Modeling and System Identification for a DC Servo

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Abstract

First, you are asked to do some background reading in control system design methodology and demonstrate your understanding of the methodology via answering some questions. Second, you develop state space and transfer function models for the case where you sense position and angular velocity for the motor shaft. Third, you are given a step response for the open-loop plant and you are asked to use a simple time-domain approach to find the plant model. Fourth, you are asked to constuct a Bode plot from multiple sinusoid input-output pairs and from this the plant model (a simple type of frequency domain based system identification).

Contents

1 Development of State Space Models and Transfer Functions

See Appendix A. Derive the differential equations for the DC servo showing each step.

1. Suppose that $x_1 = \theta$ (the output shaft position), $x_2 = \dot{\theta}$, $u = V_{in}$, and $y = \theta$ (i.e., the output is the shaft position from the encoder or potentiometer). Let the state $x = [x_1, x_2]^\top$. Find A, B, and C (in terms of the generic motor parameters given in Appendix A) to specify the state space model

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

   Use the appropriate values of the parameters to get specific numeric values for A, B, and C.

2. Assume that there are zero initial conditions. Derive

$$G(s) = \frac{\theta(s)}{V_{in}(s)}$$

   Use the appropriate values of the parameters to get the transfer function given in Appendix A. What are the pole locations?

3. Suppose that $\omega = \dot{\theta}$ and change y to be $y = \omega$ (i.e., the output is the angular rate of change of the shaft position provided by the tachometer). define C and

$$G_\omega(s) = \frac{\Omega(s)}{V_{in}(s)}$$

   for this case.

2 Time Domain Based System Identification

While the model development in the previous section that is based on principles of physics can be quite useful to specify the model for a plant, a perfect model is never found in this way (but of course very good models can sometimes be found, but these are necessarily nonlinear and stochastic and hence not as useful for systematic design methodologies). The other major approach to finding models is to use "system identification" where you inject signals into the plant and observe output signals, and then use these to specify the model that mapped one to the other. Of course, this approach also never leads to a perfect model; however, it is often quite useful since good model information can be obtained from direct experimentation with the plant. In practice, often a combination of physics-based modeling and system identification is used to determine a model.

In system identification one typically uses a discrete time model and least squares methods to determine model parameters. Here, we take a simple direct (and practical) approach by using a time-domain based approach and frequency domain based approach for continuous time systems.

In this section and the next suppose that you want to identify the model from V_{in} to $\omega = \dot{\theta}$ (from input voltage to tachometer output). Suppose that you went through the above modeling exercise and that you know from the physics that the model is basically a first order linear model with one pole and no zeros.

Suppose you went in the lab and put a unit step input into the plant and got the output shown in Figure 1.

What is the pole location and DC gain for the transfer function

$$\frac{\Omega(s)}{V_{in}(s)}$$

Note that this is not the same as the model that you derived in the last section; the plot in Figure 1 resulted from a different transfer function. All you can do to find it is to study the shape of the plot and determine the parameters from it using basics of the inverse Laplace transform.

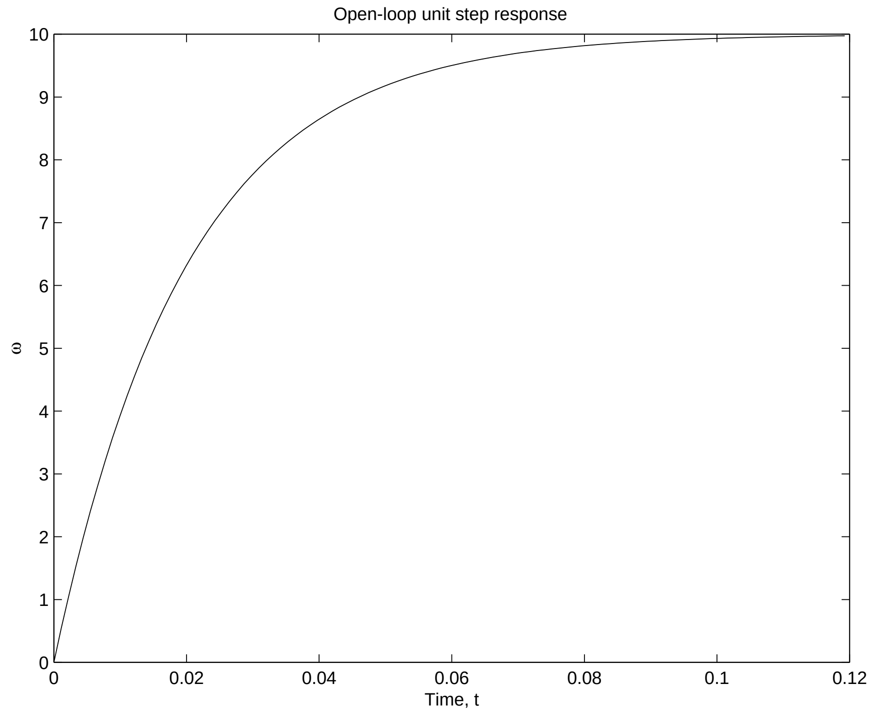

Figure 1: Step response.

3 Frequency Domain Based System Identification

Suppose you went to the lab and put three different frequency sine waves into the DC servo at the input V_{in} with frequencies 5, 50, and 500 rad/sec. You measure the output of the tachometer in each case and obtain the plots in Figures 2, 3, and 4.

1. Plot three points on a Bode magnitude ($20log(|G_\omega|)$ vs. log freqency) and phase plot (degrees vs. log freqency). Specify a straight line approximation (according to usual Bode plot techniques) to the points (to do this simply sketch it on the Bode plot for the three points). Keep in mind that the fall off is 20 dB per decade for high frequencies. Find the two straight lines first, then consider their intersection point and its relation to the third point. From this, find the pole location and the Bode gain (DC gain). Then, sketch the straight line approximation to the phase plot.

2. Specify a transfer function that would generate this Bode plot.

3. Plot the Bode plot of the transfer function that you found, with the three "experimental" points on the magnitude and phase plots. Compare. Discuss (note that this provides for a way to verify your model if you would be able to get another input-output pair for the plant you could plot it on this plot to see if the model that you identified fits data that it was not constructed well, i.e., this tests if it "predicts" experimental conditions like any good model should).

4. Then, plot the step response of the transfer function that you found and compare to the step response in Figure 1. Discuss. Note that the *same* plant was used to generate the step response in Figure 1 as the input-output pairs in Figures 2, 3, and 4.

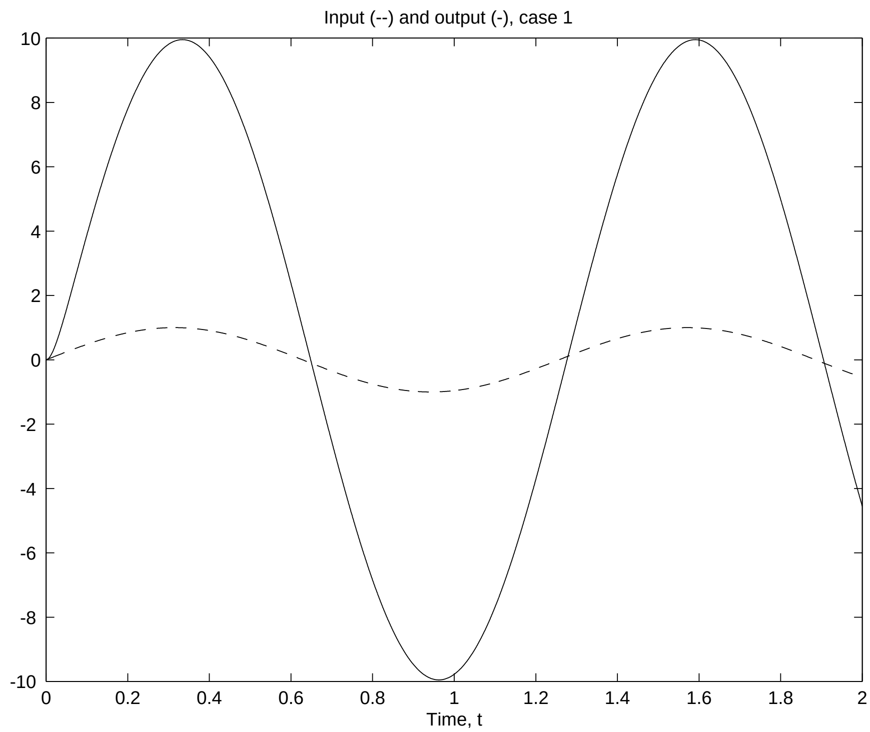

Figure 2: Input and output of plant.

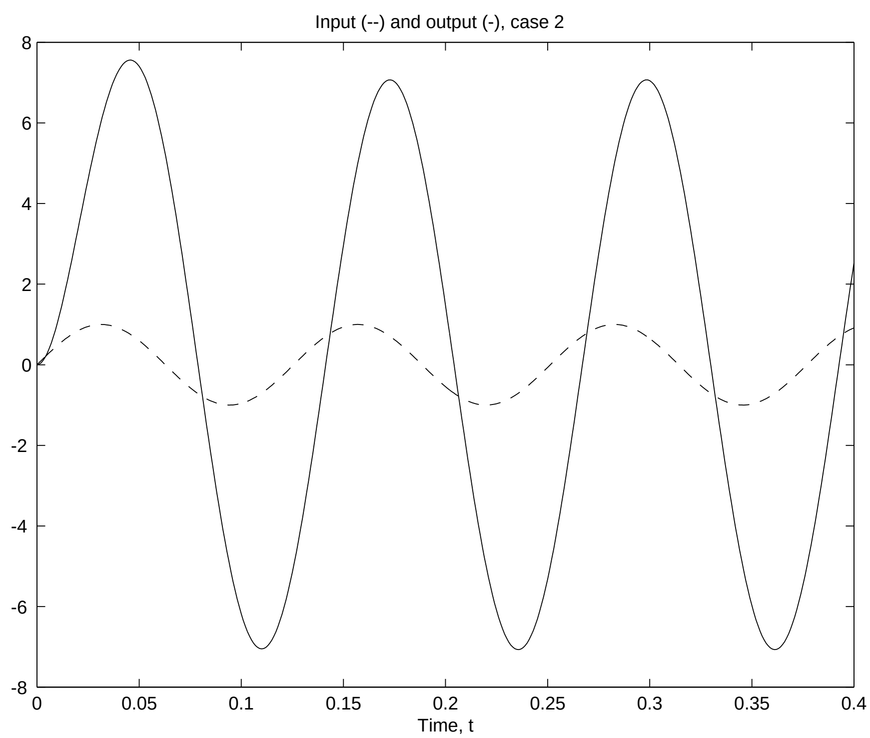

Figure 3: Input and output of plant.

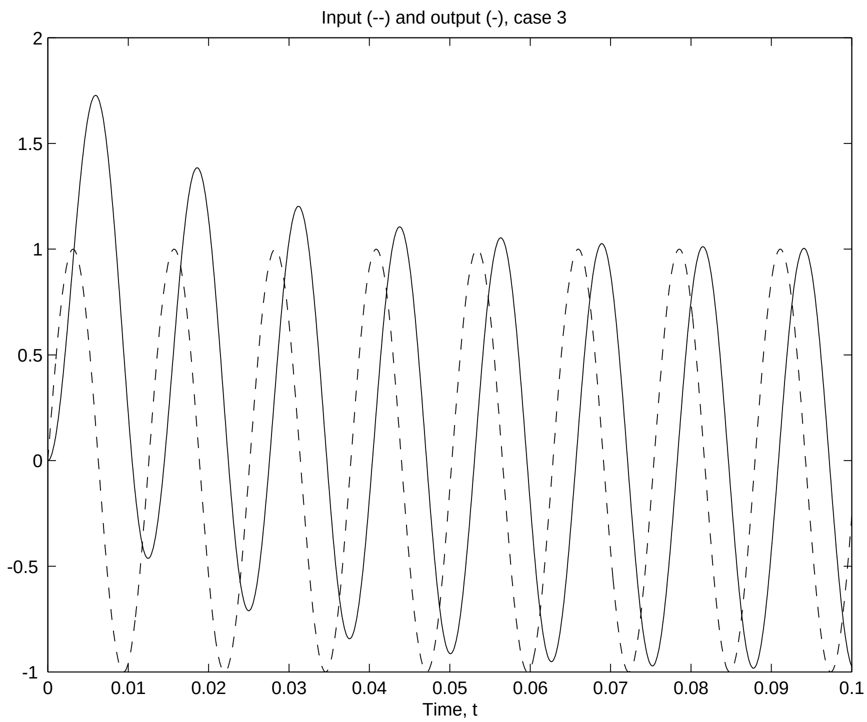

Figure 4: Input and output of plant.