

PROBLEM SET 7  
ECE 311 Autumn Quarter 2008

Assigned: November 17th

Due: November 24th in class

Instructor: Joel Johnson

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Problem 1

For each of the following charge distributions, state whether Gauss' law could be used to determine the electric field produced. If Gauss' law could be used, state the coordinate system that should be used, the field vector components that would exist, and the coordinate system variables that the field is a function of.

- (a) A finite length line charge
- (b) An infinite line charge
- (c) An infinite line charge (along the z-axis) with a charge density  $\rho_l = z^2$  C/m.
- (d) A volumetric charge density  $\rho_V = R^3$  C/cubic meter for  $2 < R < 3$ , and 0 otherwise, where  $R$  is the spherical coordinate in meters.
- (e) A volumetric charge density  $\rho_V = 2 \sin \phi$  C/cubic meter for  $2 < R < 3$ , and 0 otherwise, where  $R$  and  $\phi$  are the spherical coordinates with  $R$  in meters.

Problem 2

A surface charge distribution  $\rho_S = 2$  nC/square meter exists on the body of an infinite cylinder of radius 2.5 m; the z axis is the axis of this cylinder. Surrounding this surface charge distribution is a volumetric charge density  $\rho_V = (1 + r^2)$  nC/cubic meter which exists for  $4 < r < 5$  where  $r$  is the cylindrical coordinate in meters. We will attempt to determine the fields produced by these sources using Gauss' Law.

- (a) Choose a coordinate system for this problem. Explain your reason for this choice.
- (b) What field components should exist here, and what coordinate system variables should they be functions of?
- (c) Use Gauss' Law to find the electric field for  $r < 2.5$  m.
- (d) Use Gauss' Law to find the electric field for  $2.5 < r < 4$  m.
- (e) Use Gauss' Law to find the electric field for  $4 < r < 5$  m,
- (f) Use Gauss' Law to find the electric field for  $r > 5$  m.
- (g) Plot the amplitude of the field for  $0 < r < 6$  m. Interpret your result.

Problem 3

A point charge of 1 nC is located at the origin of a spherical coordinate system.

- Write the absolute potential produced by this point charge; use spherical coordinates.
- Evaluate the electric field from the absolute potential, and confirm that this is identical to our previous equation for the electric field of a point charge at the origin. Note it will be easiest to work in spherical coordinates, see table below for appropriate operator definitions.
- Show that the divergence of this electric field is zero.
- Sketch surfaces where  $V=0.1, 1, 3, \text{ or } 6$  Volts (“equipotential surfaces”.) Interpret your sketches.
- Sketch the direction of the electric field on the  $V=6$  Volts surface. How does the field direction relate to the equipotential surface?
- Where in your sketch from part (d) is the amplitude of the electric field largest? Where is it smallest? Interpret your answers in terms of the equipotential surfaces.
- Find the amount of work required to move a  $-2$  nC point charge from a point where  $R = 10$  m to a point where  $R = 2$  m in this field. Interpret your result. Does the path taken between the starting and ending locations matter?

$$\text{Cartesian: } \nabla V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}, \quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}, \quad \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical:

$$\nabla V = \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}, \quad \nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r}(rA_r) + \frac{1}{r} \frac{\partial}{\partial \phi}(A_\phi) + \frac{\partial A_z}{\partial z}, \quad \nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Spherical:

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \left( \frac{1}{R \sin \theta} \right) \frac{\partial V}{\partial \phi}, \quad \nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta}(A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R \sin \theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R \sin \theta)A_\phi \end{vmatrix}$$