

PROBLEM SET 6
ECE 713 Spring Quarter 2008

Assigned: May 19th
Quiz: May 30th

Instructor: Joel Johnson

Problem 1

(a) Compute and plot versus height between 50 and 300 km the number density of electrons for a Chapman layer in which electrons are lost primarily by recombination. Assume the sun is at zenith, and the peak density is 10^{11} electrons/m³ at a height of 120 km. Use a scale height appropriate for the atmosphere below 100 km altitude.

(b) Repeat part (a) assuming a solar zenith angle of 45 degrees and using the same N0 constant as in part (a).

(c) Verify that your part (b) result follows the appropriate scaling law for zenith angle changes for the point at 120 km height.

Problem 2

(a) Using your problem 1(a) results, plot the relative permittivity of the ionosphere versus altitude at 4 MHz neglecting loss and magnetic field effects. Useful constants include the mass and charge of an electron, which are $m = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C respectively.

(b) Use the program ray.m to trace rays in this ionosphere launched from a transmitter at ground level at elevation angles of 50, 40, and 20 degrees. Assume a planar Earth, and plot the rays out to distances of 500 km. Remember that ray.m needs m(h) input, which for simplicity can be approximated as the square root of the dielectric constant here. You may want to modify the ray.m program to use a continuous profile for epsilon rather than the discrete file we used earlier to reduce discretization errors.

(c) Which rays in part (b) are reflected from higher altitudes? Explain. Also find the skip distance associated with this path.

Problem 3

In this problem we will study some of the properties of ionograms for lossless, non-magnetic plasmas. Assume the electron density of the ionosphere is such that it creates a plasma frequency $\omega_N^2 = \omega_0^2 \left(1 - \left[\frac{z-80}{80}\right]^2\right)$, where $\omega_0 = 2\pi(3 \times 10^6)$ rads/sec and ω_N and z are in rads/sec and km respectively.

(a) Plot the relative permittivity of the ionosphere from 0 to 160 km altitude at 1.5 MHz and 3 MHz.

(b) Plot the lowest altitude at which normally incident waves are reflected from 1.5 to 3 MHz. This is the actual reflection height.

(c) Plot the group velocity versus altitude from 0 km up the reflection height found in part (b) at 1.5 MHz and 3 MHz.

(d) Calculate the time required for a 1.5 MHz normally incident wave to propagate from 0 km to the reflection height and return. To do this, you will need break the path into many pieces and add up the time required to travel through each piece at the group velocity for that particular height. This process results in an integration. An integral you may find useful is

$$\int \frac{1}{\sqrt{a + bx + cx^2}} dx = \frac{1}{\sqrt{c}} \ln[b + 2cx + 2\sqrt{c(a + bx + cx^2)}].$$

(e) Calculate the time in part (d) for frequencies from 1.5 to 3 MHz in a 0.1 MHz step. Use these results to create an ionogram for this plasma, with plots of virtual height versus frequency. How does your ionogram compare to those in the notes? How do your virtual heights compare to the actual reflection heights found in part (b)?

Problem 4

(a) Obtain and print out an ionogram from the Millstone observatory site in Massachusetts from the world wide web at <http://digisonde.haystack.edu>. Note the horizontal axis of this plot is the vertical frequency in megahertz while the vertical axis is the virtual height in km. Specify the time this ionogram was obtained. While using the world wide web, you may also find it interesting to visit the <http://www.ngdc.noaa.gov/stp/stp.html>, <http://www.sel.noaa.gov> and <http://www.ips.gov.au> sites which also provide information on the ionosphere.

(b) Plot a set of transmission curves for distance 1000 km and oblique frequencies of 8, 10, and 12 MHz.

(c) For which of these frequencies is it possible to obtain ionospheric reflection on this path? What are the virtual heights of reflection?

(d) Find the MUF for this path.