PROBLEM SET 6

ECE 311 Autumn Quarter 2008

Assigned: November 5th

Due: November 12th in class — Instructor: Joel Johnson

Problem 1

Point P is located at coordinates ($R=2, \theta = 30°, \phi = 210°$) in a spherical coordinate system. Point Q is located at ($r=2, \phi = 45°, z=-1$) in a cylindrical coordinate system. Find

(a) a position vector written in terms of cylindrical unit vectors from the origin to point P

(b) the coordinates of points P and Q in a Cartesian coordinate system

(c) $\overline{r_{pq}}$, a vector written in terms of Cartesian unit vectors from point P to point Q

(d) $\left|\overline{r_{pq}}\right|$, the distance between points P and Q

(e) the coordinates of point Q in a spherical coordinate system

Problem 2

This problem involves performing the integral of a scalar over a volume. A mountain takes the shape of a hemisphere (on top of the surrounding flat Earth) with radius 2 km. Note that the intersection of the hemisphere with the surrounding flat Earth is a circle; the center of this circle is defined as the origin in what follows. The mass density of the mountain in kg per kilometer cubed is known to be $\frac{1 \times 10^{12}}{1 + R^3}$ with R representing the spherical coordinate in kilometers.

(a) Choose a coordinate system for solving this problem. Explain your choice.

(b) Sketch the mass density of the mountain as a function of R.

(c) Write an integral for the total mass of the mountain. Explain the terms in your integral.

(d) Perform the integral to obtain the total mass of the mountain. Interpret your result physically. Note that $\int \frac{x^2}{1 + x^3} dx = \frac{1}{3} \ln(1 + x^3)$.

Problem 3

This problem involves computing a "flux" integral over a closed surface. The surface considered is a cylinder of length 3 m and radius 0.2 m. The cylinder's axis is the z axis of a Cartesian coordinate system, and the center of the cylinder is the origin of the coordinate system. Note the cylindrical surface considered includes 3 parts: the "body" of the cylinder and the top and bottom "end caps." The cylinder exists in the presence of a vector field $\bar{F} = \hat{y}2z + \hat{z}z$. Note when computing the flux of a vector field out of a closed surface, we need to integrate the normal (outward pointing) component of the field on the surface.

(a) Choose a coordinate system to use with this problem. Explain your choice.

(b) Determine a unit vector which is normal to the body of the cylinder in your coordinate system.

(c) Express the normal component of $\bar{F}$ on the body of the cylinder. Note this is a scalar since it is a vector component and not a vector.

(d) Write the integral for the total flux out of the cylinder's body. Explain the terms in your integral. Note the radial distance from the z-axis remains constant during an integration over the body of a cylinder.

(e) Perform the integral to obtain the total flux out of the cylinder body. Interpret your result.

(f) Repeat parts (b)-(e) for the top and bottom end caps. These can be done together because they are very similar.

Problem 4

One nC point charges are located at points P1 (x=-1 m, y=-1 m, z=0 m) and P2 (x=1 m, y=1 m, z=0 m).

(a) Find the force on the charge at point P1.
(b) Find the force on the charge at point P2. Do your answers in parts (a) and (b) represent attractive or repulsive forces? Why?
(c) Find the total electric field produced by these two charges at the point P3 (x=2 m, y=2 m, z= 2 m).
(d) If a third point charge of 0.1 nC was placed at point P3, find the force on this "test" charge due to the charges at points P1 and P2 using your answer from part (c). Interpret your result.
(e) Find the total electric field produced by the charges P1 and P2 at the point P4 (x=0 m, y=0 m, z=0 m). Explain your answer using a symmetry argument.