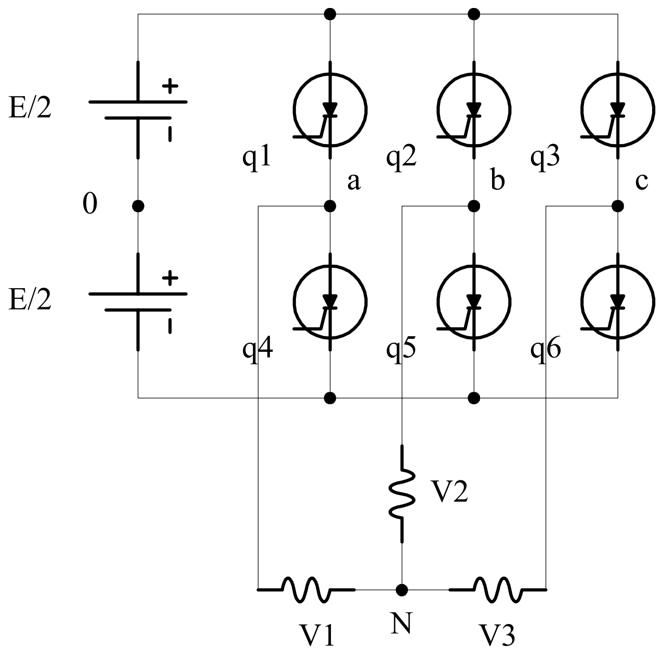
Space Vector PWM



- Each power switch can be on or off
- On = 1 off = 0

q1 = 1 q4 = 1

q4 = 1 - q1

q5 = 1 - q2

q6 = 1 - q3

state	voltage	q1	q2	q3	q4 ($\bar{q}_1$)	q5 ($\bar{q}_2$)	q6 ($\bar{q}_3$)
1	V1	1	0	0	0	1	1
2	V2	1	1	0	0	0	1
3	V3	0	1	0	1	0	1
4	V4	0	1	1	1	0	0
5	V5	0	0	1	1	1	0
6	V6	1	0	1	0	1	0
7	V7	1	1	1	0	0	0
8	V8	0	0	0	1	1	1

$$V_1 = V_{10} + V_{on}$$

$$V_2 = V_{20} + V_{on}$$

$$V_3 = V_{30} + V_{on}$$

when q1 = conducts = 1

$$V_{10} = \frac{E}{2} q_1 \qquad q_4 = 0$$

when q4 = conducts = 1

$$V_{10} = -\frac{E}{2} q_4 \qquad q_1 = 0$$

$$V_{10} = \frac{E}{2} q_1 - \frac{E}{2} q_4 \qquad q_4 = 1 - q_1$$

$$\therefore \qquad V_{10} = \frac{E}{2} q_1 - \frac{E}{2}(1 - q_1) = (2q_1 - 1)\frac{E}{2}$$

$$V_1 = (2q_1 - 1)\frac{E}{2} + V_{on}$$

similarly,

$$V_2 = (2q_2 - 1)\frac{E}{2} + V_{on}$$

$$V_3 = (2q_3 - 1)\frac{E}{2} + V_{on}$$

$$f_{abc} = T \cdot f_{012} \qquad T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad \begin{aligned} a &= -\frac{1}{2} + j\frac{\sqrt{3}}{2} = 1\angle 120^o \\ a^2 &= -\frac{1}{2} - j\frac{\sqrt{3}}{2} = 1\angle 240^o \end{aligned}$$

$$f_{012} = T^{-1} \cdot f_{abc} \qquad T^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\begin{aligned} S_{3\phi} &= [V_{abc}]^t [I_{abc}]^* \\ &= [[T]V_{012}]^t [[T]I_{012}]^* \\ &= [V_{012}]^t [T]^t [T]^* [I_{012}]^* \end{aligned} \qquad [T]^t [T]^* = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{3\phi} = 3V_{012} I_{012}^* = [V_{abc}]^t [I_{abc}]^*$$

$$V_{ref} = (V_1 + aV_2 + a^2 V_3)$$

write it in terms of peak line-to-line voltage:

$$\begin{aligned} V_{ref} &= \sqrt{\frac{3}{2}}(\widetilde{V}_1 + a\widetilde{V}_2 + a^2 \widetilde{V}_3) \\ &= \sqrt{3}\left(\frac{\widetilde{V}_1}{\sqrt{2}} + a\frac{\widetilde{V}_2}{\sqrt{2}} + a^2 \frac{\widetilde{V}_3}{\sqrt{2}}\right) \end{aligned}$$

where $\widetilde{V}_1 = \sqrt{3} V_1$ = peak line-to-line voltage

d-q Transformation

$$\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$$\begin{aligned} P_{abc} &= v_a i_a + v_b i_b + v_c i_c \\ P_{dq0} &= v_d i_d + v_q i_q + v_0 i_0 \end{aligned}$$

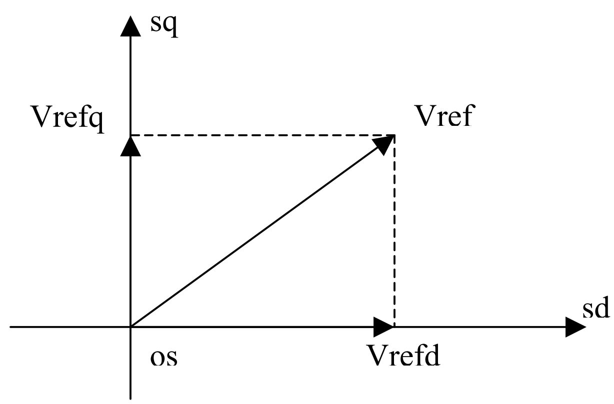
Conservative transformation or power invariant

$$\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$$P_{abc} = v_a i_a + v_b i_b + v_c i_c$$

$$P_{dq0} = \frac{3}{2}(v_d i_d + v_q i_q + v_0 i_0)$$

power variant transformation



$$V_{ref} = V_{refd} + jV_{refq}$$

$$V_{refd} = \sqrt{\frac{2}{3}}\,\mathrm{Re}\{V_1 + aV_2 + a^2 V_3\}$$

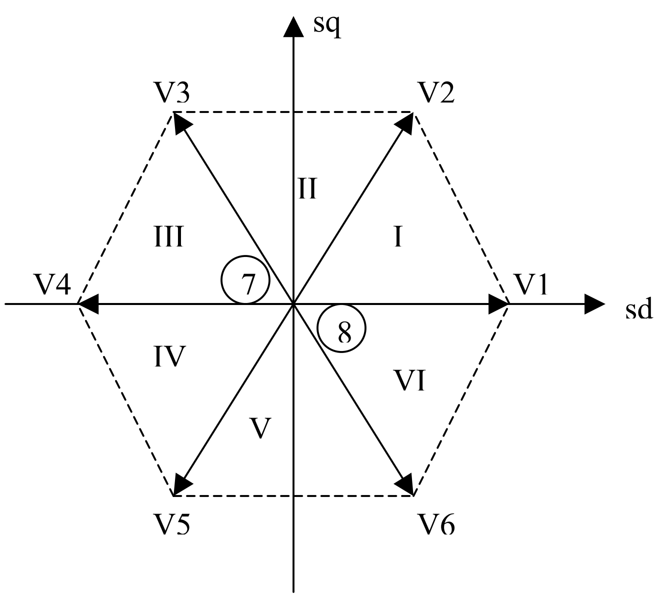
$$V_{refq} = \sqrt{\frac{2}{3}}\,\mathrm{Im}\{V_1 + aV_2 + a^2 V_3\}$$

substituting

$$V_{refd} = \sqrt{\frac{2}{3}}(q_1 - q_2/2 - q_3/2)E$$

$$V_{refq} = \frac{1}{\sqrt{2}}(q_2 - q_3)E$$

	q1	q2	q3	
V1	1	0	0	$V_{ref} = \sqrt{2/3}E$
V2	1	1	0	$V_{ref} = E/\sqrt{6} + jE/\sqrt{2}$
V3	0	1	0	$V_{ref} = -E/\sqrt{6} + jE/\sqrt{2}$
V4	0	1	1	$V_{ref} = -\sqrt{2/3}E$
V5	0	0	1	$V_{ref} = -E/\sqrt{6} - jE/\sqrt{2}$
V6	1	0	1	$V_{ref} = E/\sqrt{6} - jE/\sqrt{2}$
V7	1	1	1	$V_{ref} = 0$
V8	0	0	0	$V_{ref} = 0$

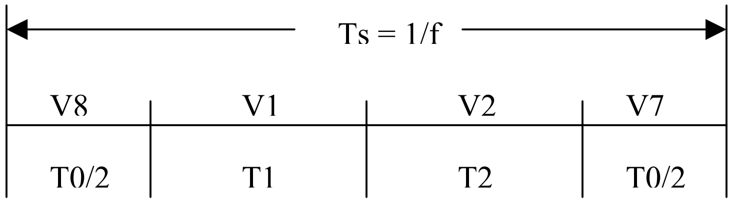


Vref rotates with ω_s.
$\alpha = \omega_s t$

6 switches x 60^o = 360^o

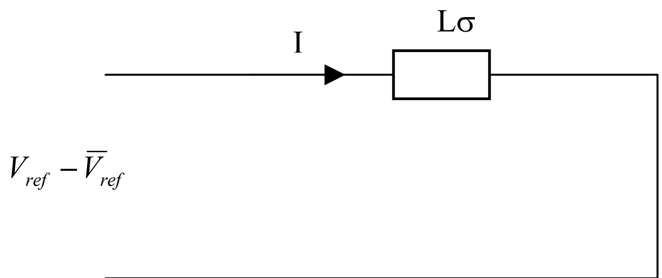
Sector	Switching sequence
I	8, 1,2, 7, 2, 1, 8
II	8, 3, 2, 7, 2, 3, 8
III	8, 3, 4, 7, 4, 3, 8
IV	8, 4, 5, 7, 5, 4, 8
V	8, 5, 6, 7, 6, 5, 8
VI	8, 1, 6, 7, 6, 1, 8

Note: One power switch is switching with each change of state.



Select T0, T1, and T2, s.t. the change in load current from the fundamental component is minimized.

Assume a simple machine,



$$\Delta I = \frac{1}{L_\sigma}\int_{t1}^{t2} (V_{ref} - \overline{V}_{ref})dt$$

t_1: beginning time of a switching state
t_2: end time of a switching state
V_{ref} : voltage phasor of SVPWM
$\overline{V}_{ref}$: reference voltage phasor

$$\overline{V}_{ref} = \int_0^{T_s} [\cancelto{0}{V_8(t)} + V_1(t) + V_2(t) + \cancelto{0}{V_7(t)}]dt$$

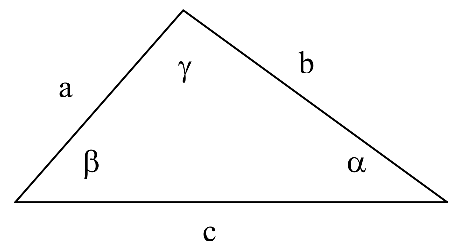
$$\overline{V}_{ref} = \int_{T_0/2}^{T_1} V_1(t)dt + \int_{T_1}^{T_2} V_2(t)dt$$

$$\Delta \bar{I} = V_{ref(1)}T_1 + V_{ref(2)}T_2 - \overline{V}_{ref}T_s$$

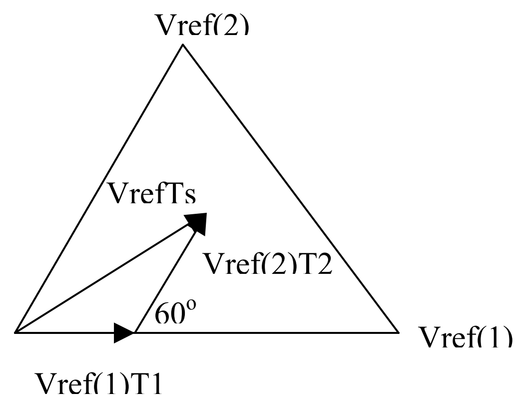
to make $\Delta \bar{I} = 0$, then

$$V_{ref(1)}T_1 + V_{ref(2)}T_2 = \overline{V}_{ref}T_s$$

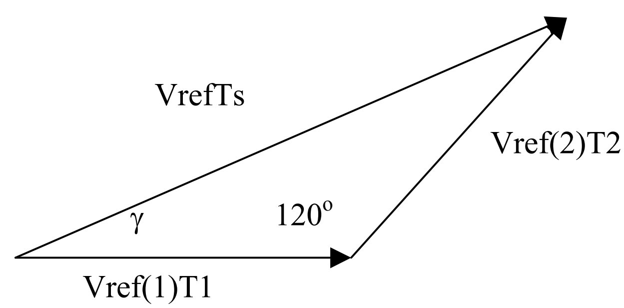
Recall



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



Vref(2)T2 is parallel with Vref(2)



$$\frac{|V_{ref(2)}|T_2}{\sin\gamma} = \frac{\overline{V}_{ref}T_s}{\sin 120}$$

$$\frac{|V_{ref(1)}|T_1}{\sin(60-\gamma)} = \frac{\overline{V}_{ref}T_s}{\sin 120}$$

$$\gamma = \omega_s t = 2\pi f t$$

T1 and T2 can be computed as

$$T_1 = T_s a \frac{\sin(60-\gamma)}{\sin 60}$$

$$T_2 = T_s a \frac{\sin\gamma}{\sin 60}$$

where

$$a = \left|\frac{\overline{V}_{ref}}{V_{ref(1)}}\right| = \left|\frac{\overline{V}_{ref}}{V_{ref(2)}}\right|$$

"a" : depth of modulation or index of modulation

$$T_0 = T_s - (T_1 + T_2)$$

- The phase of requested voltage vector identifies two nonzero voltage vectors
- The requested voltage vector can be synthesized by using fractions of the two nearest voltage vectors, which amounts to applying these two vectors one at a time, for a fraction of the switching period. The nearest zero voltage vector to the two voltage vectors is applied for the remaining switching period.

Say,

1. Apply V0 for T0/2
2. Apply Vref1 for T1
3. Apply Vref2 for T2
4. Apply V0 for T0/2