EE 713 Problem Set #1 Solutions – Spring '0[illegible]

Problem 1

(a) Direct transmission. GPS is actually slightly affected by the ionosphere, only important because it is a very high precision system. Atmospheric absorption usually negligible.

(b) Direct + Earth reflections since a car must be near the ground. Other mechanisms unlikely to have much effect, mainly multipath off of surrounding objects.

(c) AM daytime → groundwave. Most everything else is negligible.

(d) FM out of line of sight → terrain diffraction. Maybe ducting in some cases.

(e) HF → ionospheric reflection.

Problem 2

note corrected error in eq (2.33) p39 in notes

(a) from notes, $\underline{\chi}(\omega) = \int_0^\infty du\, e^{-j\omega u} f(u) = A \int_0^\infty du\, e^{-j\omega u} e^{-u/\tau_1}$

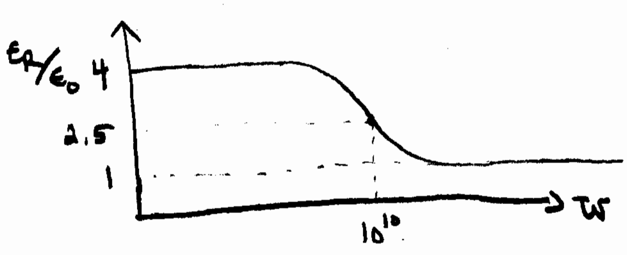
$$= A\int_0^\infty du\, e^{u(-j\omega - 1/\tau_1)} = \frac{A}{-j\omega - 1/\tau_1} e^{u(j\omega - 1/\tau_1)}\Big|_{u=0}^{u=\infty}$$

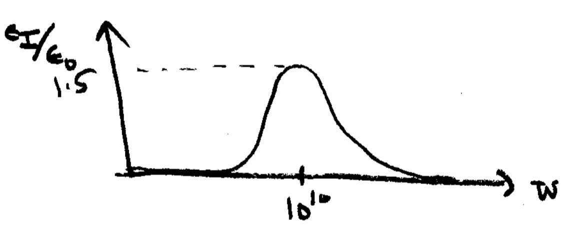
$$= \frac{A}{1/\tau_1 + j\omega} = \frac{A\tau_1}{1 + j\omega\tau_1} = \frac{3}{1 + j(\omega \times 10^{-10})}$$

(b) If $\omega \ll 10^{10}$, the $\omega \times 10^{-10}$ term is negligible compared to 1 and we get $\underline{\chi}(\omega) = 3 = A\tau_1$.

(c) $\underline{\epsilon} = \epsilon_0(1 + \underline{\chi}(\omega)) = \epsilon_0\left(1 + \frac{3}{1 + j\omega/10^{10}}\right) = \epsilon_0\left(\frac{4 + j\omega/10^{10}}{1 + j\omega/10^{10}}\right)$

$$= \epsilon_0\left(\frac{4 + j\omega/10^{10}}{1 + j\omega/10^{10}}\right)\left(\frac{1 - j\omega/10^{10}}{1 - j\omega/10^{10}}\right) = \epsilon_0\left(\frac{4 + \omega^2/10^{20} - j3\omega/10^{10}}{1 + \omega^2/10^{20}}\right) = \epsilon_R - j\epsilon_I$$





Problem 2 cont'd

(d) $f = 1\,\text{GHz} \rightarrow \omega = 2\pi \times 10^9$ rad/s

$$\underline{\epsilon} = \epsilon_0 \left(\frac{4 + \omega^2/10^{20} - j3\,\omega/10^{10}}{1 + \omega^2/10^{20}} \right) = \epsilon_0 \left(3.15 - j1.35\right)$$

$$\underline{k} = \omega\sqrt{\mu\,\underline{\epsilon}} = \frac{\omega}{c}\sqrt{3.15 - j1.35} = 37.98 - j7.796$$

Field goes as $e^{-j\underline{k}x} = e^{-j(37.98 - j7.796)x}$

In one meter this is $e^{-j37.98}e^{-7.796} = \boxed{2176^\circ \text{ phase shift},\ 4.11 \times 10^{-6} \text{ amplitude}}$

or $20 \times \log_{10}(\text{Amplitude}) \leftarrow$ this is a field! $\rightarrow -67$ dB.

Problem 3

(a) 900 MHz in free space: $k = \frac{\omega}{c} = \frac{2\pi \times 9 \times 10^8}{3 \times 10^8} = 6\pi \quad \lambda = \frac{2\pi}{k} = 1/3$ m

(b) From exponent, $\hat{k} = (\hat{y} + \hat{z})/\sqrt{2}$

(c) $\underline{\bar{H}} = \frac{\hat{k} \times \bar{E}}{\eta_0}$ for a propagating wave in free space,

$$= \frac{1}{\eta_0}\left(\hat{x}(-j\sqrt{2}) + \hat{y} - \hat{z}\right)\left(e^{-jk(y+z)/\sqrt{2}}\right), \quad \eta_0 = 120\pi$$

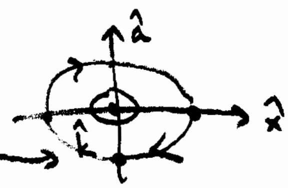
(d) Find unit vectors $\perp$ to $\hat{k}$: one is $\hat{x}$, other is $\hat{k} \times \hat{x} = (\hat{y} - \hat{z})/\sqrt{2} = \hat{a}$

Our electric field is then $\underline{E} = (\sqrt{2}\,\hat{x} + j\sqrt{2}\,\hat{a})$ so it is circular.

To decide handedness, convert to time domain at $y+z=0$

$$\bar{E}(t) = \sqrt{2}\,\hat{x}\cos(\omega t) - \sqrt{2}\,\hat{a}\sin(\omega t)$$

$\hat{k}, \hat{x}, \hat{a}$



$\rightarrow$ left handed circular!

$\hat{k}$ comes out because of our definition for $\hat{a}$

Problem 4

(a) $\langle \bar{S}(t) \rangle = \frac{1}{2} Re\{ \underline{E} \times \underline{H}^* \} = \frac{1}{2} Re\{ \hat{\theta}\, \frac{j\eta_0 k \underline{I} d}{4\pi r} e^{-jkr} \sin\theta \times \hat{\phi} \frac{(-j)k \underline{I}^* d}{4\pi r} e^{jkr} \sin\theta \}$

$= \frac{1}{2} Re\{ \hat{r} \frac{k^2 \eta_0 |\underline{I}|^2 d^2}{(4\pi)^2 r^2} \sin^2\theta \} = \frac{k^2 \eta_0 |\underline{I}|^2 d^2}{32\pi^2 r^2} \hat{r} (\sin^2\theta)$

(b) Integrate over a sphere radius R

$$P_r = \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta (R^2) \langle \bar{S}(t) \rangle \cdot \hat{r}$$

$$= \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta (R^2) \frac{k^2 \eta_0}{32\pi^2 R^2} |\underline{I}|^2 d^2 \sin^2\theta$$

$$= \int_0^\pi d\theta \int_0^{2\pi} d\phi \frac{k^2 \eta_0 |\underline{I}|^2 d^2}{32\pi^2} \sin^3\theta$$

$$= \frac{k^2 \eta_0 |\underline{I}|^2 d^2}{16\pi} \underbrace{\int_0^\pi d\theta \sin^3\theta}_{\int_0^\pi d\theta \sin\theta (1-\cos^2\theta) = \underbrace{\int_0^\pi d\theta \sin\theta}_{=2} - \underbrace{\int_0^\pi d\theta \sin\theta \cos^2\theta}_{\int_1^{-1} du\, u^2 = 2/3}}$$

for $u = \cos\theta$

$$P_r = \frac{k^2 \eta_0}{16\pi} (|\underline{I}|^2 d^2)(4/3)$$

(c) $R_{rad} = \frac{2P_r}{|\underline{I}|^2} = \frac{k^2 \eta_0}{8\pi} d^2 (4/3) = \frac{k^2 \eta_0}{6\pi} d^2 \simeq 20 k^2 d^2$

(d) $D(\theta,\phi) = \frac{P_\theta}{\eta P_{in}/4\pi r^2} = \frac{P_\theta}{P_r/4\pi r^2} = \frac{k^2 \eta_0 |\underline{I}|^2 d^2 / 32\pi^2 r^2 \sin^2\theta}{k^2 \eta_0 |\underline{I}|^2 d^2 / 64\pi^2 r^2} (3/4)$

$= 3/2 \sin^2\theta$, directivity $= 1.5$

(e) $A_e = \frac{D\lambda^2}{4\pi}$ if lossless $= \left(\frac{3}{2} \sin^2\theta\right) \lambda^2/4\pi = \frac{3\lambda^2 \sin^2\theta}{8\pi}$