PROBLEM SET 8
ECE 311 Autumn Quarter 2008

Assigned: November 24th
Due: December 5th in class   Instructor: Joel Johnson

Problem 1

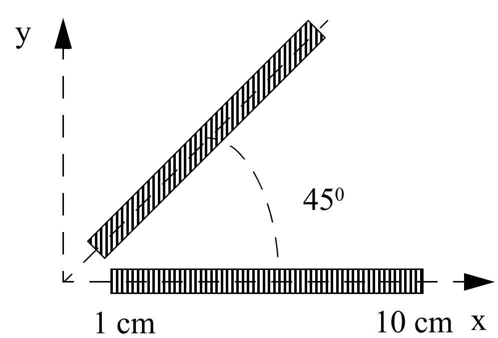
Perfectly conducting material fills the space between the planes $y = -3$ m and $y = 0$ m in a Cartesian coordinate system.

(a) Write two Cartesian unit vectors that are tangential to the surfaces of the conductor.
(b) Write a Cartesian unit vector that is normal to each surface of the conductor.
(c) An electric field $\bar{E} = -\hat{y}\left(\frac{1}{y+2}\right)$ V/m exists for $y \geq 0$. Show that this field satisfies the proper boundary conditions on the top surface of the conductor, and find the surface charge density on the top surface of the conductor.
(d) Find the electric field inside the conductor.

Problem 2

Two conductors exist in the x-y plane; the conductors are infinitely long in the z direction. One of the conductors is formed by the +x axis, and exists between $x = 1$ cm and $x = 10$ cm as shown. The second conductor is a plane rotated from the +x axis by 45 degrees as shown, and exists between $r = 1$ cm and $r = 10$ cm, where r is the cylindrical coordinate. An electric field $\bar{E} = (-\hat{\phi})\frac{60}{\pi r}$ V/m exists in the free space region between the two conductors.

(a) Show that the electric field satisfies the equations of electrostatics in free space (differential form). This will be easiest to do using operators expressed in cylindrical coordinates (see book or table on PS 7).
(b) Using the boundary conditions on the lower conductor, find the total charge on the lower conductor that is contained within the area $1 \leq x \leq 10$ cm, $0 \leq z \leq 1$ m.
(c) Perform a line integral of the electric field between the two conductors to find the potential difference between them.
(d) Use your part (b) and (c) answers to find the capacitance of this device.



Problem 3

A boundary between two dielectric media exists in the x-y plane. Above the x-y plane, the permittivity is $\varepsilon = 4\varepsilon_0$ while below the permittivity is $\varepsilon = 5\varepsilon_0$. If the electric field in region one is $\hat{x}(xy) + \hat{y}(2x) + \hat{z}(2z+1)$ V/m,

(a) Find the electric flux density in region 1.

(b) Find the electric field in region 2 at $z = 0$. Can we find the electric field for other values of z?

(c) Find the electric flux density in region 2 at $z = 0$.

(d) Find the volume charge density in region 1.

Problem 4

A circuit board designer needs to implement several parallel plate capacitors on a board (so called "chip" capacitors), and to minimize the space needed, it is desired to make all the capacitors with plate area 3 mm^2. The thickness of the board (which determines the plate separation of the capacitors because the bottom grounded side of the board is the bottom plate) is 1.5 mm. Since many different capacitances are needed, it is decided to use the dielectric constant of the board material to control capacitance. The board designer has two board materials, one with $\varepsilon = 9\varepsilon_0$ and a second with $\varepsilon = 2\varepsilon_0$. Neglect all fringing effects in this problem.

(a) Find the capacitance obtained when the capacitor is filled with the $\varepsilon = 9\varepsilon_0$ material.

(b) Find the capacitance obtained when the capacitor is filled with the $\varepsilon = 2\varepsilon_0$ material.

To obtain capacitance values between those of parts (a) and (b), the board designer is able to mix the two materials to create boards whose dielectric constant varies linearly from the bottom to the top of the capacitor: $\varepsilon(y) = (2 + by)\varepsilon_0$, where b is a constant and y ranges from 0 to 1.5 mm (the thickness of the board). We will now find the capacitance of this "inhomogeneous" capacitor.

(c) Assuming a total charge +Q on the top plate and minus Q on the bottom plate, use Gauss' Law to find the electric flux density between the two plates neglecting fringing effects.

(d) Integrate the electric field between the two plates to find the potential difference.

(e) Find the capacitance of this device. Note $\int \frac{dy}{a+by} = \frac{1}{b}\ln(a+by)$.

(f) Find the capacitance if $b = 5333$ (per meter). Verify that the value obtained is between those of parts (a) and (b).

(g) Using your part (f) capacitance, if a voltage of 50 V is placed across the capacitor, find the total electric energy stored in the capacitor.