functions in the formulation (as in [2] for the homogeneous medium case) is not required. Previous works have considered similar problems for acoustic scattering in waveguides [7], or for EM scattering from two-dimensional scatterers in the presence of layered media [8]–[10]. However, the case of EM scattering from a three dimensional scatterer in the presence of an isotropic half space apparently has not been previously presented.

In Section II, the basic theorem is formulated, and its simplified form is presented in Section III. Numerical results illustrating use of the theorem are provided in Sections IV, and Section V presents conclusions. An $e^{-i\omega t}$ harmonic time convention is assumed and suppressed throughout, where ω is the radian frequency.

II. Formulation

Without loss of generality, consider a scatterer at the interface $(z = 0)$ of a lossless isotropic half space as shown in Fig. 1. It will be clear later that the scatterer could be located at an arbitrary position above or below the interface without changing the formulation. In Fig. 1, the region above the interface, denoted as Region 1, is a homogeneous, lossless, and isotropic medium described by electric permittivity ϵ_1 and magnetic permeability μ_1. The region below the interface, denoted as Region 2, is also a homogeneous, lossless, and isotropic medium described by electric permittivity ϵ_2 and magnetic permeability μ_2. An incident plane wave illuminates this geometry in the direction of incidence $\hat{k}_i$ as shown in Fig. 1. For practical cases of interest involving fields incident in free space, the medium in Region 2 is usually denser than the medium in Region 1, which means that there is no total reflection phenomenon in the absence of the scatterer. The formulation in this section is restricted to cases when the total reflection phenomenon does not exist in the absence of the scatterer. Let S be the spherical surface at infinity, i.e., $S = S_1 + S_2$. In Fig. 1, the surfaces at infinity S_1 and S_2 denote upper and lower hemispherical surfaces of infinite radius, respectively, and $\hat{n}$ is the outward unit vector normal to these surfaces.

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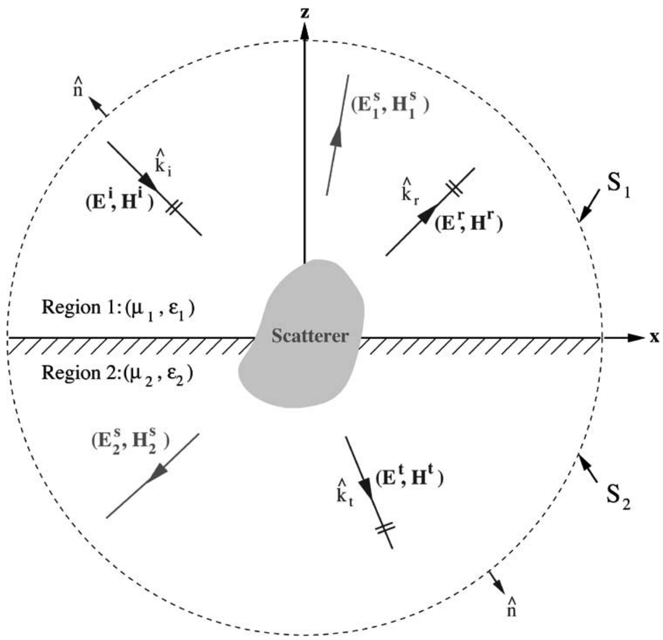

Fig. 1. Scatterer on the interface of a lossless isotropic half space is illuminated by an incident plane wave.

A. Energy Conservation in the Absence of a Scatterer

Begin by considering the problem in the absence of the scatterer. The total field consists of the incident and reflected plane waves in Region 1, and the transmitted plane wave in Region 2. Let $\mathbf{E}^i$, $\mathbf{E}^r$, and $\mathbf{E}^t$ denote the incident, reflected, and transmitted plane-wave electric fields, respectively, with corresponding magnetic fields $\mathbf{H}^i$, $\mathbf{H}^r$, and $\mathbf{H}^t$. These fields can be written as

$$\mathbf{E}^i = \hat{e}_i E_0 e^{i\mathbf{k}_i \cdot \mathbf{r}} \tag{1}$$

$$\mathbf{H}^i = \frac{1}{\eta_1} \hat{k}_i \times \mathbf{E}^i, \tag{2}$$

$$\mathbf{E}^r = \hat{e}_r E_0^r e^{i\mathbf{k}_r \cdot \mathbf{r}} \tag{3}$$

$$\mathbf{H}^r = \frac{1}{\eta_1} \hat{k}_r \times \mathbf{E}^r \tag{4}$$

$$\mathbf{E}^t = \hat{e}_t E_0^t e^{i\mathbf{k}_t \cdot \mathbf{r}} \tag{5}$$

$$\mathbf{H}^t = \frac{1}{\eta_2} \hat{k}_t \times \mathbf{E}^t \tag{6}$$

where E_0, E_0^r, and E_0^t are the plane wave amplitudes, $\hat{e}_i$, $\hat{e}_r$, and $\hat{e}_t$ are their polarization vectors, and $\mathbf{k}_i$, $\mathbf{k}_r$, and $\mathbf{k}_t$ are their propagation vectors. The propagation vectors can be further expressed as

$$\mathbf{k}_i = k_1 \hat{k}_i \tag{7}$$

$$\mathbf{k}_r = k_1 \hat{k}_r \tag{8}$$

$$\mathbf{k}_t = k_2 \hat{k}_t \tag{9}$$

with the unit vectors $\hat{k}_i$, $\hat{k}_r$, and $\hat{k}_t$ in Fig. 1 characterized by the angles (θ_i, ϕ_i), (θ_r, ϕ_r), and (θ_t, ϕ_t) in spherical coordinates, respectively. Mathematically, these unit vectors are given as follows:

$$\hat{k}_j = \hat{x} \sin\theta_j \cos\phi_j + \hat{y} \sin\theta_j \sin\phi_j + \hat{z} \cos\theta_j \tag{10}$$

where $0 \le \phi_j < 2\pi$, and $j = i, r$ or t. In this problem, θ_i and θ_r lie in the following ranges: $\pi/2 < \theta_i \le \pi$ and $0 \le \theta_r < \pi/2$. In the absence of the total reflection and with a lossless half space, θ_t is always real and lies in the following range: $\pi/2 < \theta_t \le \pi$. Here $k_j = \omega\sqrt{\mu_j \epsilon_j}$ and $\eta_j = \sqrt{\mu_j/\epsilon_j}$ denote the wavenumber and the intrinsic impedance of the medium in Region j, respectively, where $j = 1$ or 2. Finally

$$\mathbf{r} = r\hat{r} \tag{11}$$

where r is the distance from the origin to an observation point (r, θ, ϕ), and $\hat{r}$ is defined as

$$\hat{r} = \hat{x} \sin\theta \cos\phi + \hat{y} \sin\theta \sin\phi + \hat{z} \cos\theta. \tag{12}$$

Once values of E_0, $\hat{k}_i$, and $\hat{e}_i$ are specified for the incident plane wave, the remaining quantities in the above expressions are determined through the standard solution for a half-space reflection problem.

The time-average Poynting vector is

$$\langle \mathbf{S} \rangle = \frac{1}{2} \mathrm{Re} \left\{ \mathbf{E} \times \mathbf{H}^* \right\} \tag{13}$$

where Re$[\cdot]$ denotes the real part of its argument, and the superscript "$*$" denotes the complex conjugate operator. Applying energy conservation concepts and using the fact that the half space is lossless, the following equation is obtained:

$$\int_{S_1} \langle \mathbf{S} \rangle \cdot \hat{n} dS + \int_{S_2} \langle \mathbf{S} \rangle \cdot \hat{n} dS = 0 \tag{14}$$

where $dS = r^2 \sin\theta d\theta d\phi$ and $\hat{n} = \hat{r}$. Using (13) and the total field $\mathbf{E} = \mathbf{E}^i + \mathbf{E}^r$ in Region 1, the first term on the left-hand side (LHS) of (14) can be expressed as

$$\int_{S_1} \langle \mathbf{S} \rangle \cdot \hat{n} dS = W^i + W^r + W^{ir} \tag{15}$$

where

$$W^i = \frac{1}{2} \mathrm{Re} \left\{ \int_{S_1} \left(\mathbf{E}^i \times \mathbf{H}^{i*} \right) \cdot \hat{n} dS \right\} \tag{16}$$

$$W^r = \frac{1}{2} \mathrm{Re} \left\{ \int_{S_1} \left(\mathbf{E}^r \times \mathbf{H}^{r*} \right) \cdot \hat{n} dS \right\} \tag{17}$$

$$W^{ir} = \frac{1}{2} \mathrm{Re} \left\{ \int_{S_1} \left(\mathbf{E}^r \times \mathbf{H}^{i*} + \mathbf{E}^i \times \mathbf{H}^{r*} \right) \cdot \hat{n} dS \right\}. \tag{18}$$

Similarly, the second term on the LHS of (14) can be expressed as

$$\int_{S_2} \langle \mathbf{S} \rangle \cdot \hat{n} dS = W^t \tag{19}$$

where

$$W^t = \frac{1}{2} \mathrm{Re} \left\{ \int_{S_2} \left(\mathbf{E}^t \times \mathbf{H}^{t*} \right) \cdot \hat{n} dS \right\}. \tag{20}$$

Substituting (15) and (19) into (14) yields

$$(W^i + W^r + W^{ir}) + W^t = 0 \tag{21}$$

which is the equation of power conservation in the absence of a scatterer.

B. Energy Conservation in the Presence of a Scatterer

Next, consider the case when the scatterer is present. In this case, scattered field intensities $(\mathbf{E}_1^s, \mathbf{H}_1^s)$ and $(\mathbf{E}_2^s, \mathbf{H}_2^s)$ are produced by the scatterer in Regions 1 and 2, respectively. Unlike the plane waves $(\mathbf{E}^i, \mathbf{H}^i)$, $(\mathbf{E}^r, \mathbf{H}^r)$, and $(\mathbf{E}^t, \mathbf{H}^t)$, $(\mathbf{E}_1^s, \mathbf{H}_1^s)$ and $(\mathbf{E}_2^s, \mathbf{H}_2^s)$ are spherical waves on surfaces S_1 and S_2, respectively, so that

$$\mathbf{E}_1^s(\mathbf{r}) = \frac{e^{ik_1 r}}{r}\mathbf{F}_1^s(\theta, \phi) \tag{22}$$
$$\mathbf{H}_1^s(\mathbf{r}) = \frac{1}{\eta_1}\hat{r} \times \mathbf{E}_1^s(\mathbf{r}) \tag{23}$$
$$\mathbf{E}_2^s(\mathbf{r}) = \frac{e^{ik_2 r}}{r}\mathbf{F}_2^s(\theta, \phi) \tag{24}$$
$$\mathbf{H}_2^s(\mathbf{r}) = \frac{1}{\eta_2}\hat{r} \times \mathbf{E}_2^s(\mathbf{r}) \tag{25}$$

where $\mathbf{F}_1^s(\theta, \phi)$ and $\mathbf{F}_2^s(\theta, \phi)$ are the far-field scattering amplitudes associated with $\mathbf{E}_1^s$ and $\mathbf{E}_2^s$, respectively. These scattering amplitudes would be determined by solution of the EM scattering problem for a specific scatterer in the presence of a specific half space. Standard integral equations for this problem involve the "Sommerfeld" Green's functions [16], and a numerical solution would typically be required to determine the scattering amplitudes. The spherical wave forms in (22)–(25) as r approaches infinity are obtained from appropriate far-field limits of the Sommerfeld Green's functions, and are valid everywhere on surface S. For finite-sized scatterers, there are no additional "surface wave" contributions to radiated power in the far field [16], so such terms are not required in the following analysis.

Total fields $(\mathbf{E}, \mathbf{H})$ in the presence of a scatterer are then expressed as

$$\mathbf{E} = \begin{cases} \mathbf{E}^i + \mathbf{E}^r + \mathbf{E}_1^s, & \text{in Region 1} \\ \mathbf{E}^t + \mathbf{E}_2^s, & \text{in Region 2} \end{cases} \tag{26}$$
$$\mathbf{H} = \begin{cases} \mathbf{H}^i + \mathbf{H}^r + \mathbf{H}_1^s, & \text{in Region 1} \\ \mathbf{H}^t + \mathbf{H}_2^s, & \text{in Region 2.} \end{cases} \tag{27}$$

Let W^a denote the average power being *absorbed* by the scatterer. Applying energy conservation concepts and following the same procedure as in the case when the scatterer is absent, one obtains the following equation:

$$\int_{S_1} \langle \mathbf{S} \rangle \cdot \hat{n} dS + \int_{S_2} \langle \mathbf{S} \rangle \cdot \hat{n} dS = -W^a. \tag{28}$$

Note that there is a negative sign on the right-hand side (RHS) of (28), since the sum of the two terms on the LHS of (28) represents the difference between the power flowing out of the volume enclosed by the closed surface S and the power flowing into the volume. Using (26), (27), and (13), (28) can be reexpressed as

$$\left(W^i + W^r + W_1^s + W^{ir} + W_1^{irs}\right) + \left(W^t + W_2^s + W_2^{ts}\right) = -W^a \tag{29}$$

where W^i, W^r, W^{ir}, and W^t are defined as in (16)–(18) and (20), respectively, and

$$W_1^s = \frac{1}{2}\text{Re}\left\{\int_{S_1} (\mathbf{E}_1^s \times \mathbf{H}_1^{s*}) \cdot \hat{n} dS\right\} \tag{30}$$
$$W_2^s = \frac{1}{2}\text{Re}\left\{\int_{S_2} (\mathbf{E}_2^s \times \mathbf{H}_2^{s*}) \cdot \hat{n} dS\right\} \tag{31}$$
$$W_1^{irs} = \frac{1}{2}\text{Re}\left\{\int_{S_1} \left(\mathbf{E}_1^s \times \mathbf{H}^{i*} + \mathbf{E}_1^s \times \mathbf{H}^{r*} + \mathbf{E}^i \times \mathbf{H}_1^{s*} + \mathbf{E}^r \times \mathbf{H}_1^{s*}\right) \cdot \hat{n} dS\right\} \tag{32}$$
$$W_2^{ts} = \frac{1}{2}\text{Re}\left\{\int_{S_2} \left(\mathbf{E}_2^s \times \mathbf{H}^{t*} + \mathbf{E}^t \times \mathbf{H}_2^{s*}\right) \cdot \hat{n} dS\right\}. \tag{33}$$

Using (21), (29) can be rewritten as

$$-W' = W^s + W^a \tag{34}$$

where W' and W^s are defined as

$$W' = W_1^{irs} + W_2^{ts} \tag{35}$$
$$W^s = W_1^s + W_2^s. \tag{36}$$

Note that W^s in (36) represents the total power scattered by the scatterer into Regions 1 and 2, while W_a is the total power absorbed by the scatterer. Their total is the total extinction due to the presence of the scatterer, and is equal to $-W'$ by (34). The term W' contains contributions from the interaction between incident and reflected plane waves and scattered waves on the surface $S_1(W_1^{irs})$, and from the interaction between the transmitted plane wave and scattered waves on the surface S_2 (W_2^{ts}). Simplified forms for W_1^{irs} and W_2^{ts} would provide a convenient means for computation of scatterer extinction, and are sought in Section III.

III. Simplification

From (32), W_1^{irs} can be rewritten as

$$W_1^{irs} = W_1^{is} + W_1^{rs} \tag{37}$$

where

$$W_1^{is} = \frac{1}{2}\text{Re}\left\{\int_{S_1} \left(\mathbf{E}_1^s \times \mathbf{H}^{i*} + \mathbf{E}^i \times \mathbf{H}_1^{s*}\right) \cdot \hat{n} dS\right\} \tag{38}$$
$$W_1^{rs} = \frac{1}{2}\text{Re}\left\{\int_{S_1} (\mathbf{E}_1^s \times \mathbf{H}^{r*} + \mathbf{E}^r \times \mathbf{H}_1^{s*}) \cdot \hat{n} dS\right\}. \tag{39}$$

From (38), W_1^{is} can be rewritten as

$$W_1^{is} = \frac{1}{2}\text{Re}\left\{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \left(\mathbf{E}_1^s \times \mathbf{H}^{i*} + \mathbf{E}^i \times \mathbf{H}_1^{s*}\right) \cdot \hat{r} r^2 \sin\theta d\theta d\phi\right\}. \tag{40}$$

Substituting (2) and (23) into (40) and using appropriate vector identities, one obtains

$$W_1^{is} = \frac{1}{2\eta_1}\mathrm{Re}\left\{I_1^{is}\right\} \tag{41}$$

where

$$I_1^{is} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left(1+\hat{k}_i\cdot\hat{r}\right)\left(\mathbf{E}^{i*}\cdot\mathbf{E}_1^s\right) - \left(\mathbf{E}^{i*}\cdot\hat{r}\right)\left(\mathbf{E}_1^s\cdot\hat{k}_i\right)\right] r^2\sin\theta d\theta d\phi. \tag{42}$$

Substituting (1) and (22) into (42) yields

$$I_1^{is} = \left(E_0{}^* r e^{ik_1 r}\right)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\left[\left(1+\hat{k}_i\cdot\hat{r}\right)\left(\hat{e}_i\cdot\mathbf{F}_1^s\right) - \left(\hat{e}_i\cdot\hat{r}\right)\left(\mathbf{F}_1^s\cdot\hat{k}_i\right)\right] e^{-i\mathbf{k}_i\cdot\mathbf{r}}\sin\theta d\theta d\phi, \tag{43}$$

where $r\to\infty$ on S_1, and

$$\mathbf{k}_i\cdot\mathbf{r} = k_1 r\left(\sin\theta_i\cos\phi_i\sin\theta\cos\phi + \sin\theta_i\sin\phi_i\sin\theta\sin\phi + \cos\theta_i\cos\theta\right). \tag{44}$$

Applying the method of stationary phase [6] to evaluate the above double integral in (43) asymptotically, it can be shown that

$$I_1^{is} = 0 \tag{45}$$

due to the fact that there is *no* stationary phase point lying *inside* the angular domain of interest ($0\le\theta<\pi/2, 0\le\phi<2\pi$), and the end-point contributions vanish as $r\to\infty$. Substituting (45) into (41) yields

$$W_1^{is} = 0. \tag{46}$$

Similarly, W_1^{rs} in (39) can be evaluated asymptotically by using the method of stationary phase, and it is found to be

$$W_1^{rs} = -\left(\frac{1}{f\mu_1}\right)\mathrm{Im}\left\{E_0^{r*}\hat{e}_r\cdot\mathbf{F}_1^s(\theta_r,\phi_r)\right\} \tag{47}$$

where Im[$\cdot$] denotes the imaginary part of its argument. Note that the only stationary phase point (θ_s,ϕ_s) that contributes to W_1^{rs} occurs at $(\theta_s,\phi_s)=(\theta_r,\phi_r)$, and the scattering vector $\mathbf{F}_1^s$ in (47) is evaluated at (θ_r,ϕ_r). It should be pointed out that the end-point contributions for the integral W_1^{rs} in (39) vanish as $r\to\infty$. Substituting (46) and (47) into (37) yields

$$W_1^{irs} = W_1^{rs} = -\left(\frac{1}{f\mu_1}\right)\mathrm{Im}\left\{E_0^{r*}\hat{e}_r\cdot\mathbf{F}_1^s(\theta_r,\phi_r)\right\}. \tag{48}$$

From (48), when there is no reflection (i.e., Brewster angle incidence in vertical polarization), W_1^{irs} is equal to zero, since $E_0^r=0$.

Next, consider the term W_2^{ts} in (33), which can be rewritten as

$$W_2^{ts} = \frac{1}{2}\mathrm{Re}\left\{\int_{\phi=0}^{2\pi}\int_{\theta=\pi/2}^{\pi}\cdot\left(\mathbf{E}_2^s\times\mathbf{H}^{t*}+\mathbf{E}^t\times\mathbf{H}_2^{s*}\right)\hat{r}r^2\sin\theta d\theta d\phi\right\}. \tag{49}$$

Using (5), (6), (24), and (25) and performing the same asymptotic analysis as above, (49) can be simplified to

$$W_2^{ts} = -\left(\frac{1}{f\mu_2}\right)\mathrm{Im}\left\{E_0^{t*}\hat{e}_t\cdot\mathbf{F}_2^s(\theta_t,\phi_t)\right\} \tag{50}$$

where the scattering vector $\mathbf{F}_2^s$ in (50) is evaluated at (θ_t,ϕ_t). Note that the only stationary phase point (θ_s,ϕ_s) that contributes to W_2^{ts} occurs at $(\theta_s,\phi_s)=(\theta_t,\phi_t)$ for the angular domain of interest (($\pi/2<\theta\le\pi, 0\le\phi<2\pi$). End-point contributions associated with the integral W_2^{ts} in (49) again vanish as $r\to\infty$.

Finally, using (35), (48) and (50), (34) can be expressed as

$$-W' = \left(\frac{1}{f\mu_1}\right)\mathrm{Im}\left\{E_0^{r*}\hat{e}_r\cdot\mathbf{F}_1^s(\theta_r,\phi_r)\right\} + \left(\frac{1}{f\mu_2}\right)\mathrm{Im}\left\{E_0^{t*}\hat{e}_t\cdot\mathbf{F}_2^s(\theta_t,\phi_t)\right\} = W^s+W^a, \tag{51}$$

when there is no total reflection. This simplified form shows that the extinction (W^s+W^a) caused by a scatterer in the presence of a lossless half space can be determined from knowledge of the scattering amplitudes $\mathbf{F}_1^s$ and $\mathbf{F}_2^s$ in the directions of the specularly reflected and transmitted plane waves, respectively. A verification of energy conservation for an EM computation in the presence of a lossless half space involves these quantities, and is illustrated in Section IV.

IV. Numerical Illustration

Consider a scatterer in the presence of a boundary between free space (Region 1) and a dielectric half space (Region 2) with $\epsilon_2=3\epsilon_0$ and $\mu_2=\mu_0$, where ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively The scatterer is a homogeneous, lossless dielectric cube with side length 0.2 free-space wavelengths, centered 0.5 free-space wavelengths either above or below the interface. The relative permittivity and permeability of the cube are equal to 2.0 and 1.0, respectively; the incident field is vertically polarized and incident from Region 1 ($\pi/2<\theta_i\le\pi$).

The ESP5 code [17] based on the method of moments (MoM) is employed to compute scattered fields from this object numerically as the incidence angle θ_i is varied with $\phi_i=0$. To provide a reasonable accuracy, the cube was discretized by using a MoM segment size of $0.06\lambda_c$, where λ_c is the wavelength in the dielectric cube. Note this discretization implies some error in the computations so that the optical theorem will not be satisfied exactly; the level of error observed gives some suggestion as to the accuracy of the numerical computation.

Since the cube is lossless, the average absorbed power W^a in (51) is equal to zero, resulting in

$$W_1^s - \left(\frac{1}{f\mu_1}\right)\mathrm{Im}\left\{E_0^{r*}\hat{e}_r\cdot\mathbf{F}_1^s(\theta_r,\phi_r)\right\} = -W_2^s + \left(\frac{1}{f\mu_2}\right)\mathrm{Im}\left\{E_0^{t*}\hat{e}_t\cdot\mathbf{F}_2^s(\theta_t,\phi_t)\right\}. \tag{52}$$

The above form of the extended optical theorem is used for convenience in illustrating the numerical results. The terms W_1^s and W_2^s above are the total powers bistatically scattered into Regions 1 and 2, respectively. Bistatic scattered fields

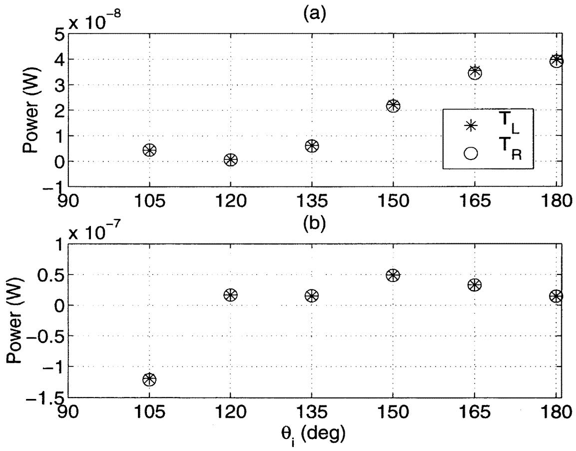

Fig. 2. T_L and T_R versus incidence angle θ_i when the cube is (a) below or (b) above the interface ($\epsilon_2 = 3\epsilon_0$ and $\mu_2 = \mu_0$).

were computed by the ESP5 code (for each incidence angle) over surfaces S_1 and S_2 in a 1 degree grid both in θ and ϕ. The corresponding powers were then numerically integrated over surfaces S_1 and S_2 to compute W_1^s and W_2^s. The remaining "cross" terms in (52) were evaluated from the fields computed in the specularly reflected and transmitted directions.

For convenience in comparison, let T_L and T_R denote the terms on the LHS and RHS of (52), respectively. Fig. 2 illustrates T_L and T_R versus incidence angle θ_i when the scatterer is either below [Fig. 2(a)] or above [Fig. 2(b)] the interface. It should be pointed out that the Brewster angle exists for this case at $\theta_i = 120°$, and no total reflection occurs, since medium 2 is denser than medium 1. The good agreement between T_L and T_R observed confirms that the ESP5 code is providing relatively good energy conservation (to within a few percent) in its predictions. Note that T_L and T_R can take either positive or negative values depending on the relationship between the scattered power and cross terms. The variations observed with observation angle are caused both by the varying half-space reflection properties and the varying response of the cubical scatterer as the observation angle is varied. Curves similar to these for thermal emission from a subsurface object have been reported in [15].

V. Conclusion

The optical theorem for an object in the presence of a lossless half space has been presented in this letter. Results show that knowledge of the far-field scattering amplitude in the specularly reflected and transmitted directions is sufficient to determine scatterer extinction properties. The theorem can be applied to evaluate the energy conservation properties of EM methods (as illustrated in the example) and to assess half-space effects on scatterer extinction behaviors. The method has also been applied in the computation of thermal emission from objects in the presence of a half space. Further extensions to include the case of critical angle incidence are in progress.

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