Dynamics of Metabolism and Decision Making During Alcohol Consumption: Modeling and Analysis

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***Abstract*—Heavy alcohol consumption is considered an important public health issue in the United States as over 88 000 people die every year from alcohol-related causes. Research is being conducted to understand the etiology of alcohol consumption and to develop strategies to decrease high-risk consumption and its consequences, but there are still important gaps in determining the main factors that influence the consumption behaviors throughout the drinking event. There is a need for methodologies that allow us not only to identify such factors but also to have a comprehensive understanding of how they are connected and how they affect the dynamical evolution of a drinking event. In this paper, we use previous empirical findings from laboratory and field studies to build a mathematical model of the blood alcohol concentration dynamics in individuals that are in drinking events. We characterize these dynamics as the result of the interaction between a decision-making system and the metabolic process for alcohol. We provide a model of the metabolic process for arbitrary alcohol intake patterns and a characterization of the mechanisms that drive the decision-making process of a drinker during the drinking event. We use computational simulations and Lyapunov stability theory to analyze the effects of the parameters of the model on the blood alcohol concentration dynamics that are characterized. Also, we propose a methodology to inform the model using data collected *in situ* and to make estimations that provide additional information to the analysis. We show how this model allows us to analyze and predict previously observed behaviors, to design new approaches for the collection of data that improves the construction of the model, and help with the design of interventions.**

***Index Terms*—Blood alcohol concentration, decision making, dynamical systems, feedback systems, Lyapunov stability analysis, metabolism, public health.**

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This paper has supplementary downloadable multimedia material available at http://ieeexplore.ieee.org provided by the authors. This includes a file with mathematical proofs and MATLAB source code. The total size of the file is 0.09 MB.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

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I. Introduction

High-risk alcohol consumption is considered a large problem in public health. For example, in the United States, around 88 000 people die annually from alcohol-related causes [1], and it is considered the third major preventable determinant of death in the United States [2]. Also, over 40% of all college students report getting drunk in the past month [3] resulting in an estimated 1800 deaths per year [4]. A drinking event is defined as the situation where an individual or a group of people naturally participate in activities related to alcohol-consumption during a specified period of time. Research has focused on comprehending the etiology of heavy drinking at the event level, since this understanding is crucial to identifying potential "leverage points" in the event to intervene and alter heavy drinking or related problems [5].

Several field studies have examined drinking events *in situ*. These studies have found that individual characteristics (e.g., motivations for drinking and drinking history) [6], [7], event-level factors (e.g., duration of drinking) [8], group activities (e.g., drinking games) [9], [10], and environmental factors (e.g., drink specials and dancing) [11], [12] impact intoxication. These findings rely on data that are obtained through questionnaires, breathalyzer readings of blood alcohol content (BAC), and observations of the environment. Their interpretation is mainly based on statistical analyzes that try to uncover correlations between these factors and high-risk drinking [8]. Although they represent a very important contribution to better understand heavy alcohol consumption, there are still major gaps in our understanding of drinking events. The available information and the tools employed in the above analyzes are limited. For example, the behavior of the BAC in an individual has been measured only before and after the drinking event, but not throughout it. Also, the statistical tools that have been used for their analysis only capture "snapshots" of the underlying dynamics of the drinking event, not its time evolution. Studying these elements is crucial to understand the dynamic and complex nature of high-risk drinking at the event level. In this paper, we set a starting point to fill these gaps.

There is a growing need for developing dynamical system models to better understand and address problems related to alcohol and, in general, to public health [13]–[15]. These models enable a comprehensive analysis of the problem and the elements involved in it, to design more effective interventions and assessment methodologies, and to formulate large-scale field experiments. Several dynamical models have been proposed to study high-risk drinking [16], [17]. They characterize how alcohol use in large groups or populations changes between different categories such as "nondrinkers," "social-drinkers," and "heavy-drinkers." Also, several simulation strategies have been proposed to recreate drinking scenarios [18], [19]. However, as far as we know, there has not yet been a model constructed that characterizes how the BAC of an individual varies over time throughout the drinking event, taking into account the dynamical interaction between physiological factors and decision-making processes, and that allows us to conduct both computational and mathematical analyzes for a more complete understanding of the modeled behaviors.

The contribution of this paper is twofold. First, we provide a model of a system that, based on observations and up-to-date collected data of behaviors in drinking events, characterizes the BAC dynamics of a drinker throughout the event. Second, We provide a mathematical analysis of the modeled behaviors, and show how this model can complement empirical research. This model is constructed as the interconnection of a component that accounts for the dynamics of the metabolic process of alcohol, and a component that describes the decision-making process that drives the individual's alcohol consumption. In the same way that physics attempts to describe how forces translate into changes in motion, our main objective is to propose a model that explains the "physics" of an individual's BAC during a drinking event: how the output of a decision-making process translates into BAC variations and how BAC variations in turn affect decision-making. We propose a methodology to construct the model using data collected *in situ*, and to find the parameters of the decision-making process that generate the BAC trajectories that fit the available data.

We envision this model to be the starting point of a body of research that looks for a more complete characterization of the dynamics of social drinking events. Technological advances have made available via portable and reliable devices for real-time data collection on drinking events that include transdermal alcohol sensors for BAC monitoring [20]–[22], and devices to estimate relative proximity between individuals and their geographical location to make inferences about their social interactions and environment [15], [23]. These sources of real-time data collection will eventually be used, along with survey and observational data, to build on our model and develop more accurate descriptions of the dynamics of drinking events. In the same way that modern control engineering derives mathematical descriptions of systems (such as electrical, mechanical, chemical, economic, and human systems) to design strategies that control them such that they have a desired behavior [24], [25], our medium-term goal is to have a mathematical description of the dynamics of a drinking event to design control strategies (i.e., interventions) at the individual, group, and environment level that manipulate them such that high-risk behaviors are minimized [26]. Current technologies, such as smartphones, could possibly assist with interventions at the individual and group level and help minimize high-risk situations [27], [28]. Fig. 1 illustrates this concept. This paper is a step toward that goal.

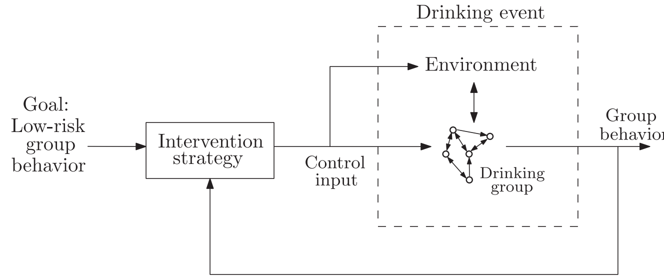


Fig. 1. Block diagram of an intervention strategy that seeks to minimize high-risk drinking behaviors. The network in the diagram represents the drinking group where each individual (small circles) interacts with other individual's behaviors.

The proposed model follows a standard mathematical framework for the analysis of linear and nonlinear systems [29]–[31]. Using this framework, we are able to model the dynamics of metabolic and decision-making processes of an individual in a drinking event. We characterize, based on data collected in laboratory settings and through a set of differential equations, the dynamics of the BAC resulting from the metabolic process of alcohol given arbitrary alcohol intake patterns, and provide its causal-loop representation (Section II-A). Also, we propose a description of the mechanisms that drive the decision-making process of a drinker during a drinking event based on data collected *in situ* and observations about the relationship between the subjective level of intoxication, BAC, and BAC rate of change (Section II-B). Using computer simulations and Lyapunov stability analysis of nonsmooth systems [29], [32] we analyze the modeled behaviors (Sections II-C and III). We finish this paper with a discussion about the utility of these results in the future design of intervention strategies for the prevention/minimization of high-risk behaviors in drinking events.

II. Model Formulation

The dynamics of alcohol consumption of an individual in a drinking event are modeled as the result of the interaction between a decision-making process, where the individual manipulates his/her drinking rate to reach a desired level of alcohol intoxication, and the metabolic process of the consumed alcohol that drives the BAC. Fig. 2 shows a block diagram that represents the functional relationships of the components of the dynamical system that characterizes the drinking behaviors of an individual during a drinking event. This system has a decision making component that, based on the current BAC and BAC rate of change, adapts the rate of alcohol intake to achieve the desired level of intoxication. The consumed alcohol is then metabolized by the body following a dynamical process that is characterized by a component whose output is the BAC and BAC rate of change. The behavior of this system might be affected by external influences such as food consumption. A system with this structure of the interconnection between components is a feedback control system [25]. A feedback control system is typically represented

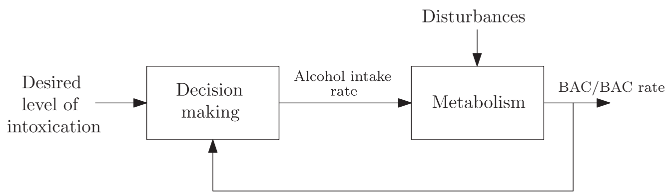


Fig. 2. Block diagram of the system that results of the interconnection between the drinker's decision-making process and metabolic process for alcohol.

by the interconnection of two basic elements: 1) a process ("plant"), which describes the relation between the variables to be controlled and the control variables ("outputs") and 2) the controller, which compares the outputs with the desired state to be achieved, and manipulates the control variables accordingly. In the system represented in Fig. 2, the process and controller are the metabolic and the decision-making processes, respectively. The variables to be controlled are the BAC and BAC rate of change, and the control variable is the alcohol intake rate. Next, we present models that characterize the dynamics of the metabolism and decision-making components of the feedback system, and show through a mathematical and computational analysis that the behaviors that they describe are consistent with observations of alcohol-consumption behaviors in individuals who are in a drinking event.

A. Metabolism

Research on pharmacokinetics of alcohol has shown that, after a single dose of alcohol has been administered, the human body metabolizes it throughout different stages such that the BAC exhibits a profile as shown in Fig. 3. Starting from an initial condition, the BAC rises and reaches a peak concentration, followed by a declining phase where the BAC reaches zero [33], [34]. Different mathematical models have been proposed to describe this behavior of the BAC. Among the most relevant models are the Widmark and Michaelis–Menten models, which characterize the alcohol elimination process after the peak in BAC has been reached [35], [36]. Also, models based on the physiology of the human body have been formulated, taking into account the interaction between organs such as the liver, kidney, and stomach [37]–[39]. Although they represent a very important contribution to the understanding of the metabolic process of alcohol, they either are nonlinear or describe only some of the stages in this process. Our aim in this paper is to formulate a model that characterizes the dynamics of all the stages in the pharmacokinetics of alcohol and at the same time it must be simple enough so that a mathematical and computational analysis can be conducted once the model is integrated into the feedback system in Fig. 2.

In the theory of dynamical systems, the impulse response of a system refers to the output when there is a very short-duration input signal whose integral has a given value. If we model the alcohol metabolic process as a system whose input is the alcohol intake rate and the output is the BAC level, then the behavior shown in Fig. 3 approximates the impulse response of a second-order dynamical system [24, Ch. 5], assuming that the system is linear and

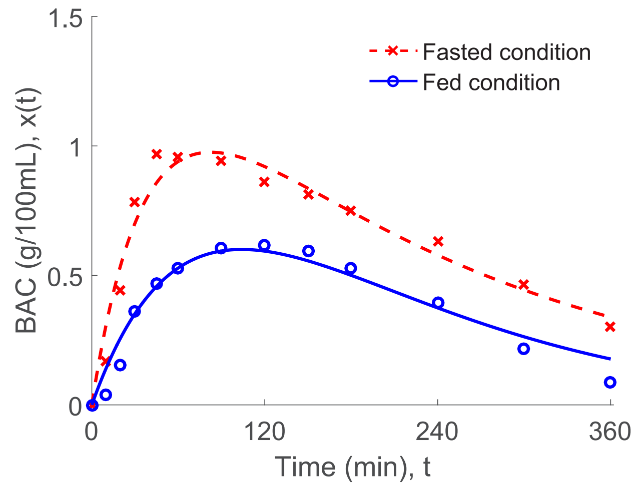


Fig. 3. Average of BAC samples measured on 12 individuals when a dose of 0.8 g ethanol/kg of weight was administered under two different situations: when they were in a fasted condition (symbol "x"), and when they were in a fed condition (symbol "o") (measurements taken from [40]). The solid and dashed lines are the estimated trajectories using the model in (1) and a least-squares curve fit. The values of the parameters for the fasted condition were $\zeta^* = 1.4$, $\beta^* = 0.0107$, and $\eta^* = 39.23$, and for the fed condition were $\zeta^* = 1.0003$, $\beta^* = 0.0096$, and $\eta^* = 21.21$. The methodology and MATLAB code used to fit the data are provided in Appendixes C and E in the supplementary file.

time invariant. This type of systems is commonly used to describe the dynamics of physical processes including electrical, chemical, and mechanical systems. Here, we use this family of systems to characterize the metabolic process as the result of the interaction between an external input and two variables that describe the intrinsic state of the system. In our case, these two variables are the BAC level and BAC rate of change.

The mathematical formulation is as follows. Let $x(t) \geq 0$ be the BAC level at time $t \geq 0$, and let $v(t) = \dot{x}(t)$ be the BAC rate of change, where $\dot{x}$ denotes the derivative of $x(t)$ with respect to t. Let $u(t) \geq 0$ be the alcohol intake (consumption) rate. Note that the amount of alcohol consumed from time 0 to time $t > 0$ is given by $\int_0^t u(\tau)d\tau$. The set of differential equations that represents a second-order dynamical system whose impulse response follows the one in Fig. 3 is:

$$\begin{aligned}\dot{x}(t) &= v(t)\\ \dot{v}(t) &= -\beta^2 x(t) - 2\zeta\beta v(t) + \beta^2 \eta u(t)\end{aligned} \tag{1}$$

where η, β, and ζ are the parameters of the model. This dynamical system can be seen from the viewpoint of Newton's second law, where there are forces that define the BAC's acceleration $\dot{v}$. The term $-\beta^2 x$ is one that forces the BAC to decrease to zero, and $-2\zeta\beta v$ affects how quickly the BAC changes. The term $\beta^2\eta u(t)$ is the scaled external input to the system, that in this case is related to the alcohol consumption rate throughout the drinking event. Fig. 4 shows the causal-loop diagram of the dynamical system in (1).

Next, we provide a theorem that characterizes the behavior of the trajectories generated by the system in (1), and show how the model fits data collected in laboratory settings.

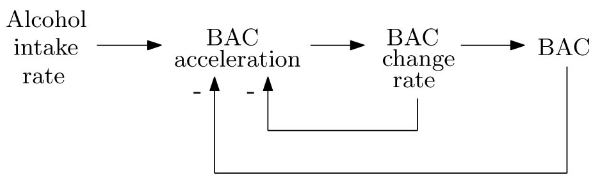


Fig. 4. Causal-loop diagram of the dynamical system that characterizes the metabolic process of alcohol in (1).

Theorem 1: Let the parameters of the model in (1) be such that $\eta > 0$, $\beta > 0$, and $\zeta > 1$. Then, a solution trajectory of the BAC for a given alcohol intake rate $u(t)$, $t \geq 0$, is

$$x(t) = g_0(t) + \frac{\beta\eta}{2\sqrt{\zeta^2 - 1}} \int_0^t \left[e^{\lambda_2(t-s)} - e^{\lambda_1(t-s)} \right] u(s) ds \quad (2)$$

where

$$g_0(t) = \frac{1}{2\beta\sqrt{\zeta^2 - 1}} \left[(\lambda_2 x_0 - v_0) e^{\lambda_1 t} - (\lambda_1 x_0 - v_0) e^{\lambda_2 t} \right]$$

and $x_0 = x(0)$, $v_0 = v(0)$, $\lambda_1 = -\beta\zeta - \beta\sqrt{\zeta^2 - 1}$, and $\lambda_2 = -\beta\zeta + \beta\sqrt{\zeta^2 - 1}$.

Corollary 1: In addition to the assumptions in Theorem 1, assume that $u(t) \in [0, +\infty)$ for all $t \geq 0$ and $x_0 = x(0) \geq 0$. Then, the trajectories generated by (1) satisfy the following properties.

1) $x(t) \geq 0$ for all $t \geq 0$.
2) Let the alcohol intake rate be an impulse with amplitude $C > 0$ (i.e., a single dose administered at $t = 0$ with amount of alcohol C), and let the initial conditions be $x(0) = 0$ and $v(0) = 0$. Then, the impulse response of the system for $t \geq 0$ is given by
$$x_\delta(t) = \frac{C\beta\eta}{2\sqrt{\zeta^2 - 1}} \left(e^{\lambda_2 t} - e^{\lambda_1 t} \right). \quad (3)$$
3) $\lim_{t\to\infty} x_\delta(t) = 0$.
4) The impulse response $x_\delta(t)$, $t \geq 0$, has a peak that occurs at time
$$t_p = \frac{1}{2\beta\sqrt{\zeta^2 - 1}} \log\left(\frac{-\zeta + \sqrt{\zeta^2 - 1}}{-\zeta - \sqrt{\zeta^2 - 1}} \right). \quad (4)$$

The proofs are based on basic concepts of linear systems and optimization [30], and they are presented in Appendix A in the supplementary file. Theorem 1 shows that the behavior of the BAC level is modeled as the integral of the alcohol intake rate, which is the accumulated ingested alcohol. The amplitude of the alcohol intake rate is modulated by the subtraction of two exponentials with different decay rates. This accounts for the elimination process of alcohol [33], [34]. The term g_0 is associated with dynamics that result from initial conditions that are different from zero (e.g., when someone has consumed alcohol before the time reference of $t = 0$). Some useful properties of this family of trajectories are presented in Corollary 1: property 1) indicates that the BAC cannot take negative values as long as the initial conditions are non-negative. Property 2) shows that the impulse response of the system corresponds to the subtraction of two exponential curves that have different decay rates. This result is consistent with the models describing the BAC profile in Fig. 3 presented in [41] and [42], which are referred as "one-compartment models." As shown in Appendix A in the supplementary file, the conditions on the parameters that are necessary to describe this behavior are that β and η have to be positive, and $\zeta > 1$, which guarantee that $\lambda_1 < \lambda_2 < 0$. Property 3) shows that the BAC decreases and tends to zero asymptotically when the input is an impulse signal, behavior resulting from the elimination stage of the metabolic process. Property 4) shows that the modeled impulse response has a peak whose occurrence time depends on parameters ζ and β, but not on η. From (4), the peak value of x_δ is

$$x_\delta(t_p) = \frac{C\beta\eta}{2\sqrt{\zeta^2 - 1}} \left(e^{\frac{\sigma_2}{2\sqrt{\zeta^2-1}} \log\left(\frac{\sigma_2}{\sigma_1}\right)} - e^{\frac{\sigma_1}{2\sqrt{\zeta^2-1}} \log\left(\frac{\sigma_2}{\sigma_1}\right)} \right)$$

where $\sigma_2 = -\zeta - \sqrt{\zeta^2 - 1}$ and $\sigma_1 = -\zeta + \sqrt{\zeta^2 - 1}$. This corresponds to maximum value of the BAC that marks the beginning of the ethanol elimination stage. Note that parameter η is associated with the height of the peak but not with its occurrence time. Fig. 5 shows the relationship between the parameters ζ and β on the time t_p when the peak occurs and its amplitude $x_\delta(t_p)$. Observe that t_p decreases as ζ and β increase, as opposed to $x_\delta(t^*)$, which increases as ζ and β increase.

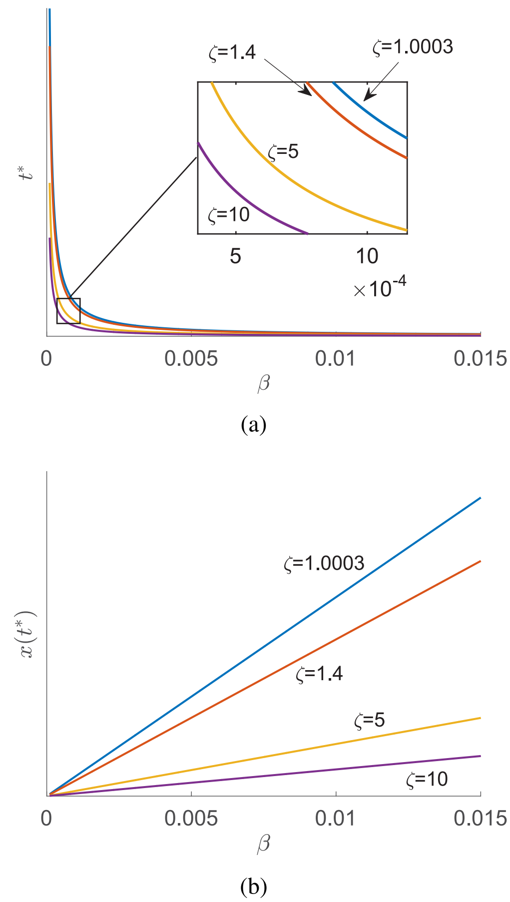


Fig. 5. (a) Time (t^*) and (b) amplitude ($x_\delta(t^*)$) of the maximum value that the BAC reaches during the metabolic process for different values of the parameters ζ and β when the input alcohol intake rate corresponds to an impulse.

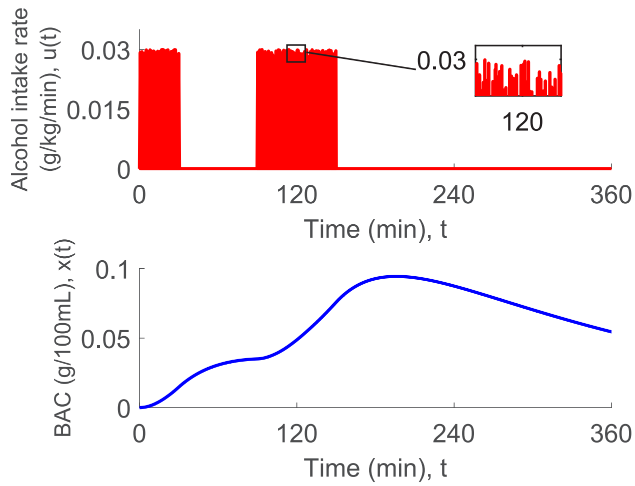


Fig. 6. Trajectory of the BAC when the alcohol intake rate is a signal that simulates a drinker taking sips whose alcohol concentration changes around 0.02 g of alcohol per kg of weight. The parameters of model associated with the metabolic process are the ones estimated in Fig. 3 in the fasted condition case. The MATLAB code to generate these signals is provided in Appendix F in the supplementary file.

Fig. 3 shows the average BAC samples collected when a dose of alcohol was administered to participants under two different situations: 1) fasted condition and 2) fed condition. The trajectories generated by the model were computed using (1) after finding the parameters that provided the best fit according to a least squares criterion for each case (fasted and fed conditions). Note that the proposed system provides a good approximation of the measured BAC response. We also simulate the case when the alcohol intake rate is not an impulse. Fig. 6 shows the solution trajectory of the BAC when the alcohol intake rate is a signal containing short duration pulses that simulate an individual taking sips of alcohol. Details of the implementation are presented in Appendix E in the supplementary file.

Although (2) provides an expression for the BAC trajectories for any alcohol intake rate function $u(t)$ with respect to time, the formulation of the system through the differential equations in (1) facilitates a mathematical analysis once it is integrated into the feedback system in Fig. 2 as we will show below.

B. Decision Making

Research that studies the factors that affect the behavior of drinkers in a drinking event has shown that the individual's alcohol intake rate is correlated with the desired level of intoxication that he/she wants to achieve. The desired level of intoxication of an individual in a drinking event is influenced by elements that depend on the characteristics of the drinker and the social and environmental influences. Research suggests that the individual's characteristics such as desired body reaction to the BAC, history of drinking, and drinking motives are individual factors that affect the alcohol intoxication level at the drinking event [6], [7]. At the group level, peer interactions have a significant impact on the behavior of a person in a drinking group. Perception of social norms [43], the way that people communicate in the group, and the strength of the interpersonal relations influence drinking behaviors [9], [10]. Also, data that have been collected *in situ* indicate that environmental factors affect consumption patterns. For example, drinking games and alcohol price "specials" can promote higher consumption of alcohol, as opposed to scenarios that are crowded, which can result in lower alcohol consumption [11], [12]. The interactions between the influences at the individual, group, and environmental levels sets the level of intoxication that the individual wants to achieve during the drinking event. In this paper, we do not model the dynamics of such interactions. We focus our analysis on the dynamics of the decision-making and metabolism processes as shown in the feedback system in Fig. 2 given the desired level of intoxication, where this desired level might change over time. That is, we focus on the individual.

Following these observations, we hypothesize that an individual manipulates his/her alcohol intake rate to reach the intended intoxication level based on his/her subjective feeling of intoxication. It has been shown that both the current state of the BAC and the BAC rate are important determinants of how a drinker perceives his/her own level of intoxication [44], [45]. The study in [44] found that the subjective intoxication level changes depending on the BAC rate of change: an individual tends to perceive levels of intoxication that are higher (lower) than the one associated with his/her current BAC when the BAC rate of change is larger (lower, respectively). Hence, we define the subjective intoxication at time t as

$$y(t) = x(t) + k_d v(t)$$

where $k_d \geq 0$ is a scaling factor that quantifies the influence of v on the perceived intoxication level. We assume that the mechanisms that an individual uses in a drinking event to manipulate his/her alcohol intake are based on his/her desire to reach the intended level of intoxication and his/her subjective level of intoxication $y(t)$. We model the decision-making process of a drinker such that the magnitude of his/her alcohol intake rate decreases when the perceived BAC $y(t)$ gets closer to the desired level of intoxication. Also, it is positive when is below the level of intoxication, and zero if it is above. A formulation that satisfies this conditions is

$$u(t) = \left[k_p\left(x^* - y(t)\right)\right]_+ \tag{5}$$

where $x^* \geq 0$ is the desired level of intoxication and $k_p > 0$ is a scaling factor that quantifies the commitment of the individual to reach the desired intoxication level x^*. The operator $[z]_+ = \max\{z, 0\}$ guarantees that $u(t) \geq 0$ for all $t \geq 0$. This expression can be rewritten as

$$u(t) = \left[k_p\left(x^* - x(t)\right) - k_p k_d v(t)\right]_+.$$

Interestingly, this decision-making strategy is typically known in feedback control systems theory as proportional-derivative control [25, Ch. 10]. The alcohol intake rate has a component that changes proportional to the error between the desired level of intoxication and the current BAC level, and a component that changes in the opposite direction of the BAC rate of change (derivative of BAC). This last component characterizes the opposition of the drinker to quick changes in the BAC.

It is important to note that the alcohol intake rate of a drinker typically has a shape over time that resembles a sequence of pulses (due to sipping and gulping), as shown in Fig. 6, whose frequency and amplitude change over time. In our model, $u(t)$ represents the resultant average alcohol intake rate that an individual can have given his/her desired level of intoxication, current BAC, and BAC rate of change (from the observations explained above).

Even though the decision-making strategy in (5) captures the basic behavior of an individual trying to achieve his/her desired level of intoxication, the assumption that the individual has a unique desired BAC level is restrictive. It can be the case that a drinker desires to achieve values of BAC that are within an interval such that they produce a desired influence on their body. For instance, a drinker who wants to have a minimal behavioral change will seek to reach values of the BAC that are between 0 and 0.04 g/100 mL [46, Ch. 13]. In order to generalize the formulation in (5) to take into account these cases, we introduce a personal preference function, $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$, that is a continuous and differentiable convex function that has a unique minimum such that, without loss of generality, $\min_y f(y) = 0$. This function quantifies the individual's preferences with respect to the BAC. Lower values of $f(y)$ correspond to more preferable levels of the BAC. The most preferable BAC levels satisfy $f(y) = 0$. The most useful property of this function in our problem is that the scaled negative derivative of f with respect to y satisfies the properties of the decision-making strategy: its magnitude decreases as the BAC gets closer to the most desirable levels, and it is positive (negative) if the current BAC is lower (higher) than the most desirable levels. In (5) it is implied that the preference function is $f(y(t)) = (1/2)(y(t) - x^*)^2$, where $-k_p df(y(t))/dy = k_p(x^* - y(t))$. Equation (5) can be generalized as

$$u(t) = \left[-k_p g(y(t))\right]_+ \tag{6}$$

where

$$g(y) = df(y)/dy$$

and $y(t) = x(t) + k_d v(t)$. The benefit of using the preference function in our formulation is that different profiles can be designed depending on the desired level of intoxication the drinker is assumed to have. Figs. 11 and 12, and Appendix D, in the supplementary file, show examples of profile functions. These preference functions have an interval of desired BAC levels rather than one single value. Also, they are steeper for BAC levels lower than the desired ones [when $f(x) = 0$] than those levels that are higher than the desired ones.

C. Combining the Metabolism and Decision-Making Processes

The model for the feedback interconnection between the metabolic and decision-making systems in Fig. 2 is obtained by substituting $u(t)$ from (6) in (1), to get

$$\begin{aligned} \dot{x}(t) &= v(t) \\ \dot{v}(t) &= -\beta^2 x(t) - 2\zeta\beta v(t) + \beta^2\eta\left[-k_p g(x(t) + k_d v(t))\right]_+. \end{aligned} \tag{7}$$

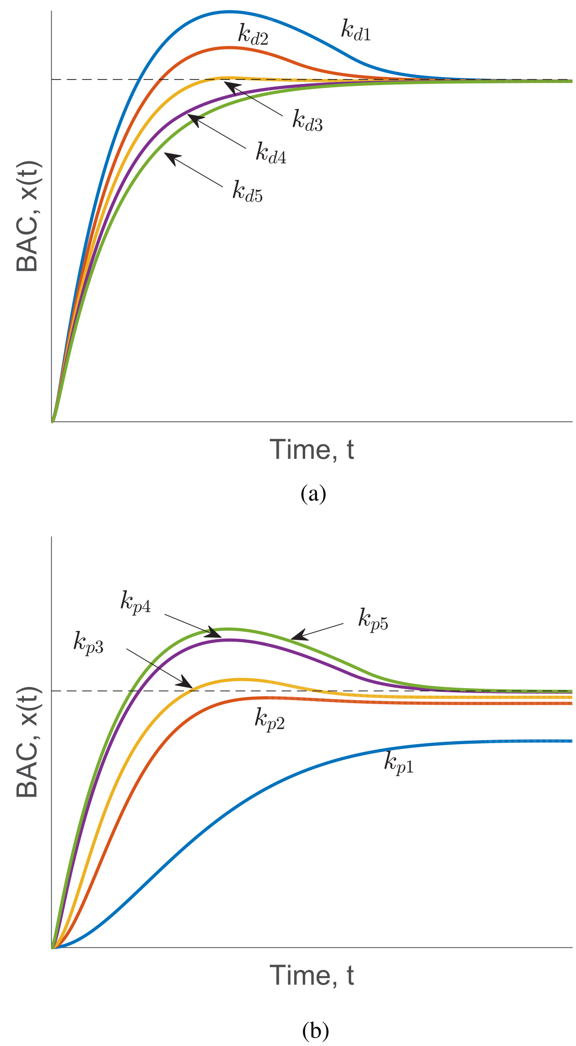


Fig. 7. (a) Trajectories of the BAC for different values of the parameter k_d and a fixed k_p. In this case, $k_{d1} < k_{d2} < \cdots < k_{d5}$. (b) Trajectories of the BAC for different values of the parameter k_p and a fixed k_d. In this case, $k_{p1} < k_{p2} < \cdots < k_{p5}$. The dashed line indicates the desired level of intoxication. The MATLAB code to generate these signals is provided in Appendix G in the supplementary file.

This captures, in one set of differential equations, the dynamics that result of the interaction between the metabolic process of alcohol and the decision-making process that drives the alcohol intake rate. The following simulations and mathematical analysis show some of the qualitative properties of the BAC dynamics for (7).

Fig. 7 shows the response of the feedback system for different values of the parameters k_p and k_d. The case in Fig. 7(a) illustrates the effect of k_d on the trajectories. Here, k_p and the initial condition are kept fixed. We can observe that as k_d increases (in the figure, $k_{d1} < \cdots < k_{d5}$), the overshoot in the BAC trajectories tends to decrease. This is due to the effect of $v(t)$ on the perception of the intoxication level. Individuals with a strong awareness of the BAC rate of change will modulate their alcohol intake rate such that their BAC trajectories increase at a slow rate. On the other hand, individuals with a small awareness of the BAC rate of change will be affected by the delayed response of the metabolic process, and therefore will have trajectories that exhibit overshoot. Fig. 8 shows

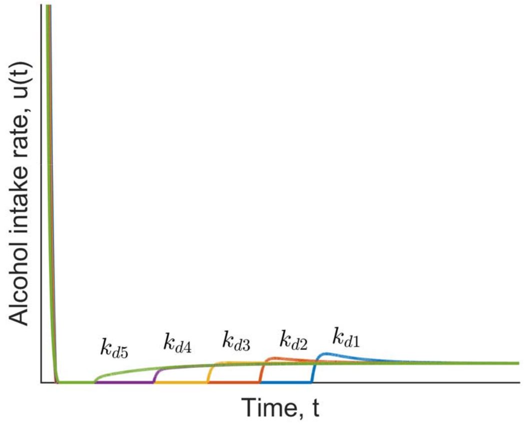
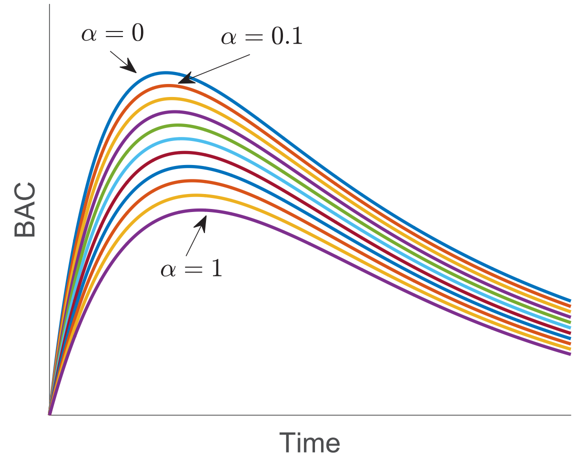
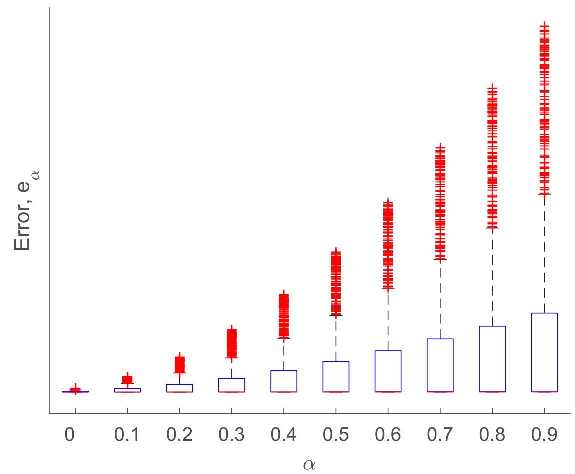


Fig. 8. Trajectories of the alcohol intake rate $u(t)$ for the cases shown in Fig. 7(a). The parameter k_p is fixed, and $k_{d1} < k_{d2} < \cdots < k_{d5}$.



(a)



(b)

Fig. 9. (a) Impulse response of metabolic processes associated with different parameters ζ, β, and η. The BAC trajectory associated with $\alpha = 0$ corresponds to the response of a metabolic process in the fasted condition in Fig. 3, and the one associated with $\alpha = 1$ corresponds to the response of the metabolic process in the fed condition. Values of α inside the interval (0, 1) are associated with responses of metabolic processes in between these two conditions. (b) Box plot of the squared error e_α defined in (8). The desired level of intoxication x^* and parameters k_p and k_d are fixed, and the initial conditions are chosen randomly. The MATLAB code to generate these plots is provided in Appendix H in the supplementary file.

the alcohol intake rate over time for each case. Observe that each individual starts with a high rate of alcohol consumption that decreases as the BAC gets closer to the desired level of intoxication. Once the BAC is around x^*, it manipulates $u(t)$ so that the BAC stays very close to the desired level.

The case in Fig. 7(b) explores the effect of k_p on the trajectories. Here, k_d and the initial condition are kept fixed. It can be observed that as k_p increases (in the figure, $k_{p1} < \cdots < k_{p5}$), the error between the desired intoxication level and the BAC decreases and the overshoot increases. The parameter k_p can be seen as the commitment of the individual to achieve his/her desired level of intoxication. Different combinations of these two parameters, k_p and k_d, provide a rich variety of responses that an individual can have during a drinking event.

In order to study the effect of the metabolism parameters ζ, β, and η, on the decision-making process, we generate different sets of metabolism parameters that produce responses that vary in between the ones for the fasted and fed conditions in Fig. 3. Let β_0, ζ_0, and η_0 the parameters associated with a metabolic process in a fasted condition, and β_1, ζ_1, and η_1 the parameters associated with a metabolic process in a fed condition. A set of parameters associated with a metabolic process in between these two conditions is defined as $\beta_\alpha = (1-\alpha)\beta_0 + \alpha\beta_1$, $\zeta_\alpha = (1-\alpha)\zeta_0 + \alpha\zeta_1$, and $\eta_\alpha = (1-\alpha)\eta_0 + \alpha\eta_1$, where $\alpha \in [0, 1]$. When $\alpha = 0$, we obtain the set of parameters associated with the fasted condition, and when $\alpha = 1$, we obtain the set of parameters associated with fed condition. For $\alpha \in (0, 1)$ we get cases in between these two conditions. Fig. 9(a) shows the impulse responses of the metabolic process for different values of α. Let $x^\alpha(t)$ the trajectory generated for the feedback system in (7) using the set of metabolism parameters β_α, ζ_α, and η_α. Let the accumulated error be defined as

$$e_\alpha = \int_0^T \left[x^\alpha(t) - x^0(t)\right]^2 dt \tag{8}$$

where $T > 0$ is the duration time of the drinking event. This corresponds to the error between the trajectory generated by the system with parameters β_α, ζ_α, and η_α, and the baseline trajectory x^0 (the one generated by the system with parameters β_0, ζ_0, and η_0). Fig. 9(b) shows the result of Monte Carlo simulations for different values of α with 1000 runs each, where k_p, k_d, and x^* (desired intoxication level) have fixed values. The initial conditions are randomly chosen following a uniform distribution on the interval $[0, 2x^*]$. Note that the difference between the trajectories increases as α increases. Even though the median is very small, the range where the error lies seems to increase exponentially with a linear increment of α. This indicates that, according to the proposed model, the decision-making process can lead to very different BAC trajectories when the metabolic process changes.

The next theorem illustrates the impact of the model parameters on the resulting BAC trajectories.

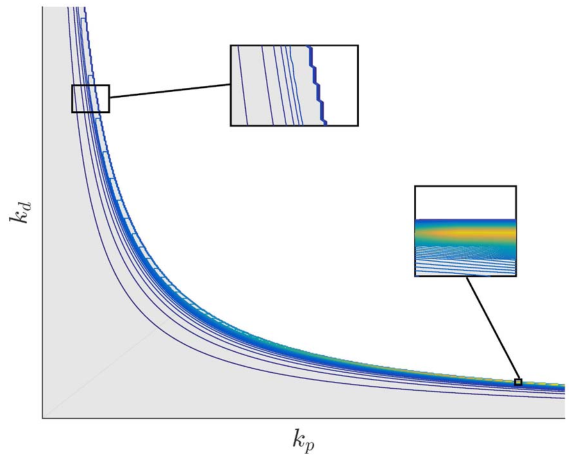


Fig. 10. Contour plot of the ultimate bound γ in (10) for different values of k_p and k_d. The shaded region shows the values of k_p and k_d that satisfy the condition in (9). Lighter colors in the contour lines indicate larger values of γ.

Theorem 2: Let the parameters of the model in (7) be such that $\eta, \beta, k_p, k_d > 0$, and $\zeta > 1$. Assume that the gradient of the preference function f is Lipschitz continuous with Lipschitz constant $L > 0$. Also, assume that

$$\frac{Lk_p k_d}{2} < \frac{\zeta}{\beta\eta}. \tag{9}$$

Then, the trajectories of the system in (7) are uniformly ultimately bounded around the point $(x = x^*, v = 0)$, with ultimate bound γ given by

$$\gamma = \max\{\gamma_1, \gamma_2\} \tag{10}$$

where

$$\gamma_1 = \frac{Mx^*}{\sqrt{2}\max\left\{\left(\frac{\zeta}{\beta} - \frac{\eta}{2}k_p k_d L\right), \beta\eta k_p L\right\}}$$
$$\gamma_2 = \frac{Mx^*}{\zeta/\beta}$$

and $M = \max\{\beta, 1/\beta\}$.

The proof of this theorem is based on Lyapunov stability analysis of nonsmooth systems [29], [32], and it is presented in Appendix B in the supplementary file. This theorem states that, under the given assumptions and for any given initial condition, it is guaranteed that the trajectories of the BAC will eventually reach values that are inside a ball centered at $(x = x^*, v = 0)$, and that has a radius (ultimate bound $\gamma \geq 0$) that depends on the parameters of the model. Saying that the BAC trajectories are "ultimately bounded" means that the individual's goals with respect to his/her BAC are the "approximately met," that is, BAC will remain closer to x^* and the BAC rate of change will remain closer to zero. A small radius implies that the trajectories of the BAC are guaranteed to reach a more localized region.

Fig. 10 shows the behavior of the ultimate bound γ in (10) for different values of k_p and k_d, where $\zeta^* = 1.4$, $\beta^* = 0.0107$, and $\eta^* = 39.23$, and $L = 0.08$. The shaded area in the (k_p, k_d) plane shows the region where k_p and k_d satisfy the condition in (9). Observe that γ_2 dominates the max operation in γ for those values of k_p and k_d that are far from the boundary. This means that the achieved precision of the individual in reaching the desired level of intoxication will depend mostly on his/her metabolism. Also, note that γ can increase for larger values of k_p. This is the situation in which the individual has aggressive alcohol consumption patterns and does not take into account the delayed response of his/her metabolism to process alcohol. Larger values of k_d allow the individual to reach values of the BAC that are close to the desired ones (i.e., small values of γ). For those values of k_p and k_d that do not satisfy the condition in (9) we are not able to guarantee, using our mathematical analysis, that the BAC trajectories will eventually reach a region close to the desired level of intoxication. An interpretation of this condition can be that if the subjective intoxication level is too sensitive to changes in the BAC rate of change and there is an aggressive commitment to reach the desired level of intoxication (large values of k_d and k_p, respectively), then we cannot have guarantees that the individual will take into account the delayed response of the metabolic process and have BAC trajectories that will tend to reach the desired BAC.

III. Estimation From Field Data

We inform the model in (7) using current available data that was collected *in situ*, and then use it to generate possible trajectories of the BAC during the drinking event under some assumptions on the parameters. We show how the model parameters k_p and k_v allow us to estimate information from the available data on BAC dynamics during drinking events. The details of the implementation are given in Appendix D in the supplementary file.

We use the field data in [8] and [47], which contains information from 1040 participants attending 30 bars. Patrons were randomly selected and interviewed, and breath alcohol samples were taken upon entering and exiting the bar. Participants were asked about their drinking plans and motivations for the night. Before entering the bar, they reported the intended level of intoxication that they wanted to reach during the drinking event. They chose one of the following categories: "not drinking," "not enough to get buzzed," "slight buzz," and "very drunk." Also, the time that each participant spent at the bar was reported. There is no information about the social interactions among participants. Using these data we construct our model to generate the BAC trajectories that start at the BAC levels when entering the bar and go through the BAC levels when exiting, having the reported duration of the drinking event. Since we have limited information to estimate all the model parameters, we make assumptions on those related to the metabolic process (parameters β, η, and ζ) to estimate those related to the decision-making process (parameters k_p and k_d).

First, we construct the profile of the preference functions based on the desired level of intoxication that the participants reported before entering the bar. Fig. 11 shows the histogram of the BAC level measured from the participants when exiting the bar given the intended level of intoxication. Note that the distribution of the BAC samples changes depending on the

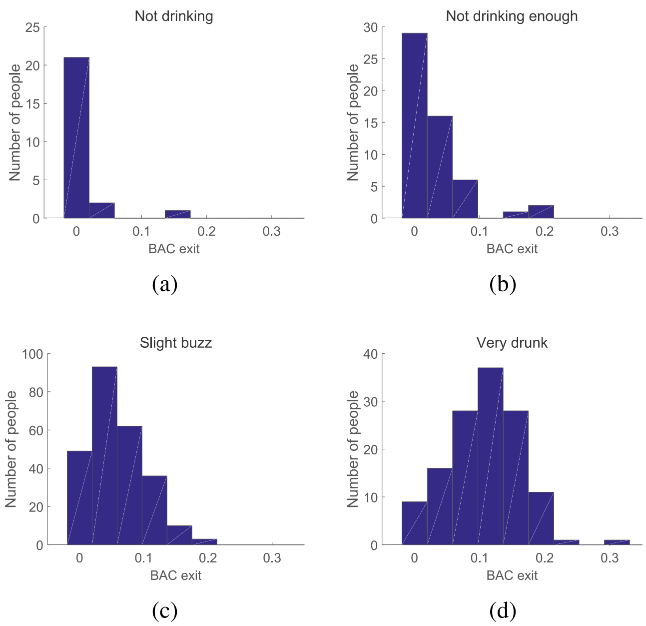


Fig. 11. Histogram of the BAC measured after exiting the bar. Each plot is associated with the preferred level of alcohol intoxication (a) not drinking, (b) not drinking enough to get buzzed, (c) slight buzz, and (d) very drunk.

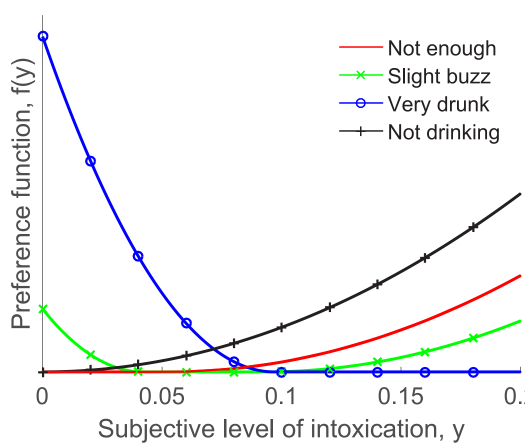


Fig. 12. Profile of the preference function for the four categories of the intended level of intoxication reported by the participants: not drinking, not drinking enough to get buzzed, slight buzz, and very drunk.

stated intentions before entering the bar. For each category of the intended level of intoxication we select the BAC interval that contains approximately 60% of the distribution. Category not enough to get buzzed is then associated with the interval $[0, 0.053]$, slight buzz with $[0.042, 0.093]$, very drunk with $[0.097, 0.3]$, and not drinking with 0. The preference function per category is constructed such that it is zero when the BAC is within the corresponding interval, and it is steeper for BAC levels that are less than this interval than for BAC levels that are greater than it. Fig. 12 shows the profile functions for each category of the intended level of intoxication.

Second, we define the strategy to find the parameters k_p and k_b such that the generated trajectory reaches the measured final BAC at the specified time (duration of the drinking event), given the initial BAC, the parameters associated with the metabolic process, and the intended level of intoxication.

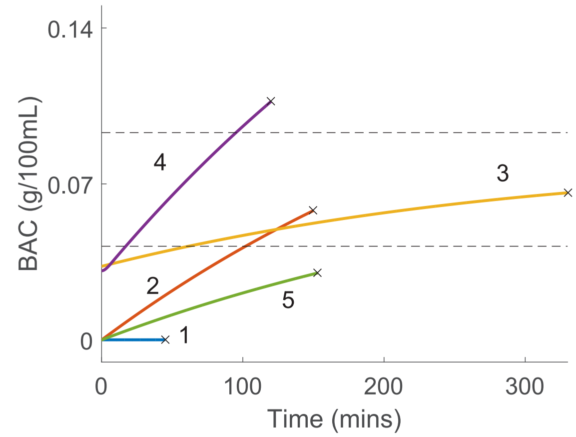


Fig. 13. Trajectories (solid lines) estimated using the proposed model for six participants that reported slight buzz as the intended level of intoxication to reach (dashed lines). Symbol x marks the end of the trajectory.

The parameters associated with the metabolism process are assumed to be the same for all the participants and have the same values obtained in Fig. 3 for the curve approximation in the fasted condition case. Let x_0 and x_f be the initial and final BAC levels measured from a participant, and let $T > 0$ be the reported duration time of the drinking event. Let $x(t; x(0), k_p, k_d)$ denote the solution trajectory of the BAC generated by the model in (7) evaluated at time $t \geq 0$ with initial condition $x(0)$ and parameters k_p and k_d. The problem of finding k_p and k_d is formulated as the optimization problem

$$\left(k_p^*, k_d^*\right) = \underset{k_p, k_d > 0}{\arg\min} \left(x_f - x(T; x_0, k_p, k_d)\right)^2. \tag{11}$$

This is a search problem that tries to find those parameters $k_p > 0$ and $k_d > 0$ such that the BAC level of the trajectory generated by the model at time T is as close as possible to the measured BAC at the exit of the bar.

Fig. 13 shows the estimated trajectories for five samples taken from participants that reported slight buzz as their desired level of intoxication to reach during the drinking event. Trajectory 1 in the figure was generated for a value of k_d^* that is significantly larger than that of k_p^*. Since the BAC and the initial and final BAC levels are almost zero, and the duration of the drinking event is short, large values of k_d are required to keep the trajectories from increasing to BAC levels that take longer to metabolize. Recall that k_d is the strength of the opposition to change in the BAC. For trajectories 3 and 5, parameters k_p^* and k_d^* have similar values. The final BAC is below or within the desired interval, and the duration of the drinking event is not as short as the other cases. On the other hand, trajectories 2 and 4 were generated for values of k_p^* significantly larger than that of k_d^*. The initial BAC level is below the preferred interval of intoxication and the final BAC is within or above it. In this case, to reach the final BAC in the specified period of time, a small awareness of how quickly the BAC is changing (i.e., lower values of k_d) is required to generate a quick response.

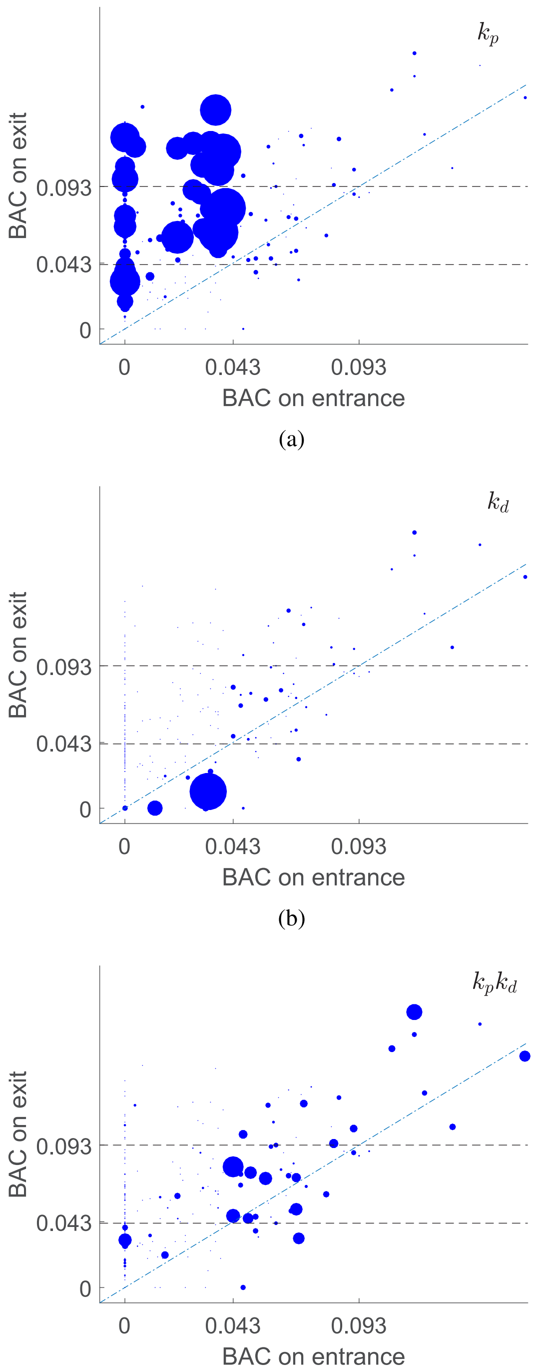


Fig. 14. (a) BAC measured on subjects before entering the bar (horizontal axis) and after leaving the bar (vertical axis). Each dot corresponds to the samples collected from an individual. The size of the marker is proportional to the value of k_p computed using the parameter estimation method in (11). Each individual reported slight buzz as the intended level of intoxication to achieve (dashed lines). The dotted line is a 45° line for reference. (b) Similar description as in (a), where the size of the marker is proportional to k_d. (c) Similar description as in (a), where the size of the marker is proportional to k_pk_d.

Fig. 14 shows the relationship between the parameters k_p and k_d estimated using (11) and the initial and final BAC measured during the drinking event for individuals that reported slight buzz as the desired intoxication level. The horizontal dashed lines indicate the individual's desired BAC interval, and the size of the markers is proportional to the parameter k_p for Fig. 14(a) and to the parameter k_d for Fig. 14(b). From Fig. 14(a), it is observe that the model predicted that those individuals that had a low BAC before entering the bar and a relatively high BAC when they left it, required higher values of k_p. This indicates that those individuals had a stronger commitment to reach the desired intoxication level, which is the region between the dashed lines. On the other hand, those individuals whose BAC before entering the bar was within, or higher than the desired BAC interval did not have to have high values of k_p to remain at that state. Also, from Fig. 14(b) it is observe that the largest values of k_d are associated with low values of the BAC at the entrance of the bar. This indicates that those individuals who want to remain outside the desired interval of intoxication must have a high sensitivity to the BAC rate of change, that is, their subjective level of intoxication $y(t) = x(t) + k_d v(t)$ easily reaches the desired level. We show in Fig. 14(c) the relationship between the product of k_p and k_d and the initial and final BAC levels. Observe that k_p and k_d tend to have relatively large values when the individual has initial and final BAC levels closer to the desired interval of intoxication. In this case, individuals have to be sensitive to decreasing variations of the BAC and have a response such that it keeps their BAC level within the desired interval. A similar analysis can be done for the other intervals of intoxication.

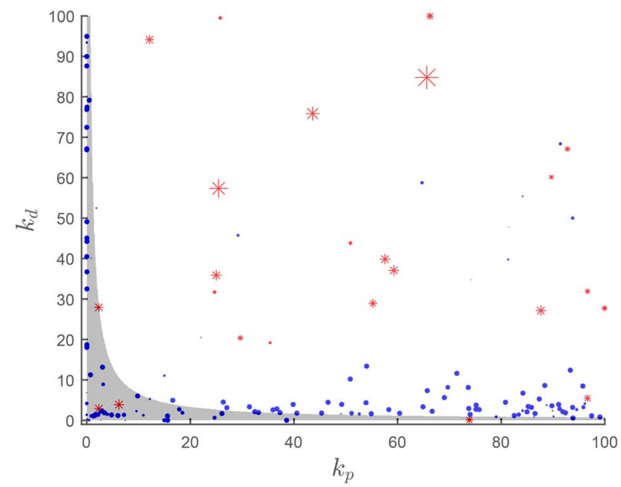


Fig. 15. Resulting values of k_p versus k_d. Each point is associated with an individual. The size of the marker is proportional to the difference between the initial BAC and the closest point of the reported desired interval. The x (red) marker indicates that the initial BAC is above the target BAC, and the "." (blue) marker indicates that the initial BAC is below the target BAC. The shaded region corresponds to the values of k_p and k_d that satisfy the condition in (9).

Additionally, we show in Fig. 15 the values of k_d versus k_p. Each point is associated with an individual, and the size of the markers is proportional to the difference between the initial BAC and the closest point of the desired interval. Samples with the x (red) marker indicate that the initial BAC is above the target BAC, and samples with the "." (blue) marker indicate that the initial BAC is below the target BAC. The shaded region corresponds to the values of k_p and k_d that satisfy the condition in (9). Observe that there is a tendency of the values of k_p and k_d to follow the shaded region. There is not an exact match since the condition in (9) is derived from a qualitative analysis of the long-term behaviors of the modeled trajectories. The results shown in Fig. 15 are computed for limited durations of the drinking event. Note that the points that do not follow this tendency are those that are associated with initial BAC values that are above the desired interval of intoxication. These values are also associated with values of the final BAC that are outside this interval. It seems that the reported desired level of intoxication changed during the drinking event. In these situations the proposed algorithm gives results that do not follow a consistent pattern.

IV. Conclusion

We developed a model that characterizes the BAC dynamics of an individual that is part of a drinking event. This model is composed of two main components: 1) a component that explains the metabolic process of alcohol in the individual for arbitrary alcohol intake patterns and 2) a decision-making component that manipulates the alcohol intake rate based on the desired level of intoxication and the current state of the BAC and BAC rate of change. We present computer simulations and a mathematical analysis of the impact of the parameters on the modeled dynamics, and propose a methodology to find the parameters of the decision-making process that allows the model to generate BAC trajectories that fit the available real data.

We showed how the model of the drinker's decision-making process, which was formulated following the observations from field studies, resembled the proportional-derivative controller from feedback control system engineering. This result provides useful insights into the mechanisms that drive the decision-making process of an individual in a drinking event. It might be the case that there are additional mechanisms that have not been uncovered yet, but are already known in the theory of feedback systems. An example is the proportional-integral-derivative controller, which in addition to the components explained in the previous sections, has a component (the integrator) that allows the system to accurately reach the desired state. This can be explored in our future work, along with personal drinking advisor development.

Currently, we are in the process of collecting real-time drinking event data that includes individual, group, and environmental level variables [20], [22]. Our objective is to study these data using tools in statistics and dynamical systems theory, and conduct a model identification process from the measured data [48]. These results will help us to validate and update our hypotheses on the modeled drinking event dynamics. Our aspiration is to gain enough understanding to start interventions at both the individual and group event levels.

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