System Identification of a Thermal Process

Lab 2: Modeling and System Identification

ECE 758: *Control System Implementation Laboratory*

1 An analog lamp driver: Darlington emitter follower

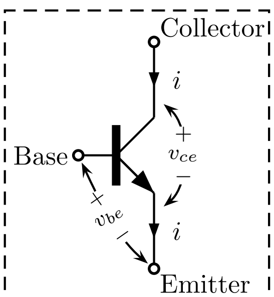
System identification of a linear time-invariant (LTI) system can be completed with time-domain or frequency-domain methods.

- *Time-domain system identification* usually involves the analysis of a plant's response to a step function on its input. Because a step function only takes two values, it can be generated by a digital output.
- *Frequency-domain system identification* finds the gain and phase shift that the system realizes for several sinusoidal input signals of different frequency. Because a sinusoidal input swings through a continuous range of input values, it can only be generated by an analog output.

Each LM35 temperature sensor is heated by an incandescent lamp that requires 100 mA at 6 V (i.e., like a 60 Ω resistor). This current is provided by a DS2003 Darlington driver chip. This DS2003 is designed for on–off control of high current devices based on a digital input (including both TTL and CMOS logic). However, as long as we only use one DS2003 channel at at time, it can be used as a linear analog amplifier for frequency-domain system identification.

1.1 Background: Bipolar junction transistors and switching

The simplified operation of a bipolar junction transistor (BJT) is shown in Figure 1.



$$\begin{cases} v_{be} > V_{BE} & \overset{\text{implies}}{\Longrightarrow} \quad v_{ce} \approx V_{CE,\text{sat}} \qquad \text{(saturation mode)} \\ v_{be} < V_{BE} & \overset{\text{implies}}{\Longrightarrow} \quad i = 0 \qquad \text{(cutoff mode)} \end{cases}$$

Figure 1: Simplified bipolar transistor with threshold V_{BE} and saturation $V_{CE,\text{sat}}$.

At room temperature, the "threshold" $V_{BE} \approx 0.7$ V and the saturation voltage $V_{CE,\text{sat}} \approx 0.2$ V. When the base–emitter voltage v_{be} is less than V_{BE}, no current flows from the collector to emitter. When v_{be} is greater than V_{BE}, collector-to-emitter current i is only limited by other elements in the circuit and the collector-to-emitter voltage $v_{ce} \approx V_{CE,\text{sat}}$. As shown in Figure 2, when the emitter of a BJT is grounded, it acts like a switch controlled by its base voltage.

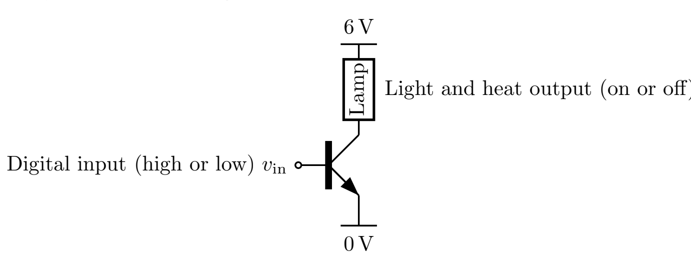


Figure 2: Bipolar transistor as lamp switch.

When v_{in} is large, almost 6 V is placed across the lamp, and it turns on. When v_{in} is low, no current flows through the lamp, and it turns off.

1.2 Background: The degenerated emitter follower as analog amplifier

When v_{be} in Figure 1 is very close to the V_{BE} threshold, the transistor is said to be in *acitve mode* and the current i is approximated as a greatly amplified version of small variations in v_{be}. By connecting this high gain amplifier in a negative feedback configuration, the BJT can be used as a *buffer* that can be used for analog control of high-current devices.

The example in Figure 3 uses so-called *emitter degeneration* to provide the necessary negative feedback to keep the transistor in its active mode. The circuit is known as an *emitter follower* because the emitter voltage follows changes in the base voltage.

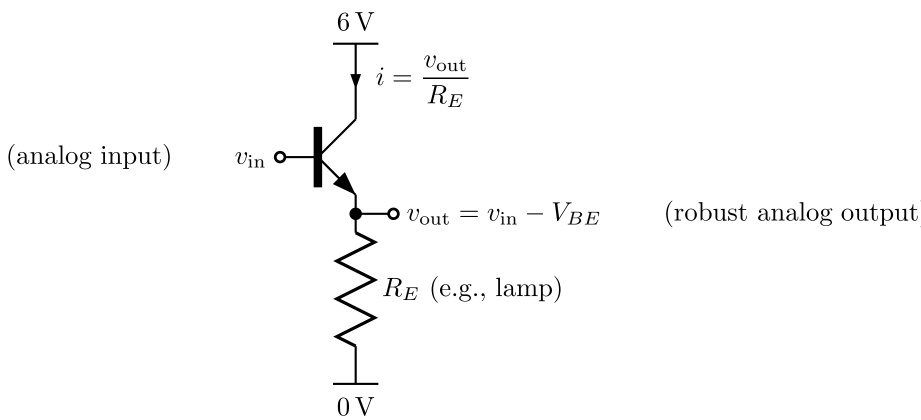


Figure 3: Transistor emitter follower. Negative feedback by R_E regulates v_{be} to V_{BE}.

To understand how negative feedback ensures that v_{be} is regulated to V_{BE}, consider two cases.

(i) When the base voltage v_{in} rises, v_{be} tends to increase greater than the V_{BE} threshold. That increase leads a large increase in current i, which results in a greater voltage drop across the emitter resistor R_E, and so the emitter voltage v_{out} *rises.* As the emitter voltage rises, the v_{be} voltage drops until it reaches the threshold V_{BE} and current i decreases.

(ii) When the base voltage v_{in} falls, v_{be} tends to decrease lower than the V_{BE} threshold. That decrease leads to a large drop in current i, which results in a lesser voltage drop across the emitter resistor R_E, and so the emitter voltage v_{out} *falls.* As the emitter voltage falls, the v_{be} voltage rises until it reaches the threshold V_{BE} and the current i increases.

So the emitter resistor R_E sets up a negative feedback system that *regulates* the v_{be} voltage to the V_{BE} threshold. As a consequence, $v_{\text{out}} = v_{\text{in}} - V_{BE}$. Moreover, the current i is set only by the input v_{in} and the resistance R_E. Smooth changes in the input v_{in} lead to smooth changes in the current i through the resistor R_E (e.g., a lamp).

The operation of an emitter follower as a linear amplifier requires two conditions on the input v_{in}.

(a) The input v_{in} cannot rise far above the collector voltage. Because the transistor is a passive device, when v_{in} rises to above the collector voltage, the transistor *saturates* once again (i.e., it ceases to act as an amplifier and begins to act like a *closed switch*). So proper operation requires that $v_{ce} > V_{CE,\text{sat}}$.

(b) The input v_{in} cannot fall below V_{BE}. Because current can only flow in the direction of the arrow on the emitter terminal, the emitter voltage cannot move below 0 V. So if $v_{\text{in}} < V_{BE}$, the transistor moves into *cutoff mode* (i.e., it acts like an *open switch*).

However, we can use the property in item (b) to turn current i off as desired.

1.3 Simplified Darlington transistor model

The circuit described in section 1.2 allows for frequency-domain system identification. However, it is not practical to use standard signaling bipolar transistors to drive high-current loads. So a common approach is to use a *Darlington transistor,* which is a special transistor that can provide high collector-to-emitter current at the price of larger V_{BE} threshold and $V_{CE,\text{sat}}$ saturation voltages.

The DS2003 packages seven high-current Darlington transistors onto a single chip. A simplified schematic, which includes the corresponding threshold and saturation voltages, is shown in Figure 4. The *flyback* connection on pin 9 is not connected (NC); it is only used when switching loads that are very inductive, like high-power DC motors.

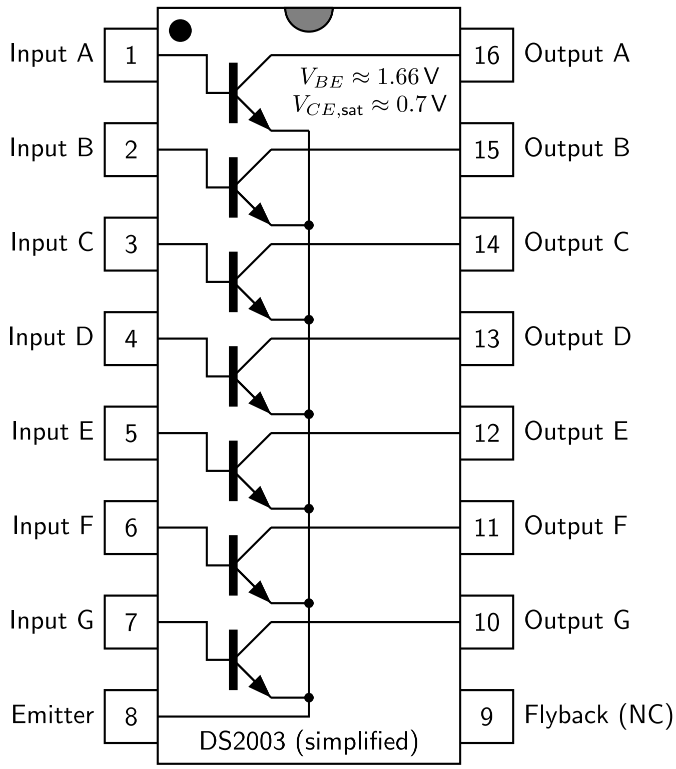


Figure 4: Simplified DS2003 high-current Darlington transistor package (top view).

This chip can be used as **seven** independent digital switches or a **single** analog amplifier.

- **Digital switching.** If the shared emitter on pin 8 is wired to ground (0 V), each input can be wired to a digital logic device. When an input goes from a low to a high logic level, the corresponding output will go from high impedance (i.e., an open circuit) to low impedance (i.e., a short circuit to $V_{CE,\text{sat}}$ above ground) and vice versa. For example, Figure 2 can be implemented using pin 16 as the collector, pin 8 as the grounded emitter, and pin 1 as the digital input v_{in} applied to the base, and this configuration can be repeated six more times to drive six other loads in a similar way.

- **Analog amplifier.** If the shared emitter on pin 8 is wired to a load, and *one* of the seven output pins is tied to a high voltage, then the voltage placed across the load will be V_{BE} less than the voltage at the corresponding input pin. For example, Figure 3 can be implemented using pin 8 as the emitter output v_{out}, pin 16 as the collector, and pin 1 as the base input v_{in}; however, none of the other input and output pins can be used.

2 The thermal process

The thermal plant that will undergo system identification is shown in Figure 5. The Darlington transistor drives a current through an incandescent lamp that produces both light and heat. Heat flow is transmitted over a channel of air until it reaches a temperature sensor. That heat flow causes a change in temperature within the sensor, and that temperature data is acquired by the computer and used to identify parameters of the system.

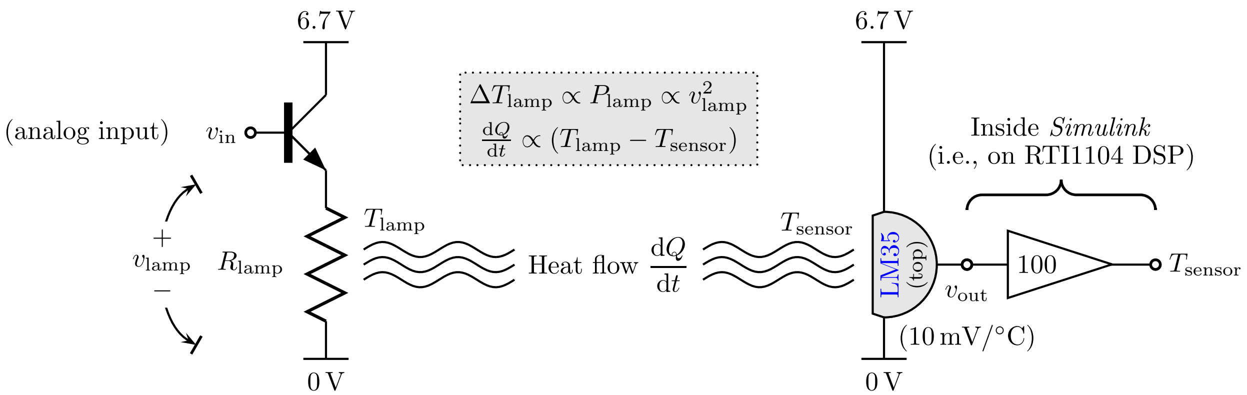


Figure 5: Simple model of thermal plant (actuator and process).

There are several complications involved with the modeling and operation of this plant. We examine some of those issues here.

2.1 Stabilizing lamp resistance with glow bias

The resistivity of the lamp's metal filament is strongly temperature dependent due to increased electron–phonon interaction with more thermal agitation. As a consequence, the resistance R_{lamp} of the lamp is much lower when it is off and cool than it is when it is on and very hot. Our model assumes that the lamp has a relatively constant resistance of approximately $(6\,\text{V})/(100\,\text{mA}) = 60\,\Omega$. To ensure this, we make sure the lamp is hot enough to glow at all times. Our assumption is that at these very high temperatures, its current–voltage relationship will be relatively linear.

It has been experimentally shown that the lamp faintly glows when 1 V is placed across it. Because our Darlington transistors regulate a $V_{BE} \approx 1.66\,\text{V}$ difference between the system input (i.e., the transistor base) and the voltage across the lamp, the input $v_{\text{in}} \geq 2.66\,\text{V}$ for the lamp to glow. However, because the lamp is only rated for 6 V, and so $v_{\text{in}} \leq 7.66\,\text{V}$. So we force

$$2.75\,\text{V} \leq v_{\text{in}} \leq 6.75\,\text{V} \qquad \text{(i.e., 4.75 V with a } \pm 2\,\text{V swing)} \tag{1}$$

so that the lamp will be operated safely in its linear range. Our input signal will always have some DC offset, and hence frequency-domain system identification must use peak-to-peak measurements after offset transients have died out.

2.2 Nonlinear power dissipation

For small changes in temperature, the change in temperature due to self heating is proportional to the corresponding power dissipation. The power dissipated across lamp resistance R_{lamp} is the product of its current and voltage. In particular,

$$P_{\text{lamp}} = \frac{v_{\text{lamp}}^2}{R_{\text{lamp}}}$$

where v_{lamp} is the voltage across the lamp. So we cannot use v_{in} or v_{lamp} as the input to our system. We must treat P_{lamp} as the input to our system or else the quadratic nonlinearity will be significant. For example, a 2 V step function on v_{lamp} must be viewed as a $4\,\text{V}^2/R$ step function on P_{lamp}. Similarly, because

$\sin(\omega t)^2 = 0.5 + 0.5\cos(2\omega t)$ (i.e., positive power is dissipated for both current directions), applying a pure sinusoid to v_{lamp} introduces a second harmonic in the power channel, so v_{in} must be adjusted so that P_{lamp} is sinusoidal.

Using power dissipation as an input

Because of the nonlinear relationship between voltage and power, the power signal P_{lamp} must be viewed as the input to the LTI system instead.

- **Step responses.** So a v_{lamp} step from 0 V to 5 V is modeled as a step on v_{lamp}^2 (which is linearly proportional to the power signal) from $0\,\text{V}^2$ to $25\,\text{V}^2$.
- **Frequency responses.** For frequency-domain system identification, a signal meeting the *spirit* of Equation (1) that generates an offset sine wave on P_{lamp} must be used. An example signal is

  $$v_{\text{in}} = \sqrt{\frac{5^2+1^2}{2} + \frac{5^2-1^2}{2}\sin(\omega t)} + 1.66\,\text{V}$$

  which results in

  $$v_{\text{lamp}}^2 = \frac{5^2+1^2}{2} + \frac{5^2-1^2}{2}\sin(\omega t).$$

  So the v_{lamp}^2 input sinusoid has a peak-to-peak amplitude of $(5^2-1^2)\,\text{V}^2 = 24\,\text{V}^2$. An example signal that actually *meets* Equation (1) is

  $$v_{\text{in}} = \sqrt{\frac{(6.75-1.66)^2+(2.75-1.66)^2}{2} + \frac{(6.75-1.66)^2-(2.75-1.66)^2}{2}\sin(\omega t)} + 1.66\,\text{V} \qquad (2)$$

  which results in a sinusoidal power-to-conductance signal v_{lamp}^2 with a peak-to-peak amplitude of $(5.09^2-1.09^2)\,\text{V}^2 = 24.72\,\text{V}^2$.

Simpler treatment of quadratic nonlinearity for offset sinusoids

Because

$$(A + \overbrace{B\sin(\omega t)}^{\text{Sinusoid}})^2 = A^2 + 2AB\sin(\omega t) + B^2\sin^2(\omega t) = A^2 + 0.5B^2 + \overbrace{2AB\sin(\omega t)}^{\text{Scaled sinusoid}} + \overbrace{0.5B^2\cos(2\omega t)}^{\text{Second harmonic}},$$

a large offset $A > B$ can causes the first ω-harmonic in the cross term to dominate the second ω-harmonic. In the context of power dissipation, so long as the current through the heating element is always positive and far from zero, increases and decreases in current correspond to similar increases and decreases in dissipation (and temperature). So linearity can be restored with a large offset.

A sinusoidal lamp voltage signal meeting Equation (1) is

$$v_{\text{in}} = 4.75\,\text{V} + (2\,\text{V})\sin(\omega t) \qquad (3)$$

which corresponds to the lamp power-to-conductance signal v_{lamp}^2 of

$$\begin{aligned}((4.75\,\text{V} - 1.66\,\text{V}) + (2\,\text{V})\sin(\omega t))^2 &= (3.09\,\text{V})^2 + 0.5(2^2) + 2(3.09\,\text{V}^2)(2)\sin(\omega t) + 0.5(2\,\text{V})^2\cos(2\omega t)\\ &= (11.5481\,\text{V}^2) + (12.36\,\text{V}^2)\sin(\omega t) + (2\,\text{V}^2)\cos(2\omega t).\end{aligned}$$

So, because of the large DC offset, the second ω-harmonic (i.e., the $\cos(2\omega t)$ term) is relatively small compared to the first ω-harmonic (i.e., the $\sin(\omega t)$ term). In particular, the second harmonic has 16% of the signal amplitude and 2.6% of the signal power of the first harmonic. In fact, the signal in Equation (3) is very similar to the signal in Equation (2), and the ω-harmonic has the same amplitude in both signals.

So, instead of generating the square root of an offset sinusoid, we generate a voltage sinusoid with enough offset that the power signal can be approximated by a scaled version of the voltage signal. However, P_{lamp} must still be viewed as the input. For best results, the restrictions in Equation (1) can be adjusted to increase the corresponding DC offset and decrease the corresponding sinusoidal amplitude. For example, signals like

$$v_{\text{in}} = (5.5\,\text{V}) + (2\,\text{V})\sin(\omega t) \qquad \text{or} \qquad v_{\text{in}} = (6.5\,\text{V}) + (1\,\text{V})\sin(\omega t) \qquad (4)$$

may lead to much more linear behavior of the actuator and channel.

2.3 Linear model of a thermal process

We assume that the LM35 has a Biot number less than 0.1 so that we can use a *lumped system model* of heat transfer. That is, we assume that the temperature across the LM35 sensor has a uniform temperature across its body. Under this assumption, the relationship between heat flow $\mathrm{d}Q/\mathrm{d}t$ and temperature T_{sensor} is *capacitive*; that is,

$$\text{(heat flow is analogous to a \textit{current})} \qquad \frac{\mathrm{d}Q}{\mathrm{d}t} = C\,\frac{\mathrm{d}T_{\text{sensor}}}{\mathrm{d}t} \qquad \text{(temperature is analogous to a \textit{voltage})} \tag{5}$$

where C is the heat capacity of the LM35. This relationship is analogous to the

$$i_C = \frac{\mathrm{d}q_C}{\mathrm{d}t} = C\,\frac{\mathrm{d}v_C}{\mathrm{d}t}$$

relationship of a capacitor where i_C is the current flowing into the capacitor (i.e., change in charge over time) and v_C is the voltage across the capacitor. As heat flows into the LM35 body, it generates a proportional change in temperature just as voltage rises across a capacitor when electric current flows into it. Just as the capacitor equation is based on the assumption of uniform voltage across the capacitor, the heat capacitance equation is based on the assumption of uniform temperature across the body. In fact, this is the meaning of having a low Biot number.

So, because:

(i) the actuator of our system is tantamount to a simple gain,

(ii) the heat flow across the channel is proportional to the temperature difference between the lamp and the sensor, and

(iii) the temperature of the sensor rises linearly with constant heat flow,

then the v_{lamp}^2–T_{sensor} system is assumed to be *linear* (e.g., step changes in the input should result in an response that shows exponential decay of the old state). In particular, an intuitive electrical model of the system is shown in Figure 6 where K is the product of the collected proportionality constants in the system, R represents the thermal resistance (i.e., the linear relationship between temperature difference and heat flow), and C represents the heat capacity of the temperature sensor.

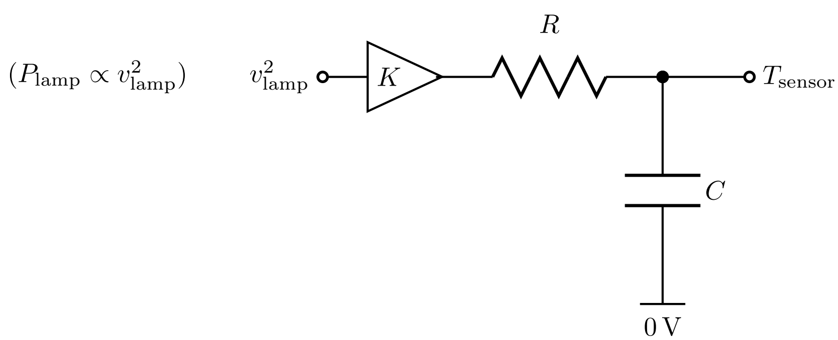


Figure 6: Electrical model of the thermal plant from Figure 5.

Hence, the LTI system will have a first-order low-pass transfer function of the form

$$H(s) \triangleq \frac{T_{\text{sensor}}(s)}{V_{\text{lamp}}(s) * V_{\text{lamp}}(s)} = \frac{K}{sRC+1},$$

which explains why temperature step responses often appear to have *exponential decay.* System identification allows for experimentally finding the specific gain K and time constant RC quantities.

3 Summary: implementation of thermal system identification

The experimental setup for thermal process system identification is shown in Figure 7. Good values for time-domain system identification step functions and frequency-domain system identification sinusoidal functions are given.

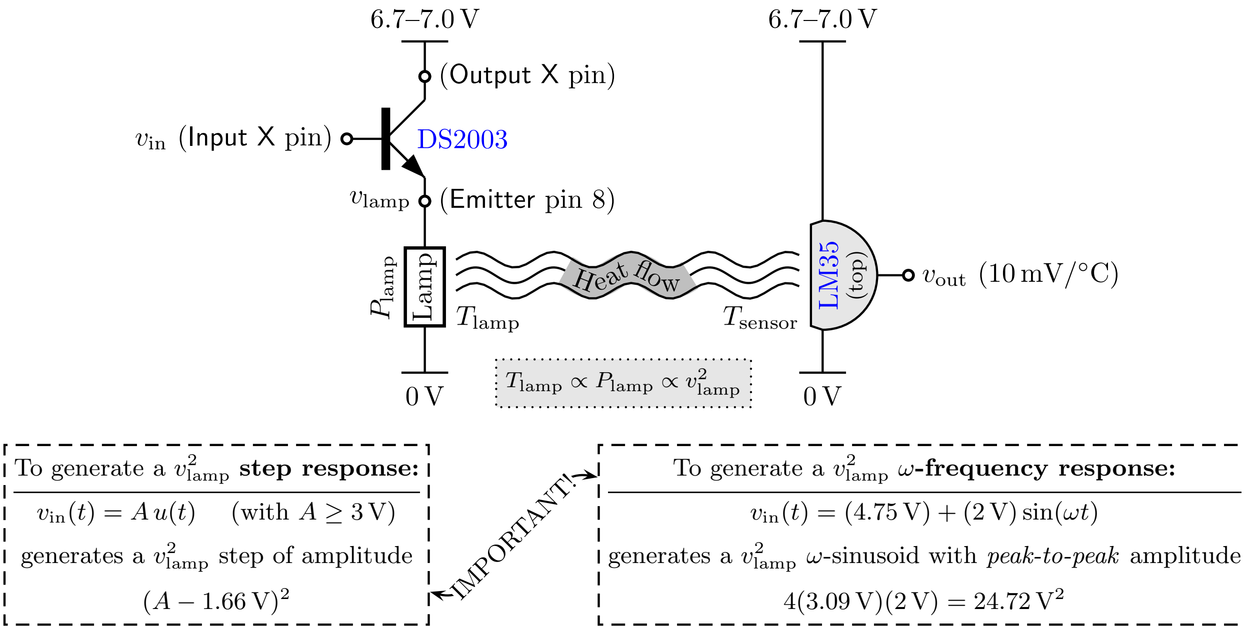


Figure 7: Thermal process experimental setup. Only use one DS2003 channel at a time.

The simplified pinout for the DS2003 Darlington transistor driver chip is shown in Figure 8.

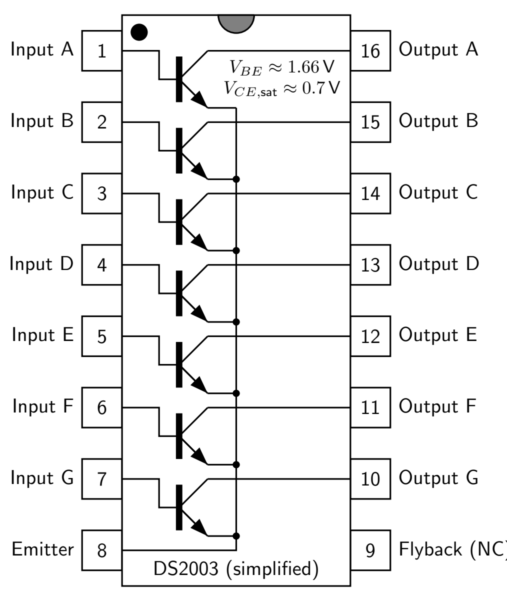


Figure 8: Simplified DS2003 high-current Darlington transistor package (top view).

The step function given in Figure 7 does not include the glow biasing described in section 2.1. This choice was deliberate. Adding the additional offset for lamp resistance stabilization requires additional experiment time for those transients to die out, and the nonlinearities in the lamp resistance are negligible for a step response anyway.

4 Data analysis

As discussed in section 2.2, this plant has nonlinear behavior unless the **power signal** P_{lamp} is viewed as the input. In the special cases of

- step responses, and
- frequency responses using significantly offset sinusoids,

signals of the appropriate shape can be applied directly to v_{in}. However, when calculating the *gain* of each test, the corresponding P_{lamp} magnitude must be used instead.

4.1 Processing step-response data

If step-response data was generated with

$$v_{\text{in}}(t) = A\,u(t),$$

then the *input* to the v_{lamp}^2–T_{sensor} LTI system is

$$v_{\text{lamp}}^2(t) = (A - 1.66\,\text{V})^2\,u(t),$$

and so the DC *gain* should be calculated as

$$\boxed{|H(0)| = \frac{\max\{T_{\text{sensor}} \text{ step response}\} - (T_{\text{sensor}} \text{ ambient temperature})}{(A - 1.66\,\text{V})^2}.}$$

Using this method, step responses of different input amplitudes should all have the same gain.

4.2 Processing frequency-response data

If the frequency-response data was generated with

$$v_{\text{in}}(t) = A + B\sin(\omega t) \qquad (\text{where } A > B),$$

then the *input* to the v_{lamp}^2–T_{sensor} LTI system can be assumed to have *peak-to-peak amplitude*

$$4(A - 1.66\,\text{V})B,$$

and so the gain at frequency ω should be calculated as

$$|H(j\omega)| = \frac{\text{Peak-to-peak } \omega\text{-amplitude in } T_{\text{sensor}} \text{ data}}{4(A - 1.66\,\text{V})B}.$$

In particular, for

$$v_{\text{in}}(t) = (4.75\,\text{V}) + (2\,\text{V})\sin(\omega t) \qquad (\text{where } A > B),$$

the gain at frequency ω should be calculated as

$$\boxed{|H(j\omega)| = \frac{\text{Peak-to-peak } \omega\text{-amplitude in } T_{\text{sensor}} \text{ data}}{24.72\,\text{V}^2}.}$$

Using this method, the data should look almost linear and be nearly consistent with the step-response data. However, because a $4\,\text{V}^2$ second harmonic exists in the power channel, **the peak-to-peak data may be up to 16% higher than it would be in a truly linear system.** Hence, you should expect frequency-domain identification gains that are **higher than those predicted by your step-response identification**[1]**.** Unless a *true* sinusoid is generated on v_{lamp}^2, the quadratic nonlinearity will corrupt the experiment. Alternatively, the data can be improved by increasing the offset-to-amplitude ratio, as in Equation (4).

[1] Similarly, the frequency-identified corner frequency may be lower than the step-identified corner frequency because of the impact of the second harmonic rolling off more quickly than the first harmonic.