TABLE I

$\frac{1}{\mu}$	Split	$T_r(P_c)$	$T_r(W_2\mu^2)$	$T_r(P_c - K)$
1	1-3	.9527	4.29×10^{-23}	2.396×10^{-9}
1	3-1	1.1045	.6721	.1517
10	3-1	1.0742	.0067	.0045
10^2	3-1	1.0861	$.0067 \times 10^{-2}$	$.0040 \times 10^{-2}$
10^3	3-1	1.0878	$.0067 \times 10^{-4}$	$.0080 \times 10^{-2}$
10^4	3-1	1.0879	$.0067 \times 10^{-6}$	$.0010 \times 10^{-5}$
10^5	3-1	1.0879	$.0067 \times 10^{-8}$	$.0017 \times 10^{-6}$

13th Annu. Allerton Conf. Circuit and System Theory, Univ. Illinois, Urbana, IL, Oct. 1975, pp. 48–57.
[4] R. A. Yackel and P. V. Kokotovic, "A boundary layer method for the matrix Riccati equation," *IEEE Trans. Automat. Contr.*, vol. AC-18, pp. 17–24, Feb. 1973.
[5] P. Sannuti and P. V. Kokotovic, "Near optimum design of linear systems by a singular perturbation method," *IEEE Trans. Automat. Contr.*, vol. AC-14, pp. 15–22, 1969.
[6] P. V. Kokotovic and A. H. Haddad, "Controllability and time-optimal control of systems with slow and fast modes," *IEEE Trans. Automat. Contr.*, vol. AC-20, pp. 111–113, Feb. 1975.

On a Canonical Form in "Design of Sensitivity Reducing Compensators Using Observers"

BRUCE KROGH AND J. B. CRUZ, JR.

***Abstract*—A correct method is presented for obtaining the "plant canonical form" of the above paper.[1]**

In the above paper,[1] an incorrect lemma from [1, Lemma 3.4] was quoted to define a canonical form for the plant matrices (A,B,C). The following lemma correctly states how this canonical form is achieved.

Lemma: Given matrices $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{l\times n}$, $0<m$, $l\leqslant n$, $\operatorname{rank}(B)=m$, $\operatorname{rank}(C)=l$, with

$$m_1=\operatorname{rank}(CB),\qquad 0\leqslant m_1\leqslant \min(l,m)$$

there exist nonsingular matrices $L\in\mathbb{R}^{l\times l}$, $M\in\mathbb{R}^{m\times m}$, and $T\in\mathbb{R}^{n\times n}$ such that

$$LCT=\left[I_l \;\vdots\; 0\right] \tag{1}$$

and

$$T^{-1}BM=\left[B_1^{*\prime} \;\vdots\; B_2^{*\prime}\right]'$$

where

$$B_1^*=\begin{bmatrix} I_{m_1} & 0\\ 0 & 0\end{bmatrix}\in R^{l\times m};\quad B_2^*=\begin{bmatrix} 0 & I_{m-m_1}\\ 0 & 0\end{bmatrix}\in R^{(n-l)\times m}. \tag{2}$$

Proof: Since $\operatorname{rank}(CB)=m_1$, there exists a nonsingular L (nonunique) such that

$$LCB=\begin{bmatrix} X_1\\ \hdashline 0_{(l-m_1)\times m}\end{bmatrix} \tag{3}$$

where $X_1\in R^{m_1\times m}$ has rank m_1. Let $\{p_i'|p_i\in R^n,\ i=1,\cdots,l\}$ be the (linearly independent) rows of LC. Let $\mathfrak{N}(\cdot)$ denote the null space of the matrix argument. Then by (3) we can select vectors s_j, $j=1,\cdots,n-l$, such that $\{p_i\}_{i=1}^{m_1}\cup\{s_j\}_{j=1}^{l-m_1}$ is a set of basis vectors for $R^n-\mathfrak{N}(B')$ and $\{p_i\}_{i=m_1+1}^{l}\cup\{s_j\}_{j=l-m_1+1}^{n-l}$ is a set of basis vectors for $\mathfrak{N}(B')$. Define

$$S=\begin{bmatrix} s_1'\\ \cdot\\ \cdot\\ \cdot\\ s_{n-l}'\end{bmatrix}$$

and let

$$T^{-1}=\begin{bmatrix} LC\\ \hdashline S\end{bmatrix};$$

then T^{-1} is nonsingular since its rows are linearly independent and (1) follows since

$$LC=\left[I_l\;\vdots\;0\right]T^{-1}.$$

Furthermore,

$$T^{-1}B=\begin{bmatrix} X_1\\ \hdashline 0_{(l-m_1)\times m}\\ \hdashline X_2\\ \hdashline 0_{(n-l-m+m_1)\times m}\end{bmatrix}$$

where $X_2\in R^{(m-m_1)\times m}$ and

$$\operatorname{rank}\begin{bmatrix} X_1\\ \hdashline X_2\end{bmatrix}=\operatorname{rank}(T^{-1}B)=\operatorname{rank}(B)=m.$$

Defining $M\triangleq\begin{bmatrix} X_1\\ \hdashline X_2\end{bmatrix}^{-1}$ implies (2) for $T^{-1}BM$. Q.E.D.

Replacing (11) and (12) in Krogh and Cruz[1] with the above lemma, the plant canonical form is defined as $(T^{-1}AT, T^{-1}BM, LCT)$. It is easily verified that the results of [1] upon which the above paper[1] was based are invariant under nonsingular transformations of the state, input, and output spaces. Hence, the results of the above paper[1] retain the generality of application to arbitrary linear time-invariant systems.

REFERENCES

[1] W. J. Naeije and D. H. Bosgra, "Comparison sensitivity design of multivariable output feedback systems," in *Proc. 4th IFAC Symp. Multivariable Technological Systems*, Fredericton, N.B., Canada, 1977, pp. 127–135.

Manuscript received November 2, 1978. This work was supported in part by the U.S. Air Force under Grant AFOSR 78-3633, in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy, and U.S. Air Force) under Contract DAAG-29-78-C-0016, and in part by the National Science Foundation under Grant ENG-74-20091.
The authors are with the Decision and Control Laboratory, Coordinated Science Laboratory, University of Illinois, Urbana, IL 61801.
[1]B. Krogh and J. B. Cruz, Jr., *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 1058–1062, Dec. 1978.

A New Lower Bound on the Cost of Optimal Regulators

G. LANGHOLZ

***Abstract*—A new lower bound on the quadratic cost of linear regulators is derived using some recently established results on norm bounds for the algebraic matrix Riccati and Lyapunov equations. The bound thus obtained**

Manuscript received July 8, 1978.
The author is with the School of Engineering, Tel-Aviv University, Tel-Aviv, Israel, on leave at the Department of Electrical Engineering and Computer Science, University of California at Santa Barbara, Santa Barbara, CA 93106.