Distributed Optimization and Control

Using Only a Germ of Intelligence

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Outline

- Foraging theory
- Chemotactic behavior (foraging strategy) of *E. coli*
- Bacterial foraging for distributed optimization
- Bacterial foraging for distributed control
- Biomimicry of intelligent foraging
- Stability analysis of foraging swarms
- Concluding remarks

Foraging Theory

- Animals search for and obtain nutrients to maximize

$$\frac{E}{T}$$

where E is energy obtained per time T

- Foraging constraints: Physiology, predators/prey, environment

→ Evolution optimizes foraging

- Foraging strategy: Find patch, decide whether to enter it and search for food, when to leave patch?

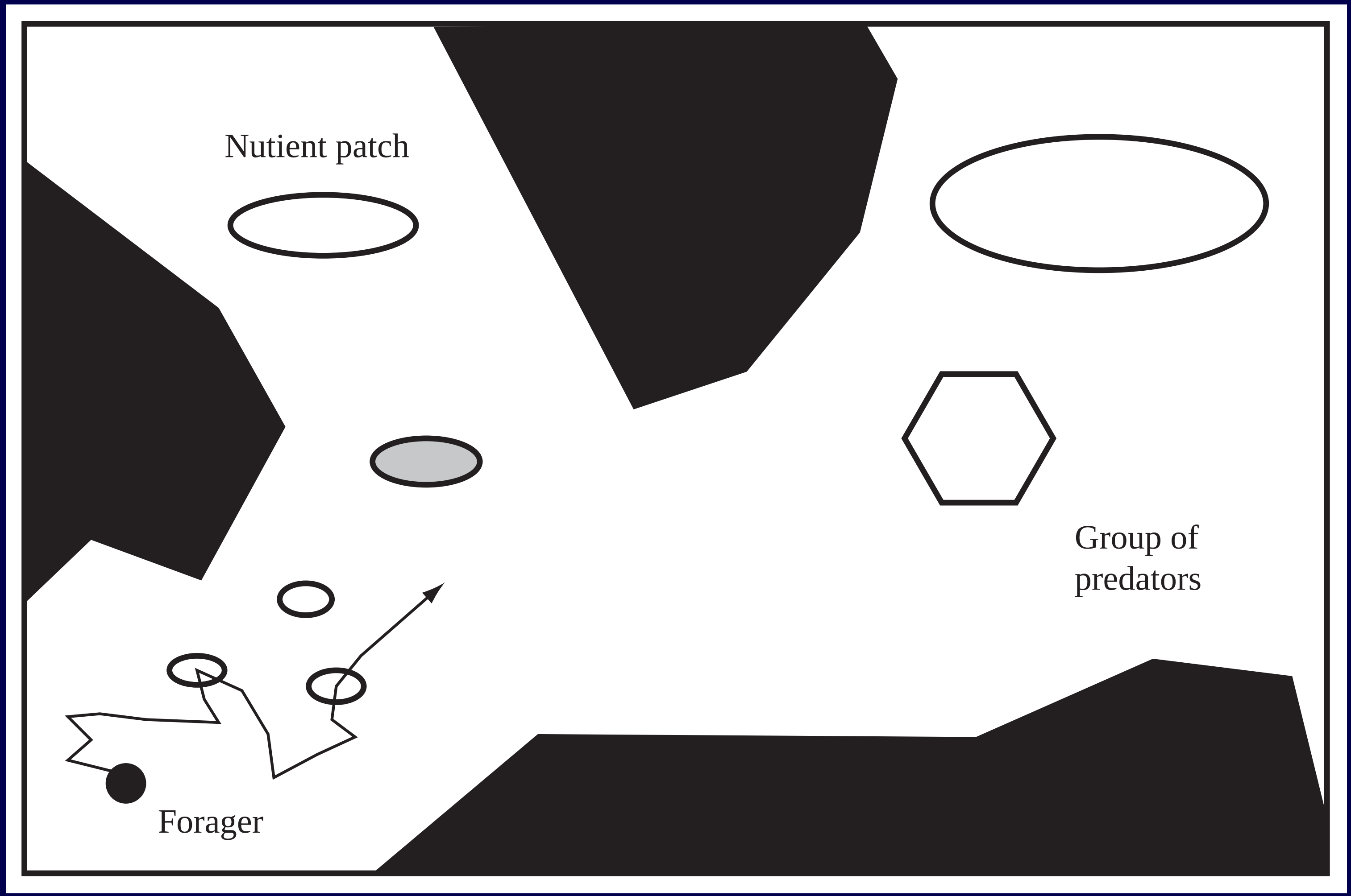

Figure 1: Foraging landscape and scenario.

- Use dynamic programming to find “optimal policies.”
- Search strategies for foraging: cruise (tuna fish), saltatory (birds, fish, insects), and ambush (snakes)

→ Social foraging: Need communications but individuals can gain advantages (more sensors, “gang-up” on large prey, protection, collective intelligence).

- Examples: Bees, ants, fish, birds, wolves, humans

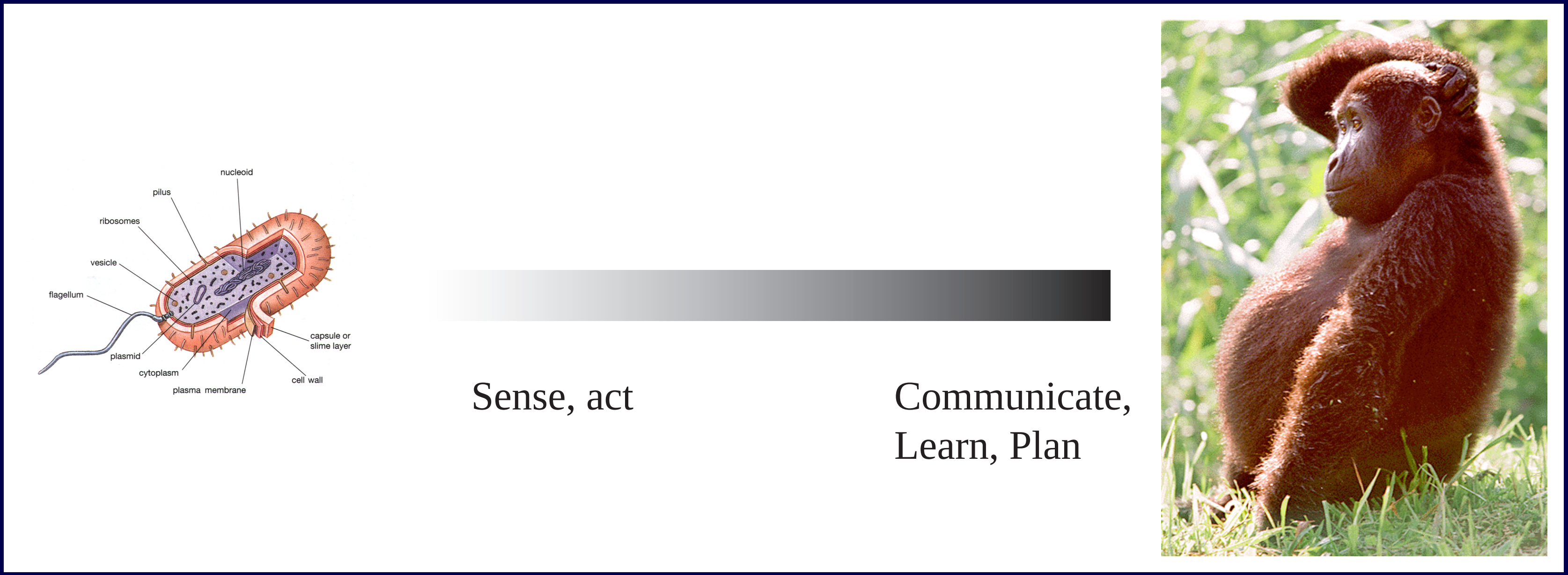

Figure 2: Cognitive spectrum for foraging.

- Entire spectrum interesting from an engineering perspective.
- Let’s start at the bottom...

Chemotactic (Foraging) Behavior of *E. coli*

- *E. coli*: Diameter: $1\mu m$, Length: $2\mu m$

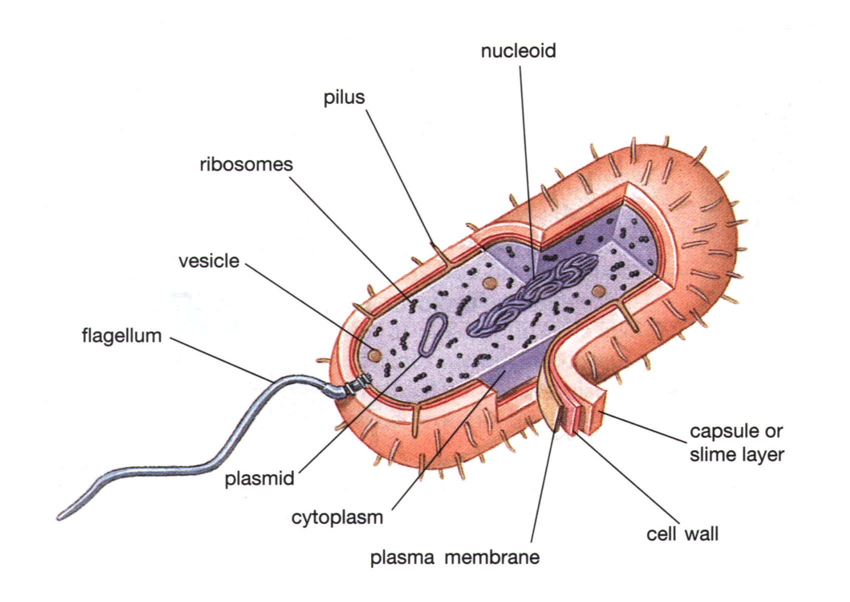

Figure 3: *E. coli* bacterium (from [2]).

- Can reproduce (split) in 20 min.

- *E. coli* in action... (from C. Morton-Firth, Cambridge Univ.)

Motility and Chemotaxis

- Motility via reversible rigid $100 - 200$ rps spinning flagella each driven by a biological “motor”

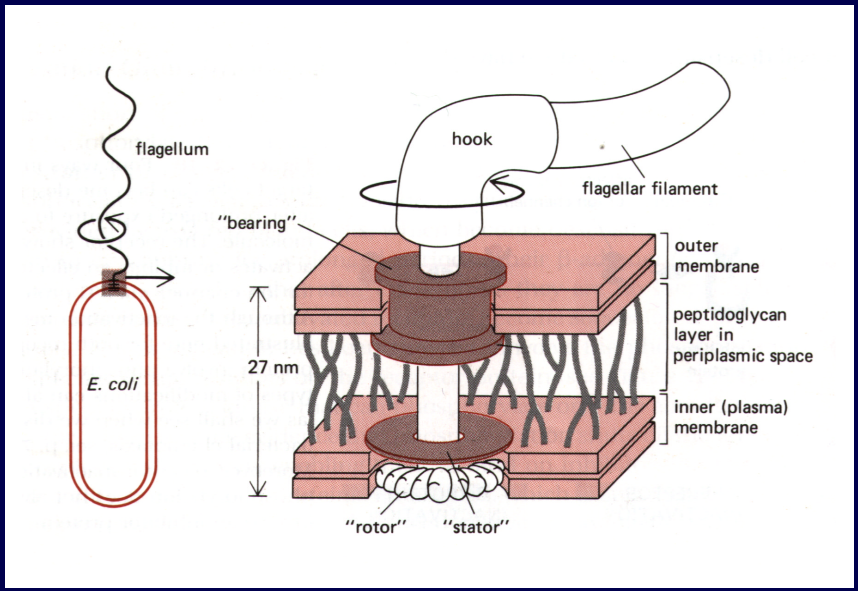

Figure 4: *E. coli* biological “motor” (from [1]).

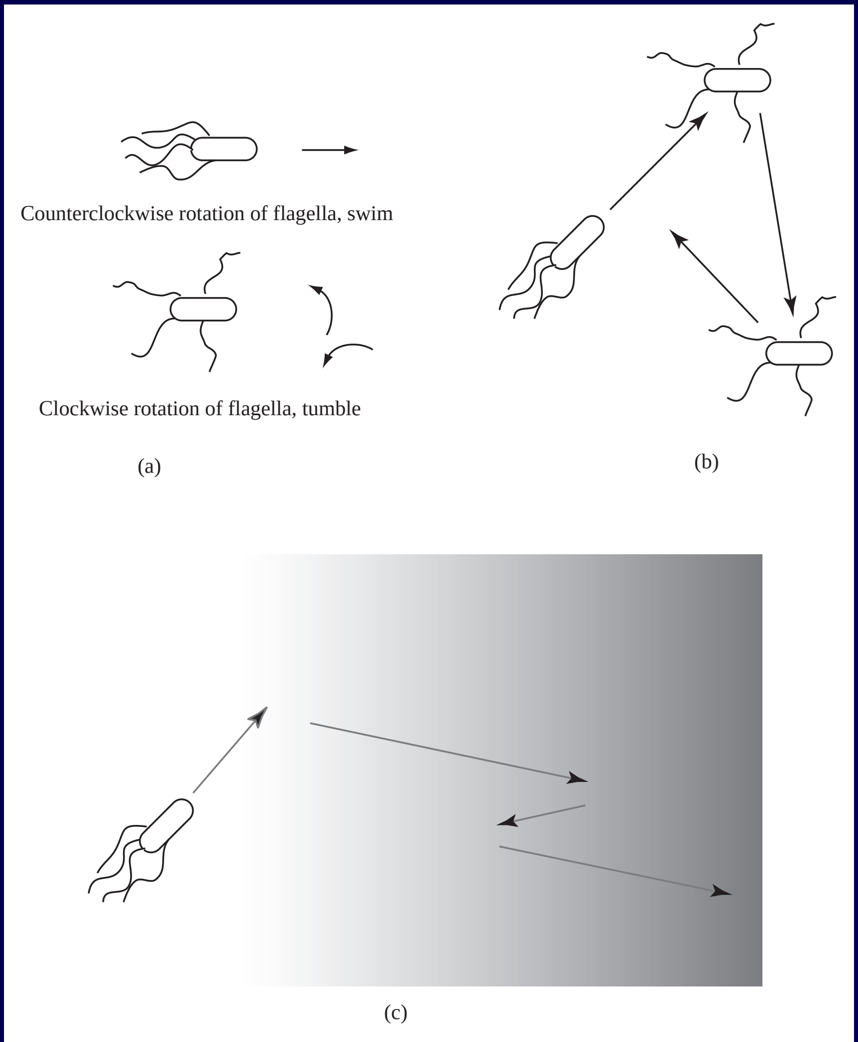

Figure 5: Chemotactic behavior.

Decision Making in Foraging

1. If in neutral medium alternate tumbles and runs

   $\Rightarrow$ Search

2. If swimming up nutrient gradient (or out of noxious substances) swim longer (climb up nutrient gradient or down noxious gradient)

   $\Rightarrow$ Seek increasingly favorable environments

3. If swimming down nutrient gradient (or up noxious substance gradient), then search

   $\Rightarrow$ Avoid unfavorable environments

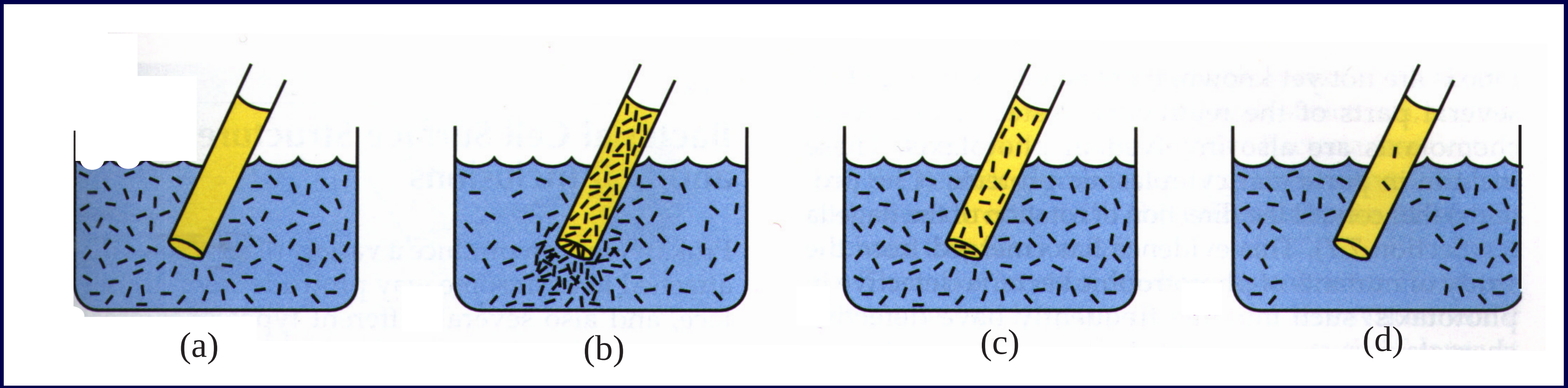

Figure 6: Capillary experiment (from [5]).

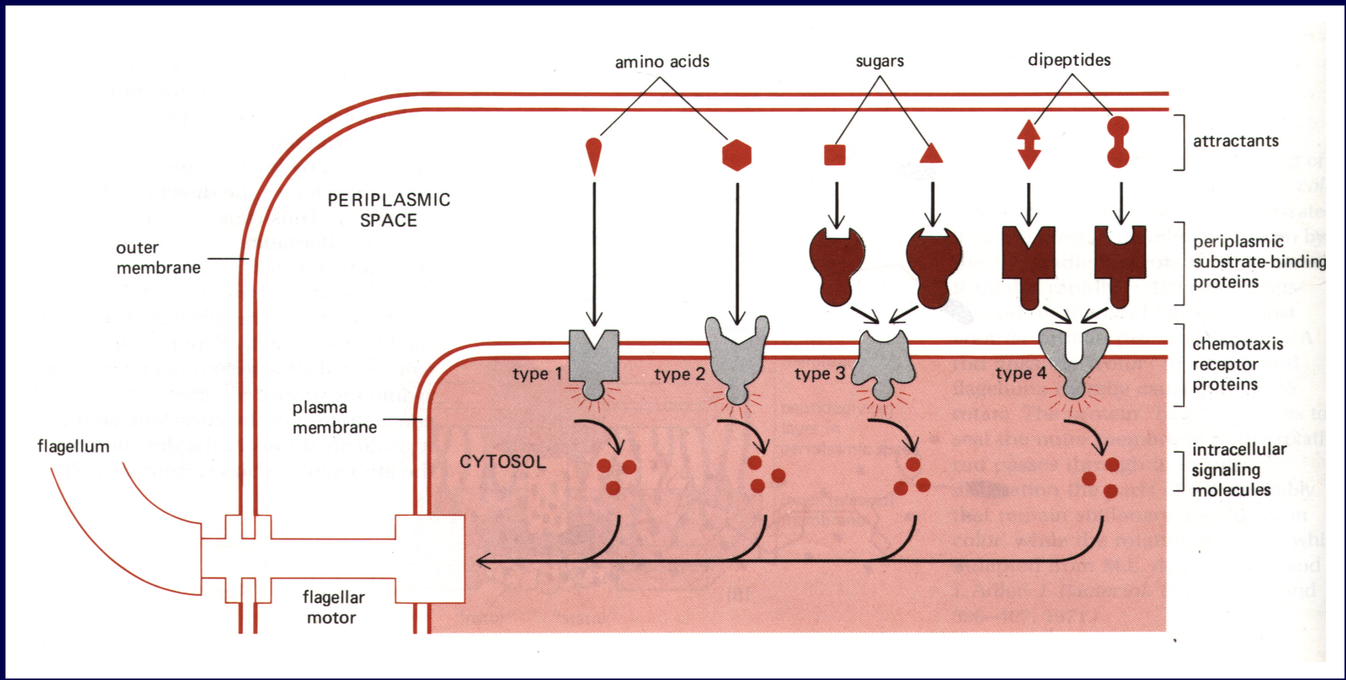

Figure 7: Sensing and control in *E. coli* (from [1]).

- The sensors are very sensitive, and overall there is a “high gain.”
- Averages sensed concentrations and computes an approximation to a *time* derivative.

→ Probably the best understood sensory and decision-making system in biology (understood/simulated at molecular level).

Elimination/Dispersal and Evolution

- Bacteria often killed and dispersed (can be viewed as part of their motility)

- Mutations in *E. coli* affect, e.g., reproductive efficiency at different temperatures, and occur at a rate of about 10^{-7} per gene, per generation.

- *E. coli* occasionally engage in a type of “sex” called “conjugation” (Figure 8)

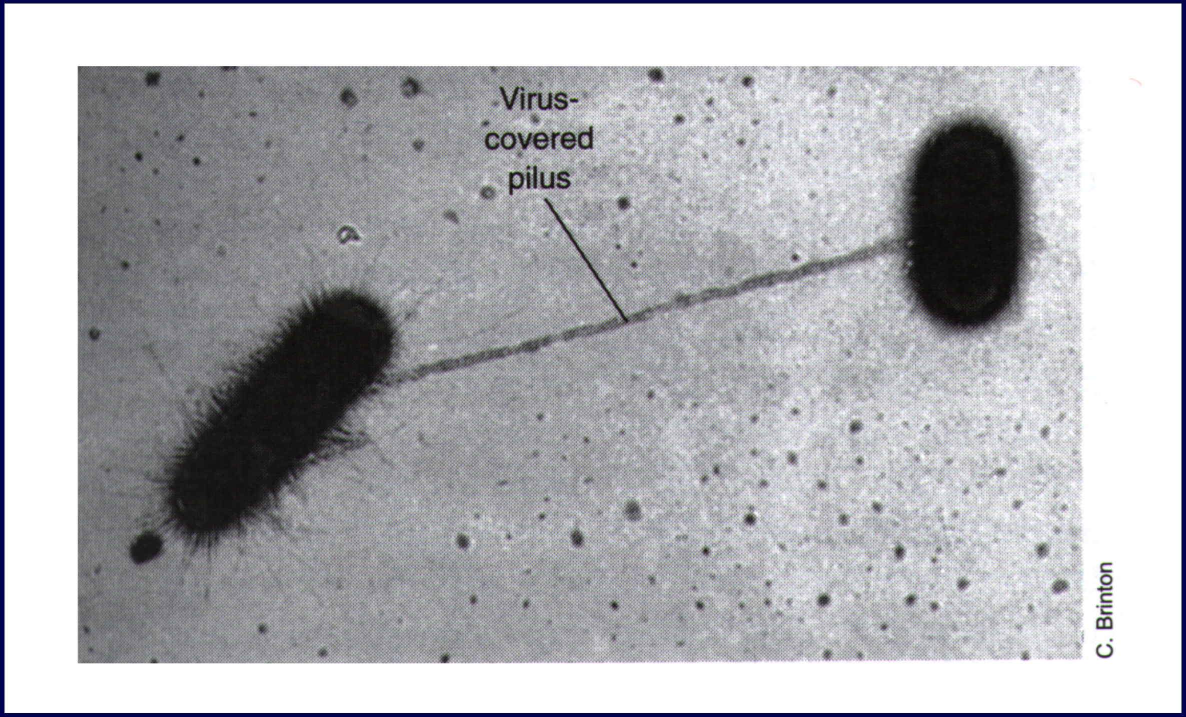

Figure 8: Conjugation in *E. coli* (from [5]).

Other Taxes

1. Change cell shape and number of flagella based on medium!

2. Oxygen (aerotaxis), light (phototaxis), temperature (thermotaxis), magnetic flux lines (magnetotaxis)

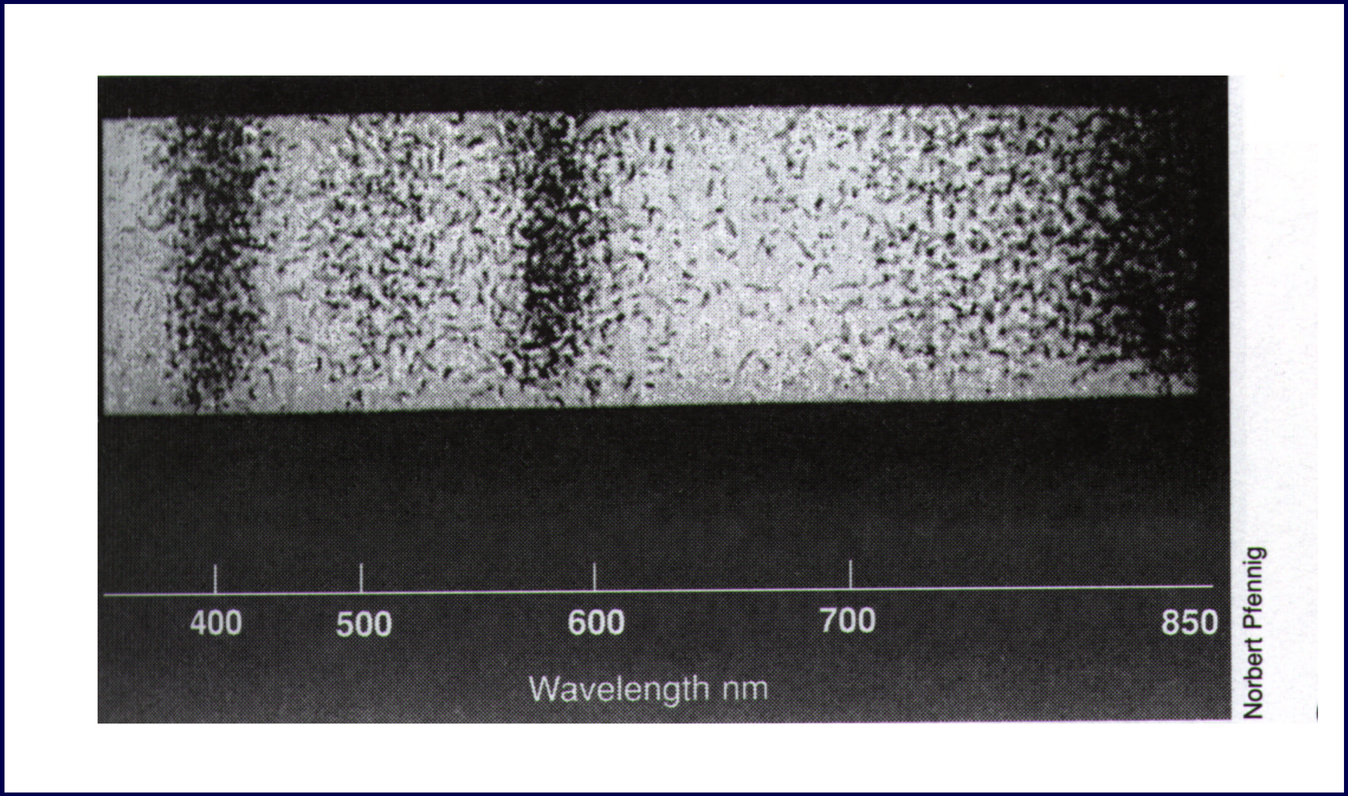

Figure 9: Phototaxis behavior of the phototropic bacterium *Thiospirillum jenense* (from [5]).

Swarms

- *E. coli* and *S. typhimurium* can form intricate stable spatio-temporal patterns in certain semi-solid nutrient media
  - Radially eat their way through the medium.
  - Cell-to-cell attractant signals.
  - The bacteria protect each other.

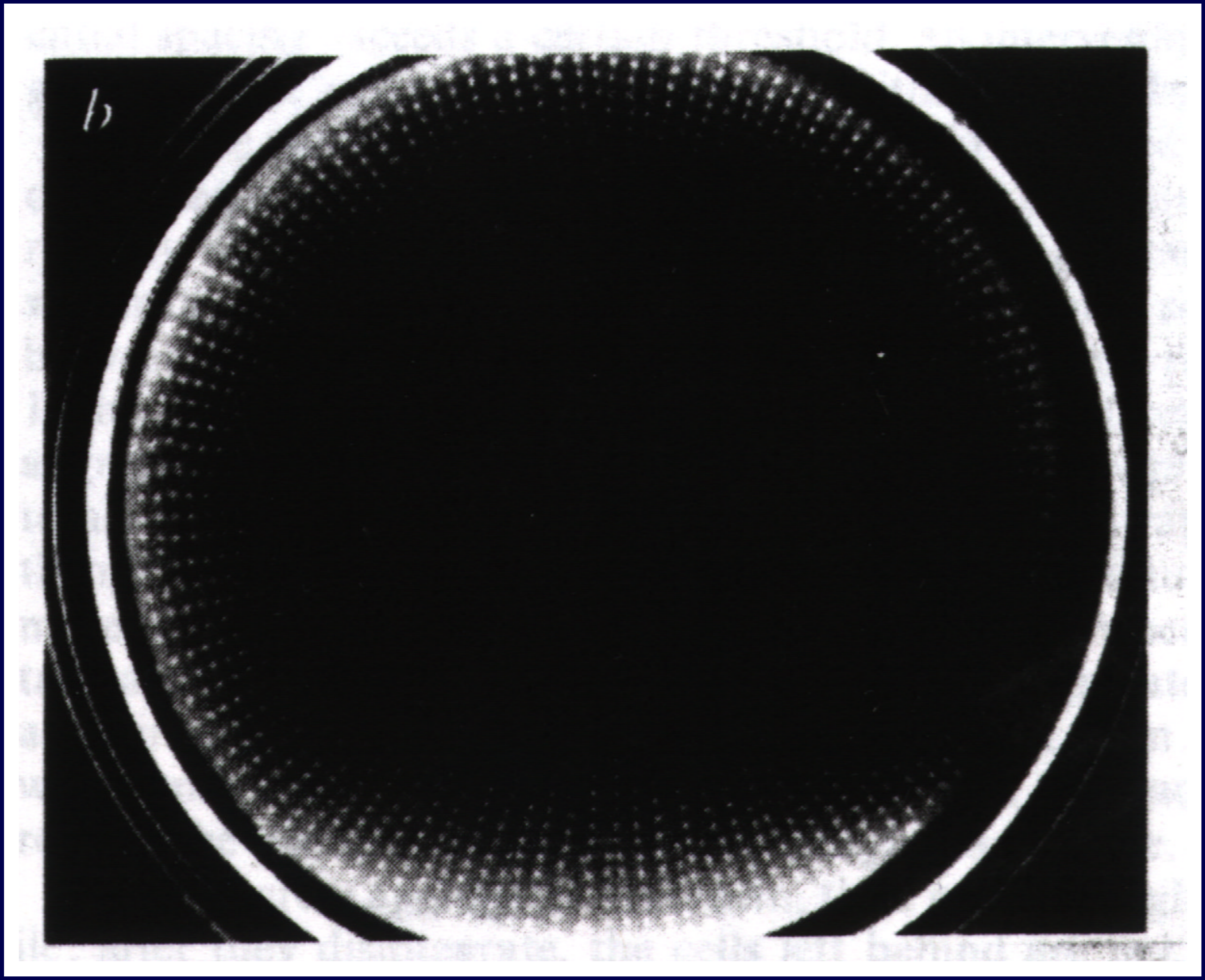

Figure 10: Swarm pattern of *E. coli* (from [3]).

Bacterial Swarm Foraging for Optimization

- Find the minimum of

$$J(\theta), \ \theta \in \Re^p$$

when we do not have $\nabla J(\theta)$.

➔ Suppose θ is the position of a bacterium, and $J(\theta)$ represents an attractant-repellant profile so:

1. $J > 0 \Rightarrow$ noxious
2. $J = 0 \Rightarrow$ neutral
3. $J < 0 \Rightarrow$ food

- Let

$$P(j,k,\ell) = \{\theta^i(j,k,\ell) | i = 1, 2, \ldots, S\}$$

be the set of all S bacterial positions at the j^{th} chemotactic step, k^{th} reproduction step, and ℓ^{th} elimination-dispersal event.

- Let $J(i,j,k,\ell)$ denote the cost at the location of the i^{th} bacterium $\theta^i(j,k,\ell) \in \Re^p$.

- Let N_c be the length of the lifetime of the bacteria as measured by the number of chemotactic steps.

- To represent a tumble, a unit length random direction, say $\phi(j)$, is generated; then we let

$$\theta^i(j+1,k,\ell) = \theta^i(j,k,\ell) + C(i)\phi(j)$$

so $C(i) > 0$ is the size of the step taken in the random direction specified by the tumble.

- If at $\theta^i(j+1,k,\ell)$ the cost $J(i,j+1,k,\ell)$ is better (lower) than at $\theta^i(j,k,\ell)$, then another chemotactic step of size $C(i)$ in this same direction will be taken, and repeat that up to a maximum number of steps, N_s.

→ Cell-to-cell signaling via an attractant:

1. Attractants are essentially “food” for other cells (chemotactically attracted to it)
2. Use $J_{cc}^{i}(\theta)$, $i = 1, 2, \ldots, S$, to represent locally secreted food.

- Repel? Via local consumption, and cells are not food for each other. Again, use $J_{cc}^{i}(\theta)$.
- Example: Consider the $S = 2$ case...

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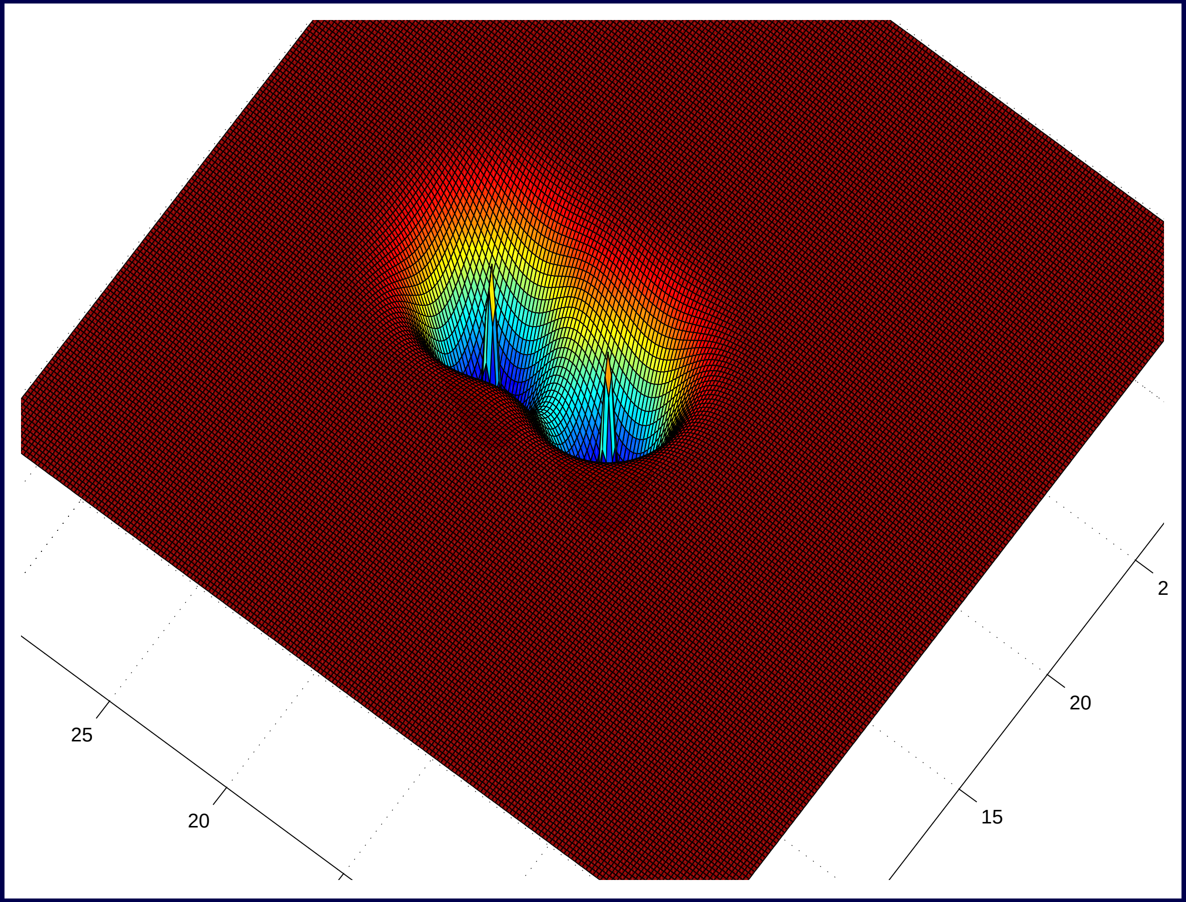

Figure 11: Example cell-to-cell attractant model, $S = 2$.

- For swarming consider minimization of

$$J(i,j,k,\ell) + J_{cc}(\theta)$$

so cells try to find nutrients, avoid noxious substances, and try to move towards other cells, but not too close to them.

- The $J_{cc}(\theta)$ function dynamically deforms the search landscape to represent the desire to swarm.
- Take N_{re} reproduction steps.

- For reproduction, healthiest bacteria (ones that have lowest accumulated cost over their lifetime) split, and then kill other unhealthy half of population.
- Let N_{ed} be the number of elimination-dispersal events and for each elimination-dispersal event each bacterium in the population is subjected to elimination-dispersal with probability p_{ed}.

- Biologically valid model? Capturing gross characteristics of chemotactic hill-climbing and swarming.

Example: Function Optimization

- Find minimum of function in Figure 12 ($[15, 5]^\top$ is the global minimum point, $[20, 15]^\top$ is a local minimum).
- Standard ideas from optimization theory can be used to set the algorithm parameters.

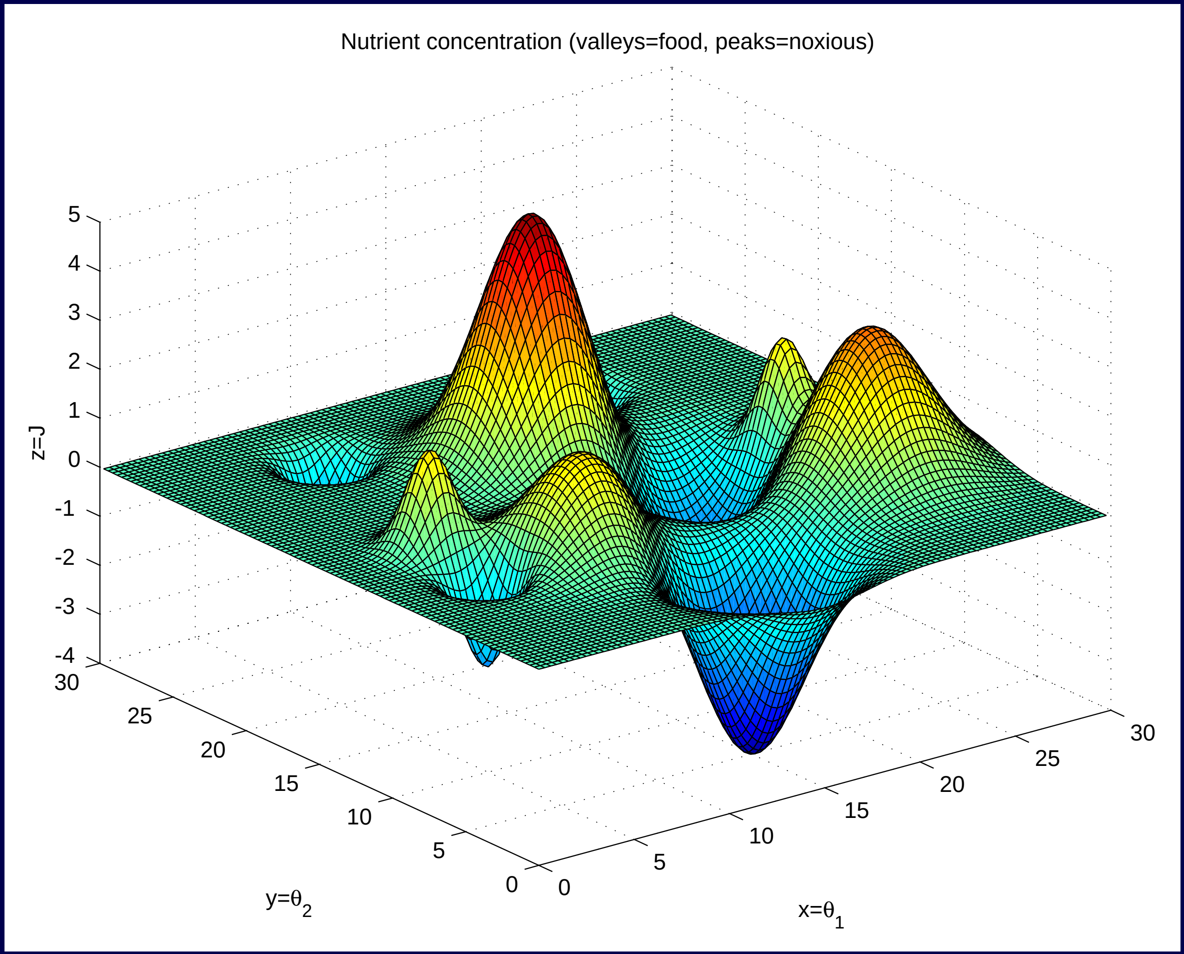

Figure 12: Function with multiple extremum points.

→ No swarming:

- $S = 50$, $N_c = 100$, $C(i) = 0.1, i = 1, 2, \ldots, S$, $N_s = 4$ (a biologically-motivated choice)
- $N_{re} = 4$, $N_{ed} = 2$, $p_{ed} = 0.25$,
- Random initial bacteria distribution.

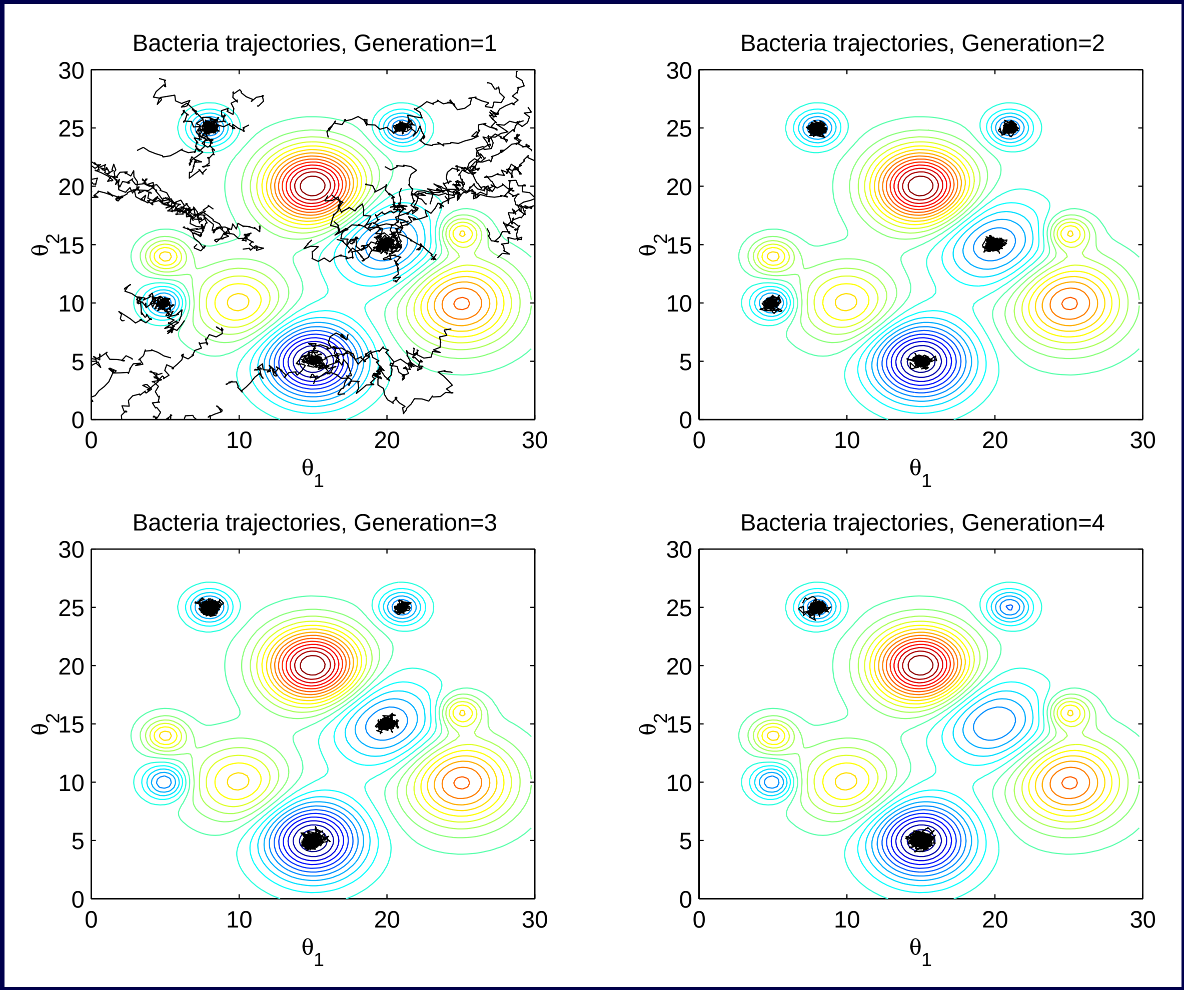

Figure 13: Bacterial motion trajectories, generations 1-4.

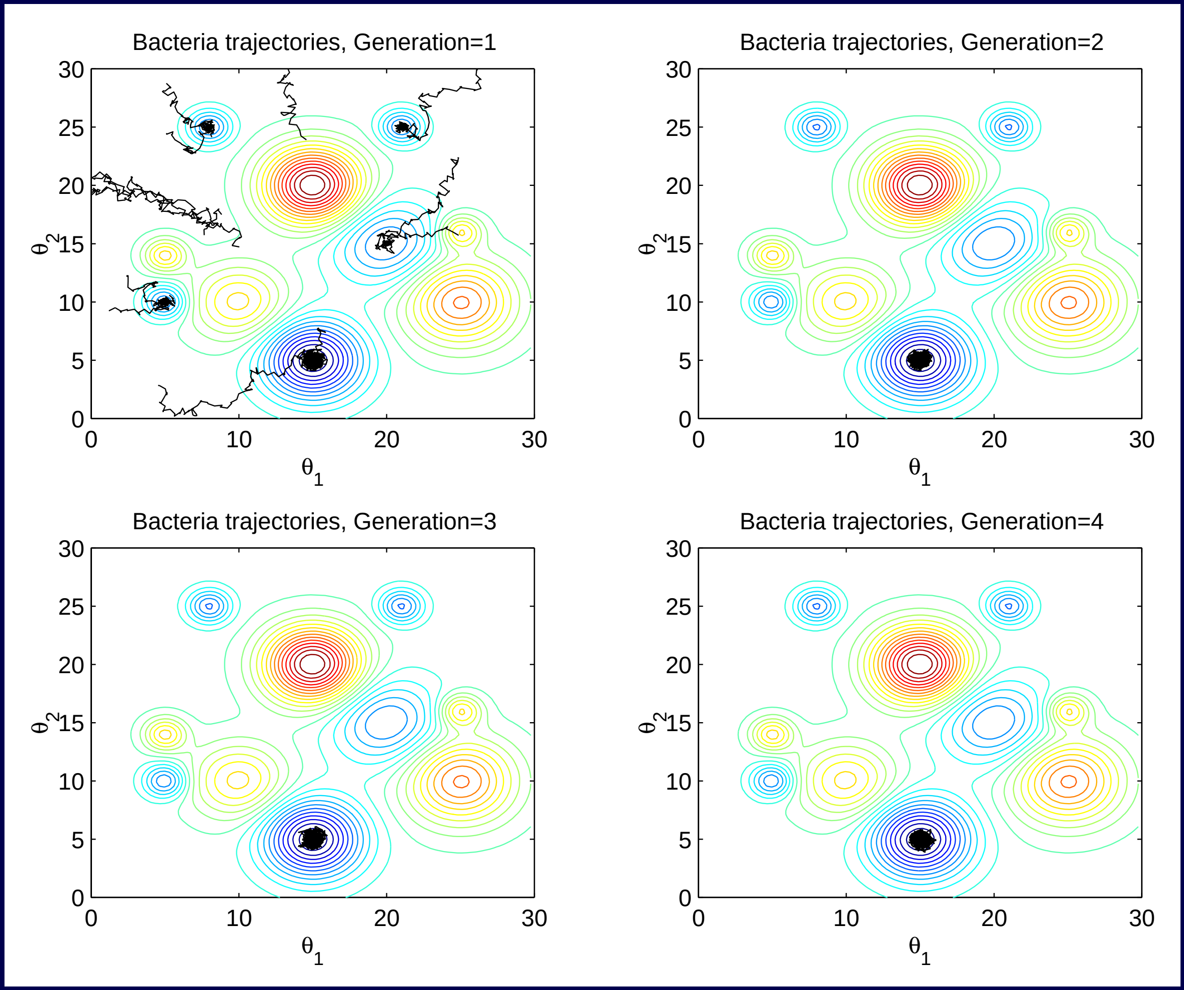

Figure 14: Bacterial motion trajectories, generations 1-4, after an elimination-dispersal event.

→ Swarm effects:

- Emulate Figure 10 by considering optimization over Figure 15.
- Initially, place all cells at the peak $[15, 15]^\top$.

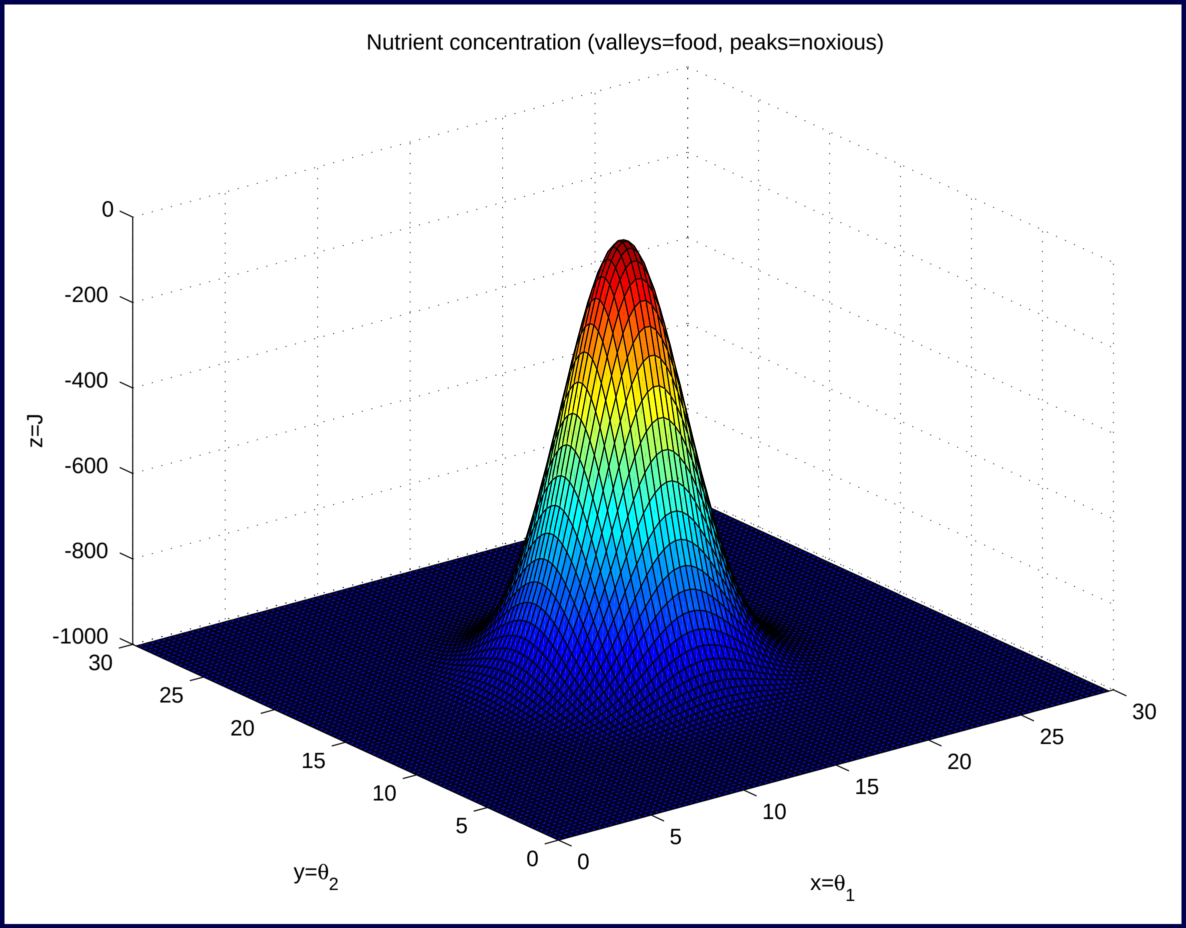

Figure 15: A nutrient surface for testing swarming.

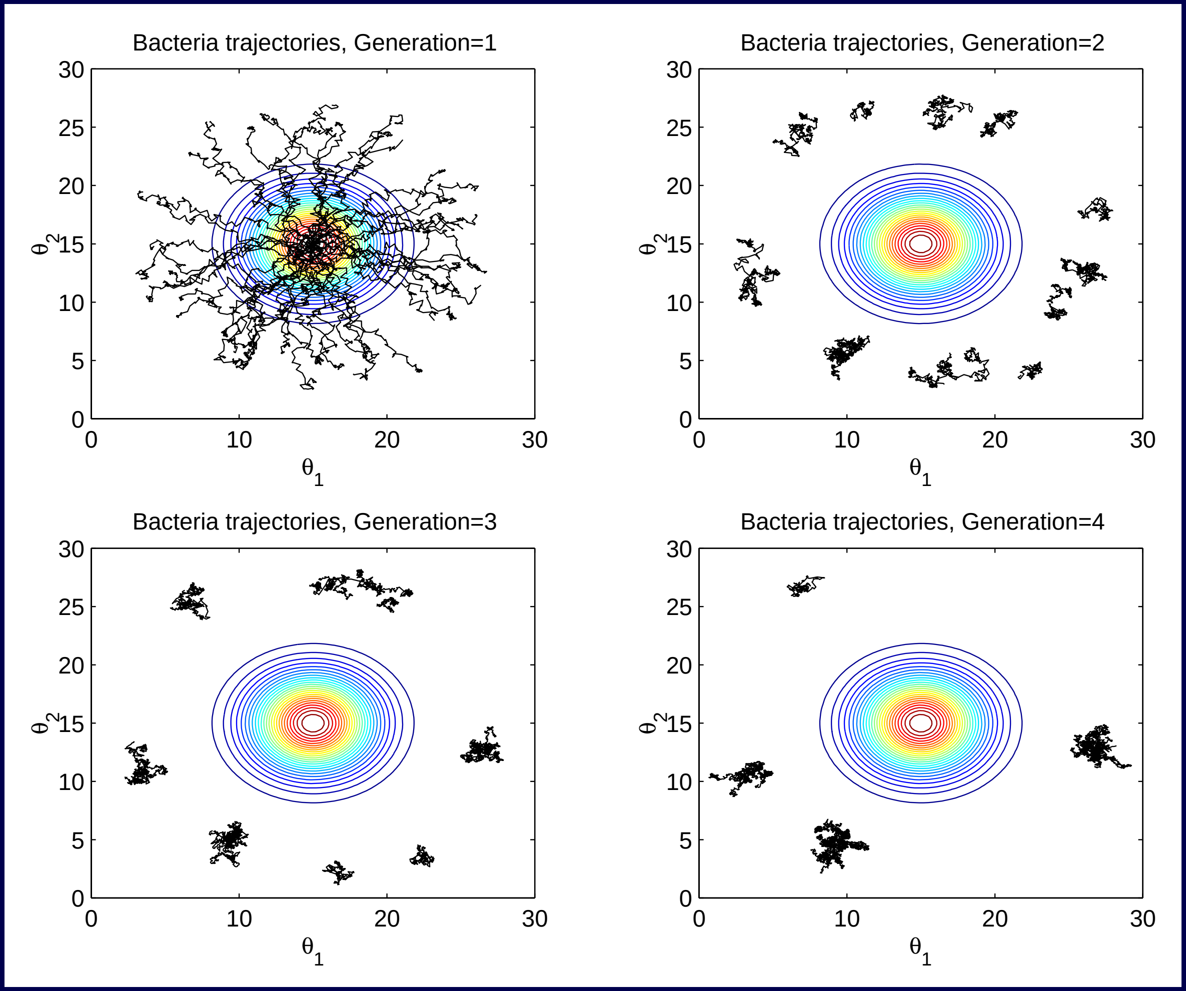

Figure 16: Swarm behavior of *E. coli* on a test function.

Take a Step Up the Cognitive Spectrum for Foraging

- *Archangium violaceum* foraging for *Sarcina* (*Myxobacteria* web page, M. Dworkin, Univ. Minnesota).

- *M. xanthus*: Social and adventurous swarming (web page of Dale Kaiser, Stanford Univ.)

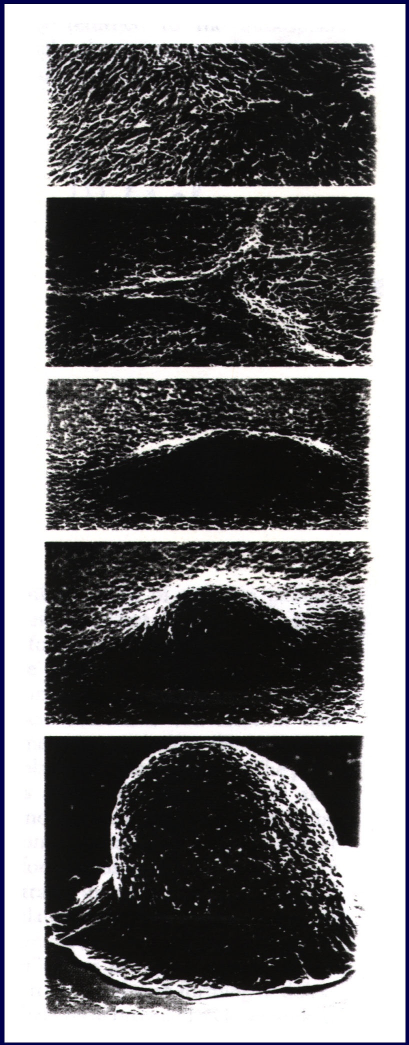

Figure 17: *M. xanthus* mound formation (from [4]).

- Cellular automata-based optimization
- Resulting swarm dynamics "emerge":

1. Formation (aggregation) events
2. Size
3. Location
4. Motility (move faster as individuals than in groups)

→ Balance between desire to individually forage and to form swarm aggregates is delicate.

Discussion

- Optimization methods: Related to stochastic approximation, genetic algorithms. Comparative analysis important! (J. Spall)

→ Evolution made foraging search strategies "optimal" for the environment of the bacteria (class of cost functions)—perhaps not our engineering problems!

- What is the value? To be determined, but for now: Science, metaphor for engineering and control?

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Uninhabited Autonomous Air Vehicles

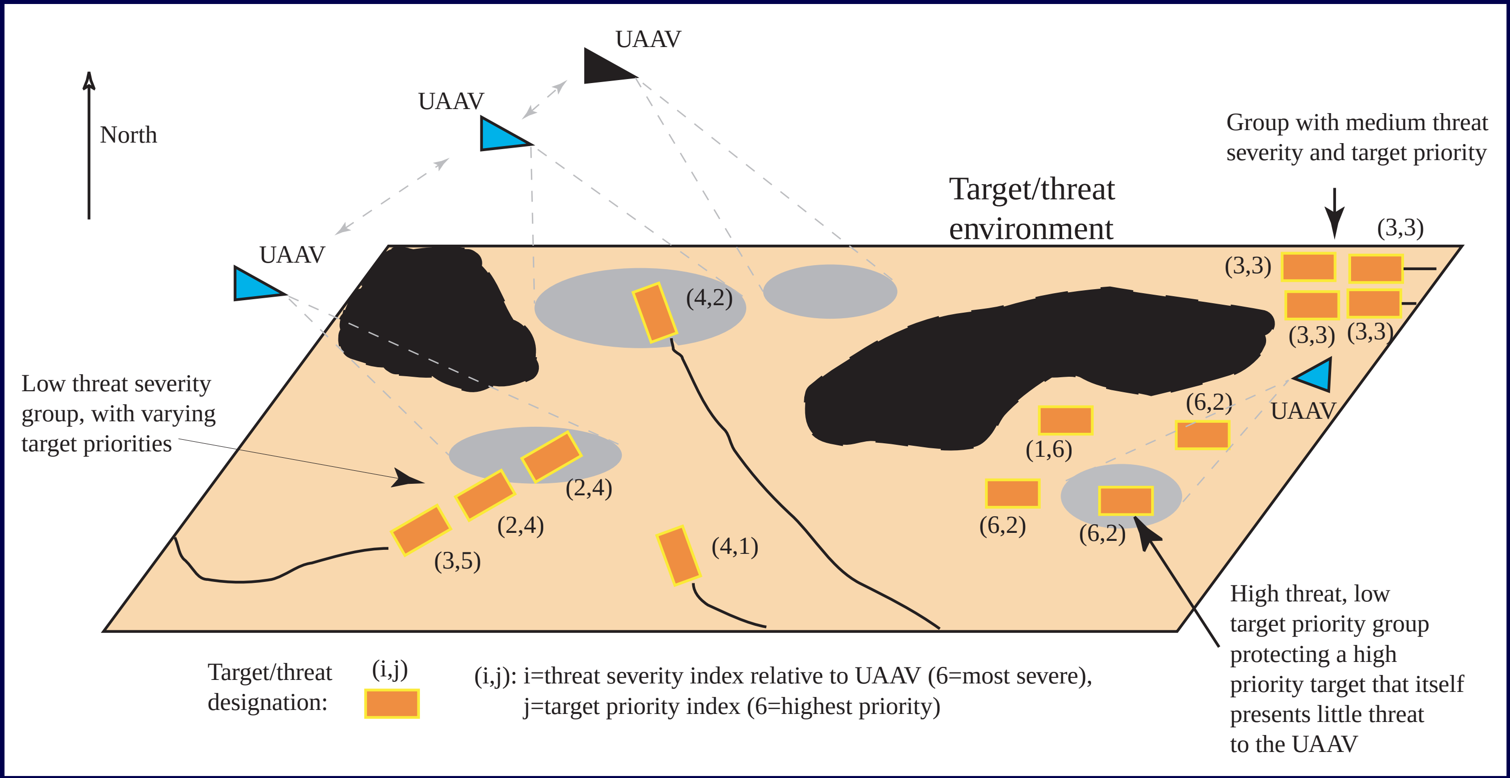

Figure 18: UAAV scenario (with M. Polycarpou).

- Fuel/time constraints
- Sensor range/accuracy may be low
- Communication constraints: Locality, bandwidth, and delays
- On-board functionality: Computer, signal processing, and control. How much?
- Vehicle dynamics constrain movements
- Target/threats may move/evade

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- *E. coli* “vehicles”—a nanotechnologist’s dream!

- Use an *E. coli* (*M. xanthus*) search strategy?
- Bacterial sensing, locomotion, and decision-making strategies are limited.
- Their foraging is optimized for a certain environment, probably not this one!

→ Foraging principle: Optimization/search is a central concept.

→ Evolutionary principle: Vehicle and environment dictate cooperative strategy.

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Intelligent Foraging for Distributed Coordination and Control (M. Baum)

- What if our forager has capabilities for planning, attention, learning, and sophisticated communications?
- Learning/planning approach: construct cognitive maps, predict using these, and share the maps
  - Relevant optimization theory: Pattern search and real-time surrogate model (response surface) methods.

Distributed Learning and Planning for UAVs

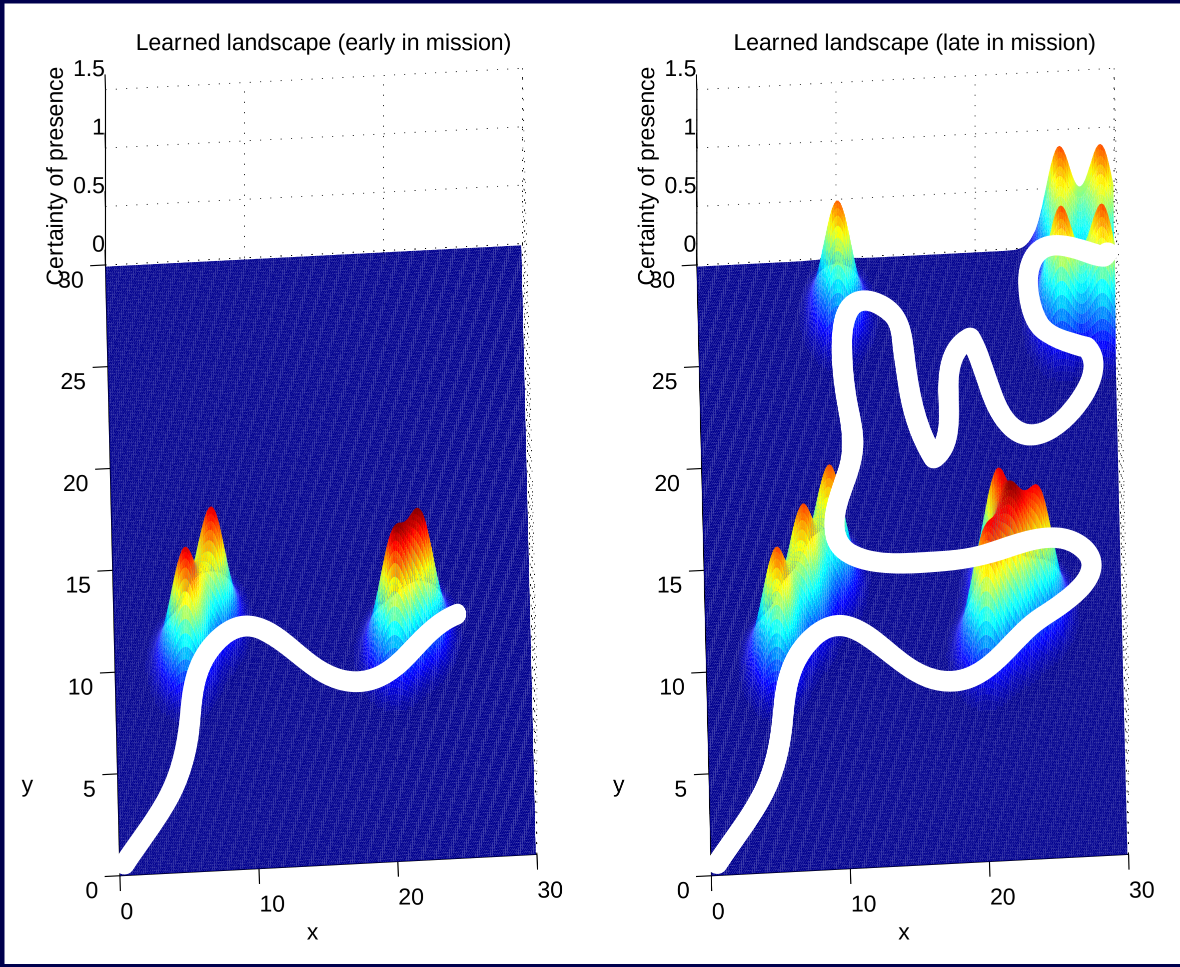

Figure 19: UAAV learning a foraging landscape.

- Other maps: Target priority, threat severity, ...
- Distributed Learning and Coordination: How to coordinate learning via sharing of maps? When to seek more information (risky) vs. when to focus on gathering more information in a previously visited area?
- Distributed Planning: On shared maps.

→ Theoretical Challenges: Stability, convergence, robustness

Stable Foraging Swarms (Y. Liu)

- Swarm aggregation and disintegration: Results from dominance of local attractive and repelling relationships between organisms, environmental effects, and organism characteristics.
- Cohesion: During vegetative swarming (feed better in group), and for protection
- Disintegration: Plentiful small prey, and scatter for survival (safety in a group?)

- One-dimensional vehicular swarm (platoon, formation?) with swarm member:
  1. Position: $x^i(t)$, $i = 1, 2, \ldots, N$ (can add dynamics)
  2. Moves at times: T^i, $i = 1, 2, \ldots, N$ (infinite, can have $T^i \cap T^j \neq \phi$)
  3. Proximity sensor: Immediate measurements within $\epsilon > 0$
  4. Neighbor sensor: Provides i^{th} member $x^{i-1}(\tau^i_{i-1}(t))$, $0 \leq \tau^i_j(t) \leq t$.

5. Goal: Achieve inter-swarm member distance $d > 0$ (or neighborhood)
6. Locomotion: Compute $e^i(t) = x^i(t) - x^{i-1}(t)$ and use $g(e^i(t) - d)$ (sector-bounded) to represent attract/repel features
7. Asychronism: Total or partial (sense and act within B time units)

- N-member swarm model: Swarm member $i = 2$, example

$$
\begin{aligned}
x^2(t+1) \;=\; & max\{x^1(t)+\epsilon, \\
& min\{x^2(t)-g(x^2(t)-x^1(\tau_1^2(t))-d), \\
& x^3(t)-\epsilon\}\}
\end{aligned}
$$

- Suppose $x^i(0) - x^{i-1}(0) > \epsilon$

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- Characterize stability via inter-swarm member distances
- Goal: Convergence to within d (or a neighborhood).
- Stationary edge member: Convergence...
  1. Total asynchronism $\Rightarrow$ asymptotic convergence
  2. Partial asynchronism $\Rightarrow$ finite-time convergence
- Totally mobile swarms: Leader-follower rules in terms of $J_c(x)$. Convergence?

- Swarm behavior: Follow the edge-leader

- Convergence?

  1. Need: Constraints on rates of movement, partial asynchronism, convergence to a neighborhood
  2. For general $J_c(x)$ must allow swarm splits/joins
  3. Key Relationship: Adventurous-cohesion balance (rates of movement related to communication delays, dynamics, and inter-swarm member neighorhood that can get convergence to)
  4. Generalize? 2, 3-dimensional cases, robustness, learning/planning.

Concluding Remarks

- ✔ You can do a lot with a germ of intelligence!
- ✔ Biomimicry of intelligent foraging for distributed optimization and control.
- ✔ Theoretical foundations (stability, optimization) are very important.
- ✔ Relevant engineering applications...

References

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