Foraging Swarms: From Biology to Engineering Applications

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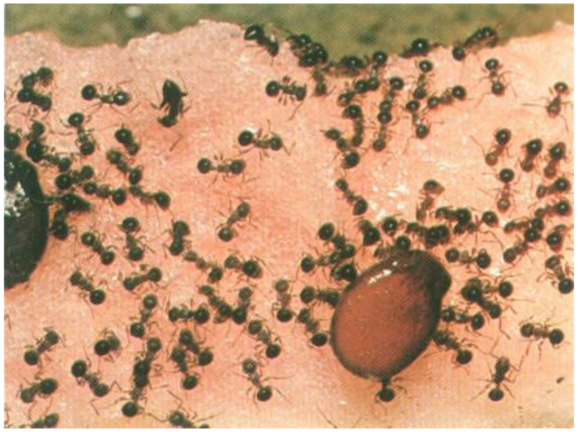
Swarms

- Biological swarms... foraging, seeking protection, etc.




- Science: "Emergent behaviors/intelligence," etc.

- Vehicular swarms... formation/pattern/group (satellites, aircraft, ground/undersea vehicles).

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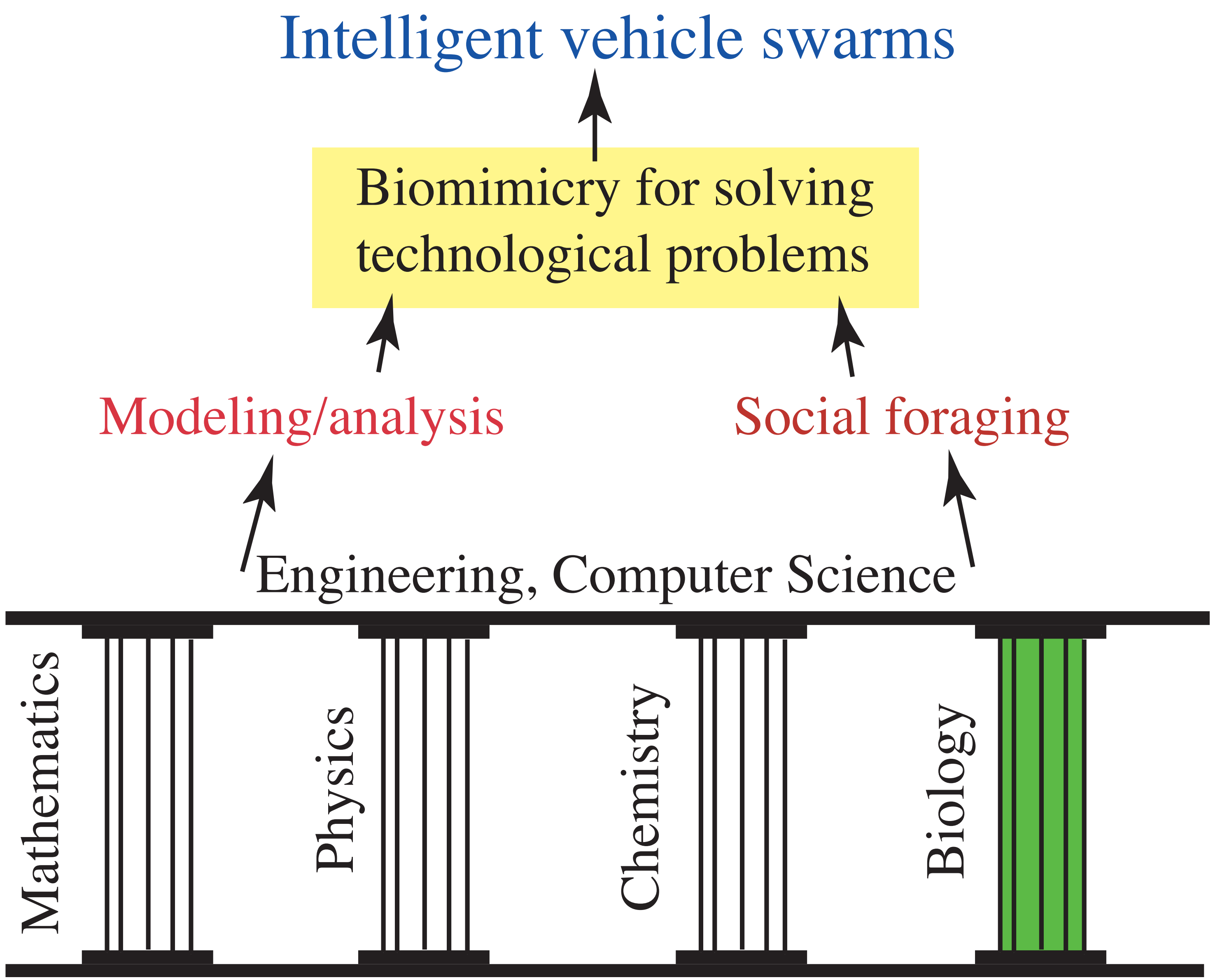

Intelligent vehicle swarms
Biomimicry for solving technological problems
Modeling/analysis
Social foraging
Engineering, Computer Science
Mathematics
Physics
Chemistry
Biology

Philosophy...

- Biomimicry: Organisms designed (evolved) to solve technological problems?
- Mathematics/Physics: Models not perfect, analysis limited, need ideas?
- Exploit best of both!
- ★ Contributions? Technology? Science?

Foraging Theory

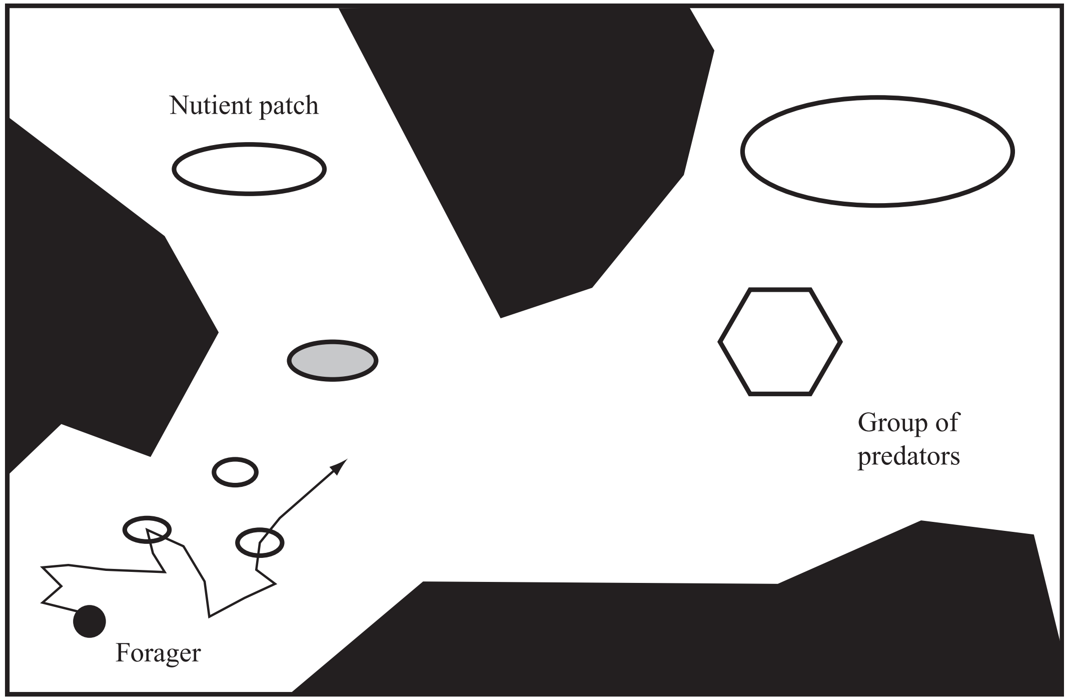
- Animals search for and obtain nutrients to maximize

$$\frac{E}{T}$$

where E is energy obtained per time T

- Foraging constraints: Physiology, predators/prey, environment

➞ Evolution optimizes foraging



- Search/foraging strategies, use dynamic programming to find "optimal policies."
- Social foraging: Costs, but get "collective intelligence"

Chemotactic Behavior of ***E. coli***

- *E. coli*: Diameter: $1\mu m$, Length: $2\mu m$

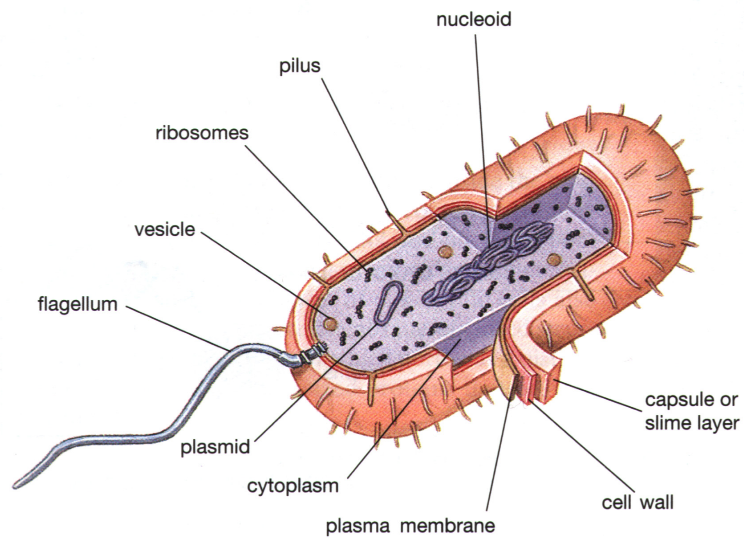
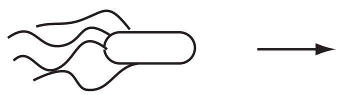
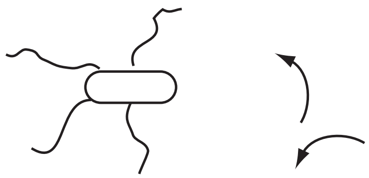


Figure 1: *E. coli* bacterium.

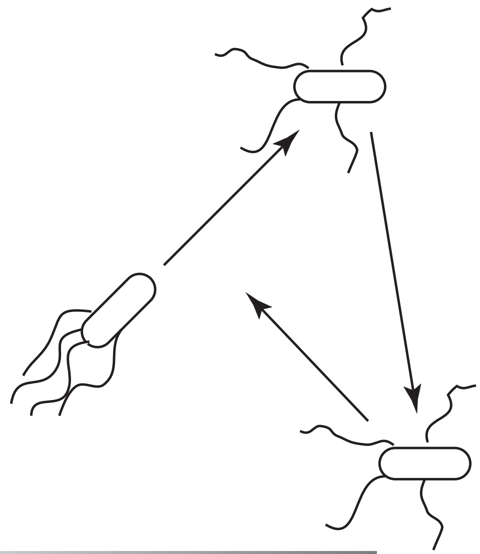
- Sensors/actuators/controller, an autonomous underwater vehicle – "nanotechnologist's dream"!

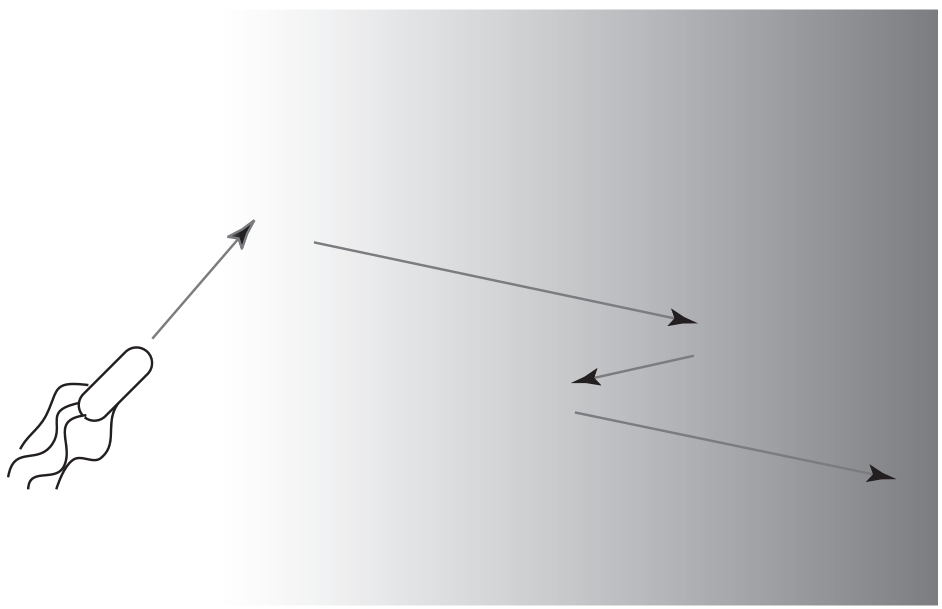


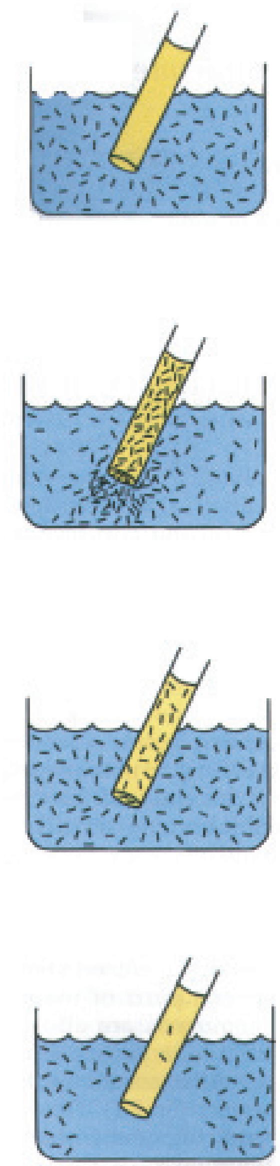
Counterclockwise rotation of flagella, swim



Clockwise rotation of flagella, tumble

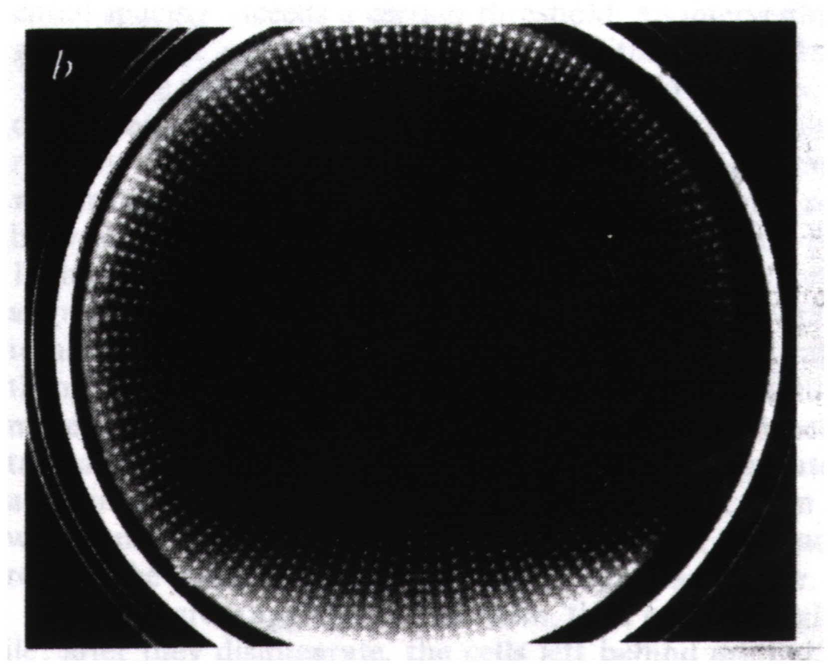






Swarms

➙ *E. coli* and *S. typhimurium* can form intricate stable spatio-temporal patterns in certain semi-solid nutrient media



- Eat radially, cell-to-cell attractant signals.

Bacterial Swarm Foraging as Optimization

- Find the minimum of

$$J(\theta), \ \theta \in \Re^p$$

when we do not have $\nabla J(\theta)$.

➙ Suppose θ is the position of a bacterium, and $J(\theta)$ represents an attractant-repellant profile so:

1. $J > 0 \Rightarrow$ noxious
2. $J = 0 \Rightarrow$ neutral
3. $J < 0 \Rightarrow$ food

➙ Set of bacteria (positions):

$$P(j,k,\ell) = \left\{\theta^i(j,k,\ell) | i = 1,2,\ldots,S\right\}$$

at the j^{th} chemotactic step, k^{th} reproduction step, and ℓ^{th} elimination-dispersal event.

- Let $J(i,j,k,\ell)$ denote the cost at the location of the i^{th} bacterium $\theta^i(j,k,\ell) \in \Re^p$.
- Let $\phi(j)$ be a random vector of unit length and $C(i)$ be a step size, then

$$\theta^i(j+1,k,\ell) = \theta^i(j,k,\ell) + C(i)\phi(j)$$

➙ If go down then continue for a few steps, if not then generate random vector

- ➙ Swarming: Add on inter-bacterial nutrient profiles for each bacterium
- ➙ Optimization model:
  - Chemotaxis for stochastic gradient climbing
  - Attraction/repulsion for social aspect, inter-agent effects → parallel optimization characteristics
  - Elimination/dispersion, evolution
- ➙ Biologically valid model?
- ➙ A good engineering optimization method?

- See: "Biomimicry of Bacterial Foraging for Distributed Optimization and Control" [5]

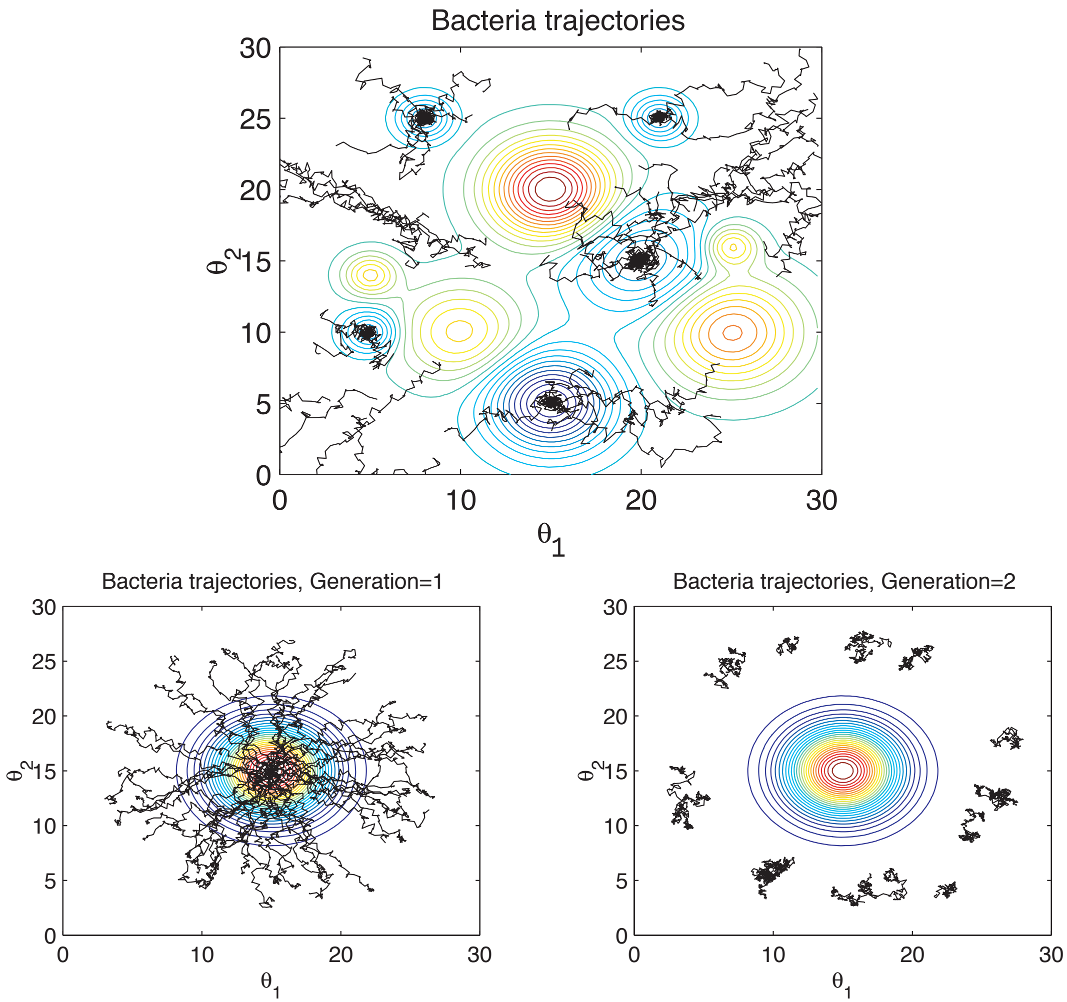
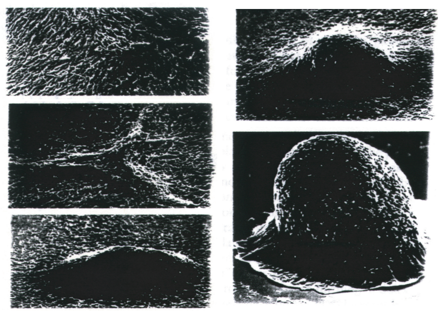
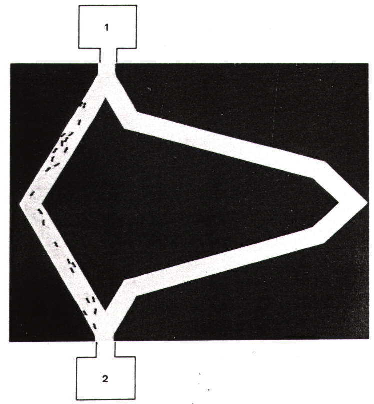
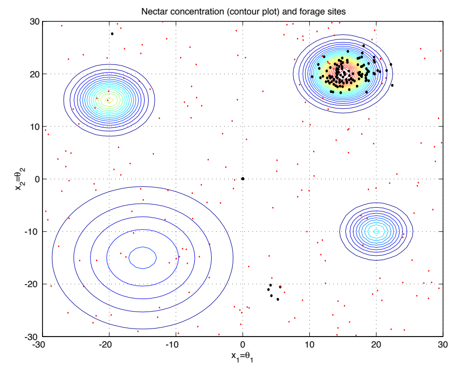


Figure 2: Function optimization, swarm behavior.

Other Social Foraging Models...







- *M. xanthus:* Optimization on noisy surfaces, cellular automaton approach [3]
- Ant colony optimization methods (e.g. shortest path)
- Social foraging of honey bees: Optimal resource allocation model 

Intelligent Social Foraging

- Learning/attentional/planning/"social" approach:
  - Construct representation as "cognitive maps" (learn)
  - Focus on parts of the map (attention)
  - Predict using these (plan)
  - Share the maps (communications → "social")

Stable "Dumb" Foraging Swarms: Concepts & Challenges

- Literature: Biology, physics, autonomous vehicles (Beni, Leonard, Murray, Morse, ...),
- Here: Lyapunov stability anlaysis of cohesion

- N "agents:"

$$\begin{aligned} \dot{x}^i &= v^i \\ \dot{v}^i &= \frac{1}{M_i} u^i \end{aligned}$$

- Agent to agent interactions – "attract-repel" to seek "comfortable" inter-agent distances.

- Attract: Term in u^i like $-k_a\,(x^i - x^j)$, $k_a > 0$
- Repel: Term in u^i like

$$k_r \exp\left(\frac{-\frac{1}{2}\|x^i - x^j\|^2}{r_s^2}\right)\left(x^i - x^j\right)$$

$k_r > 0$, $r_s > 0$

- An "equilibrium" inter-agent distance?

Environment Model

- ➙ Move along negative gradient of a "resource profile" (e.g., nutrient profile) $J(x)$, $x \in \Re^n$.
  - Plane: $J(x) = J_p(x) = R^\top x + r_o$
  - Quadratic: $J(x) = J_q(x) = \frac{r_m}{2}\|x - R_c\|^2 + r_o$
- ➙ Sensor noise $\leftrightarrow$ noise on profile

Stability Analysis of Swarm Cohesion Properties

- Swarm center, velocity:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x^i \qquad \bar{v} = \frac{1}{N}\sum_{i=1}^{N} v^i$$

- Agent objective: Move to $\bar{x}$ and $\bar{v}$ (time-varying)
- Error system: $e_p^i = x^i - \bar{x}$, $e_v^i = v^i - \bar{v}$

$$\begin{aligned} \dot{e}_p^i &= e_v^i \\ \dot{e}_v^i &= \frac{1}{M_i}u^i - \frac{1}{N}\sum_{j=1}^{N}\frac{1}{M_j}u^j \end{aligned}$$

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Cohesive Social Foraging in Noise: Constant Noise Bounds

- Noise: $\|d_p^i\| \leq D_p$, $\|d_v^i\| \leq D_v$, $\|d_f^i\| \leq D_f$
- Agents can sense: v^i and...

$$\hat{e}_p^i = e_p^i - d_p^i$$
$$\hat{e}_v^i = e_v^i - d_v^i$$
$$\nabla J_p\left(x^i\right) - d_f^i$$

- Agents steer themselves (use J_p):

$$
\begin{aligned}
u^i &= -M_i k_a \hat{e}_p^i - M_i k_a \hat{e}_v^i - M_i k_v v^i \\
&+ M_i k_r \sum_{j=1, j\neq i}^{N} \exp\left(\frac{-\frac{1}{2}\|\hat{e}_p^i - \hat{e}_p^j\|^2}{r_s^2}\right)\left(\hat{e}_p^i - \hat{e}_p^j\right) \\
&- M_i k_f \left(\nabla J_p\left(x^i\right) - d_f^i\right)
\end{aligned}
$$

- Effects on error: $\hat{e}_p^i - \hat{e}_p^j = (x^i - x^j) - \left(d_p^i - d_p^j\right)$
- What are the effects of noise?
- Stability/cohesion possible?

➞ Consider terms of: $\dot{e}_v^i = \dot{v}^i - \dot{\bar{v}}$

- Symmetry gives repel term in $\dot{\bar{v}}$ as zero, and:

$$\dot{\bar{v}} = -k_v \bar{v} + \underbrace{k_a \bar{d}_p + k_a \bar{d}_v + k_f \bar{d}_f - k_f R}_{z(t)}$$

$$\|z(t)\| \leq \left\|k_a \bar{d}_p\right\| + \left\|k_a \bar{d}_v\right\| + \left\|k_f \bar{d}_f\right\| + \|k_f R\| \leq \delta$$

$$\delta = k_a D_p + k_a D_v + k_f D_f + k_f \|R\|$$

- Exponentially stable system with a time-varying but bounded input $z(t) \rightarrow \bar{v}(t)$ is bounded:

  1. For some positive constant $0 < \theta < 1$ and some finite T we have

  $$\|\bar{v}(t)\| \leq \exp\left[-(1-\theta)k_v t\right]\|\bar{v}(0)\|, \ \forall\ 0 \leq t < T$$

  2. Also, we have the bound

  $$\|\bar{v}(t)\| \leq \frac{\delta}{k_v \theta}, \ \forall\ t \geq T$$

Remarks:

- Fix δ, θ: $k_v \uparrow \Rightarrow$ (faster, smaller bound)
- $D_p + D_v + D_f \uparrow \Rightarrow \delta \uparrow \Rightarrow$ bound $\uparrow$ (e.g., the average velocity could oscillate).
- Average sensing errors change direction of the group's movement relative to nutrients (can get lost).

➙ $N \to \infty \Rightarrow$ could have $\bar{d}_p \approx \bar{d}_v \approx \bar{d}_f \approx 0 \to$ "Grunbaum's principle" of social foraging (compare to $N = 1$ case). Groups can climb noisy gradients better.

➙ Sensor noise leads to "group inertia" (e.g., bee swarms) 

- Let $E^i = [{e_p^i}^\top,\ {e_v^i}^\top]^\top$ and $E = [{E^1}^\top, {E^2}^\top, \ldots, {E^N}^\top]^\top$

Theorem 1: Swarm trajectories will converge (in finite time) to the compact set

$$\Omega_b = \left\{ E : \ \left\| E^i \right\| \leq 2\frac{\lambda_{max}(P)}{\lambda_{min}(Q)}\beta, i = 1, 2, \ldots, N \right\}$$

$$\beta = 2k_a\left(D_p + D_v\right) + 2k_f D_f + k_r r_s (N-1)\exp\left(-\frac{1}{2}\right)$$

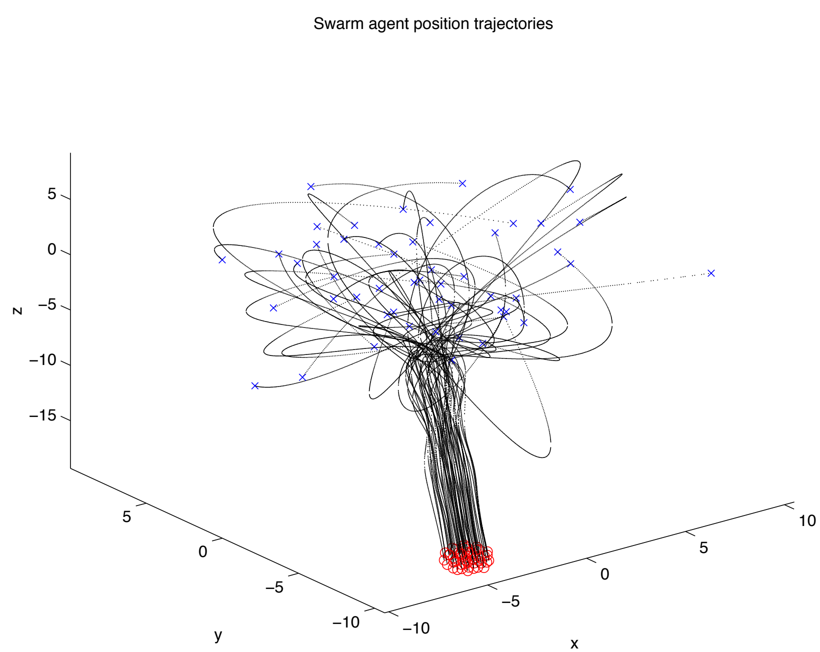
- Proof outline:

1. Lyapunov function $V(E) = \sum_{i=1}^{N} V_i\left(E^i\right)$,
   $V_i\left(E^i\right) = {E^i}^\top P E^i$

2. We have $\lambda_{max}(P)$, the maximum eigenvalue of P,

$$\dot{V}_i \leq -\lambda_{min}(Q)\left(\left\|E^i\right\| - \frac{2\lambda_{max}(P)}{\lambda_{min}(Q)}\|g^i(E)\|\right)\left\|E^i\right\|$$

3. $\|g^i(E)\| < \beta$? Finite repel!



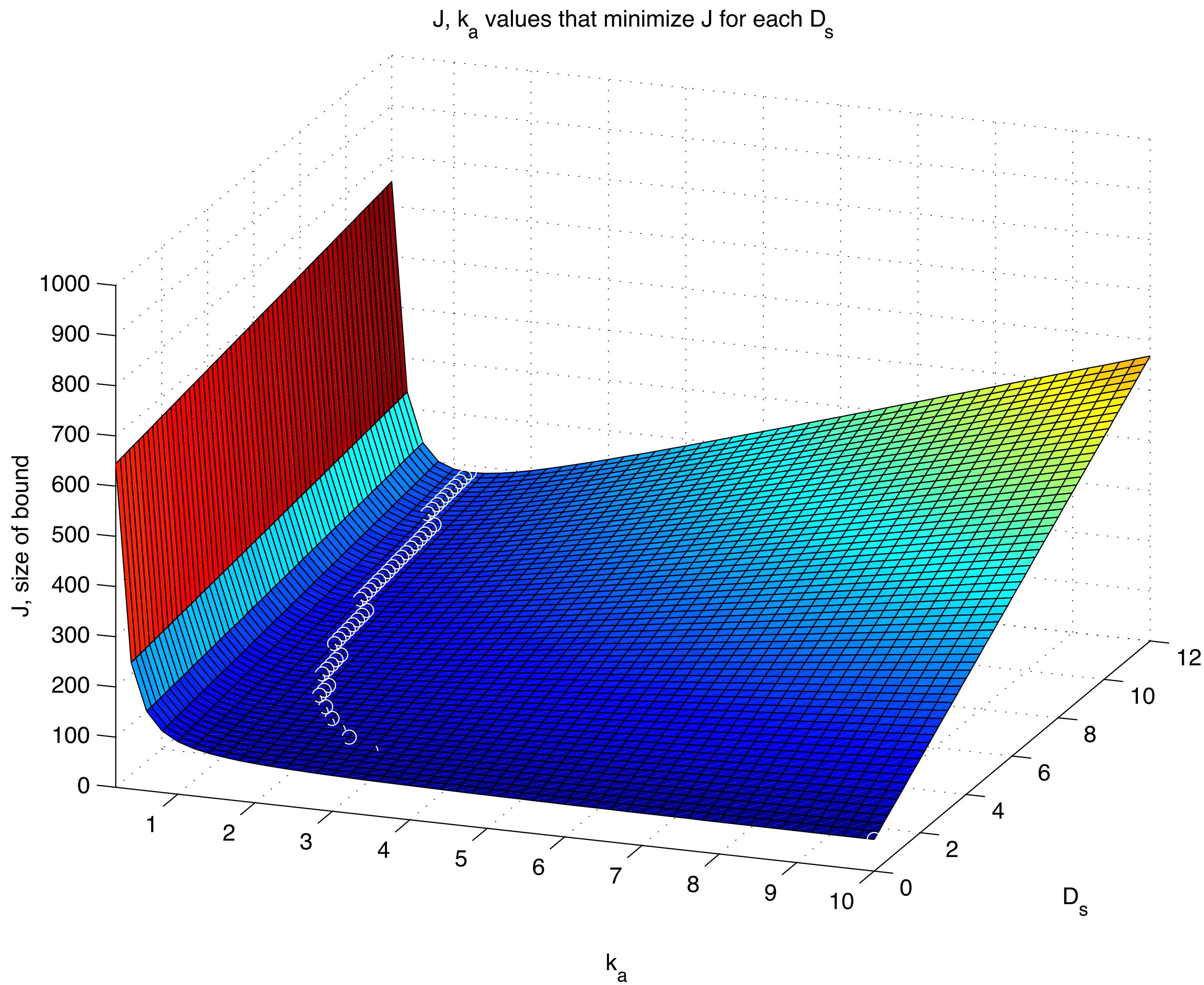
➙ Remarks: Effect of parameters on $|\Omega_b|$?

- No sensing errors ($D_p = D_v = D_f = 0$), choose $Q = k_a I$:

$$\Omega_b = \left\{ E : \; \left\| E^i \right\| \le 2k_r r_s (N-1) \frac{\lambda_{max}(P)}{\lambda_{min}(Q)} \exp\left(-\frac{1}{2}\right), \; i = 1, 2, \ldots, N \right\}$$

  - N, k_r, r_s fixed: $k_a \uparrow \Rightarrow |\Omega_b| \downarrow$, up to a point (collisions).
  - Fixed N, k_a, and k_r: $r_s \uparrow \Rightarrow |\Omega_b| \uparrow$.
  - Fixed k_r, k_a, and r_s: $N \to \infty \Rightarrow |\Omega_b| \to \infty$ (line), but average errors could be small.

- Sensing errors:
  - $D_p \uparrow D_v \uparrow D_f \uparrow \Rightarrow |\Omega_b| \uparrow$ (no R effect)
  - Fix noise at some level, effect of k_a?
  - Choose $Q = k_a I$, let $D_s = D_p + D_v$.
- Let $J = \frac{1}{2}|\Omega_b|$ and suppose that parameters are chosen (by evolution) to minimize this.


J, k_a values that minimize J for each D_s
J, size of bound
1000
900
800
700
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
k_a
0
2
4
6
8
10
12
D_s

Cohesive Social Foraging in Noise: Extensions

- More general noise (work with Yanfei Liu):

$$
\begin{aligned}
\|d_f\| &\leq D_f \\
\|d_p^i\| &\leq D_{p_1} \left\| E^i \right\| + D_{p_2} \\
\|d_v^i\| &\leq D_{v_1} \left\| E^i \right\| + D_{v_2}
\end{aligned}
$$

- Geometric meaning?
- Conditions for swarm cohesion?
- Non-identical agents
- Trajectory tracking

Cohesive Social Foraging, No Noise

- Goal: Show connections to optimization perspective
- Modify above theory to get:

$$\Omega_b' = \left\{ E : \left\| E^i \right\| \leq \frac{2k_r r_s (N-1)}{k_a} \exp\left(-\frac{1}{2}\right), \; i = 1, 2, \ldots, N \right\}$$

- Choose $V^o(E) = \sum_{i=1}^{N} V_i^o\left(E^i\right)$

$$V_i^o\left(E^i\right) = \frac{1}{2} k_a {e_p^i}^\top e_p^i + \frac{1}{2} k_a {e_v^i}^\top e_v^i + k_r r_s^2 \sum_{j=1, j \neq i}^{N} \exp\left(\frac{-\frac{1}{2} \|e_p^i - e_p^j\|^2}{r_s^2} \right)$$

  - Not a standard Lyapunov function

- View u^i as being chosen to minimize $V^o(E)$

- LaSalle's Invariance Principle: If $E(0) \in \Omega$ (invariant set) then $E(t)$ will converge to the largest invariant subset of

$$\Omega_e = \{E : \; e_v^i = 0, \; i = 1, 2, \ldots, N\} \subset \Omega$$

- Hence $e_v^i(t) \to 0$ as $t \to \infty$.
- Follow profile? $\bar{v}(t) \to -\frac{k_f}{k_v} R$ and $v^i(t) \to -\frac{k_f}{k_v} R$ for all i as $t \to \infty$ (group follows the profile)

Additional work...

- "Stability Analysis of Swarms," [1]
- "Stability Analysis of M-Dimensional Asynchronous Swarms with a Fixed Communication Topology," [4]
- Model/analyze bee swarms, [2]

★ Current work with Yanfei Liu (CDC/TAC):

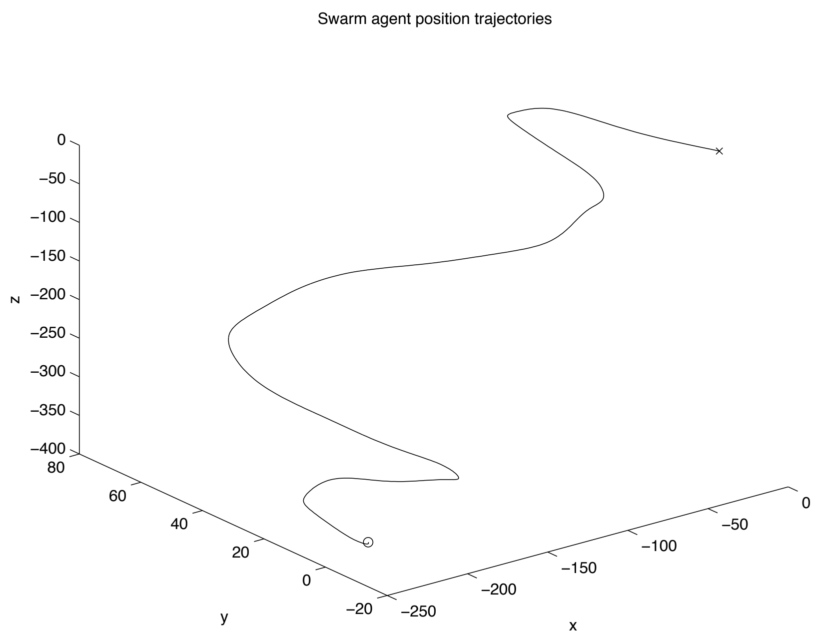
- General noise conditions
- Network effects (delays, topology, reconfiguration)
- Why should we be able to get a result?

Biology: Cooperative Foraging?

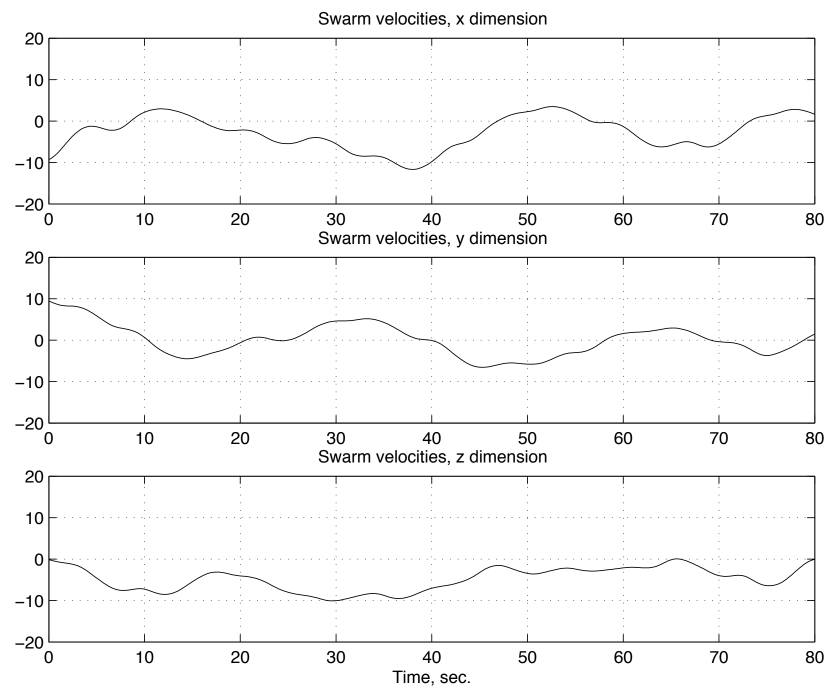
- ★ Groups can climb noisy gradients better than individuals (some organisms can forage more successfully in groups than by themselves–Grunbaum)
- ★ In getting your next meal it is best to cooperate!

➙ Why cooperate?

1. Gain since individuals exploit group information about best direction to go
2. Lose since group moves slower to better sources
3. Overall is there a gain? Apparently so...

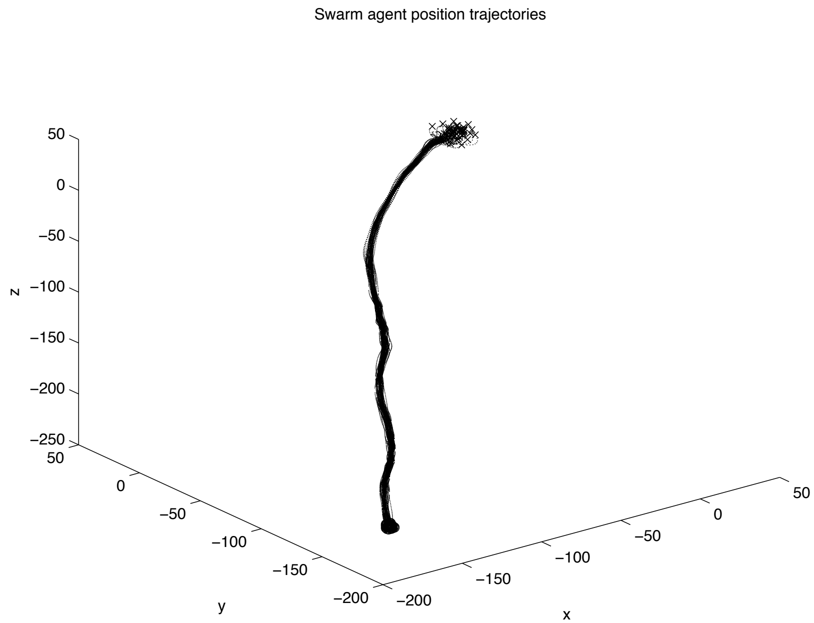


(a) Agent positions.

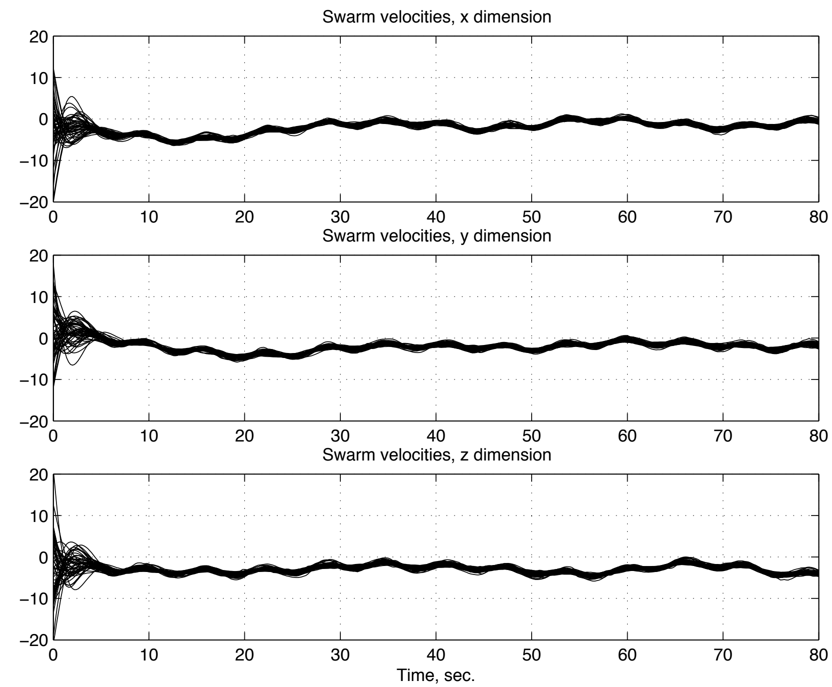


(b) Agent velocities.

Figure 3: Linear noise bounds case, plane profile $(N = 1)$.



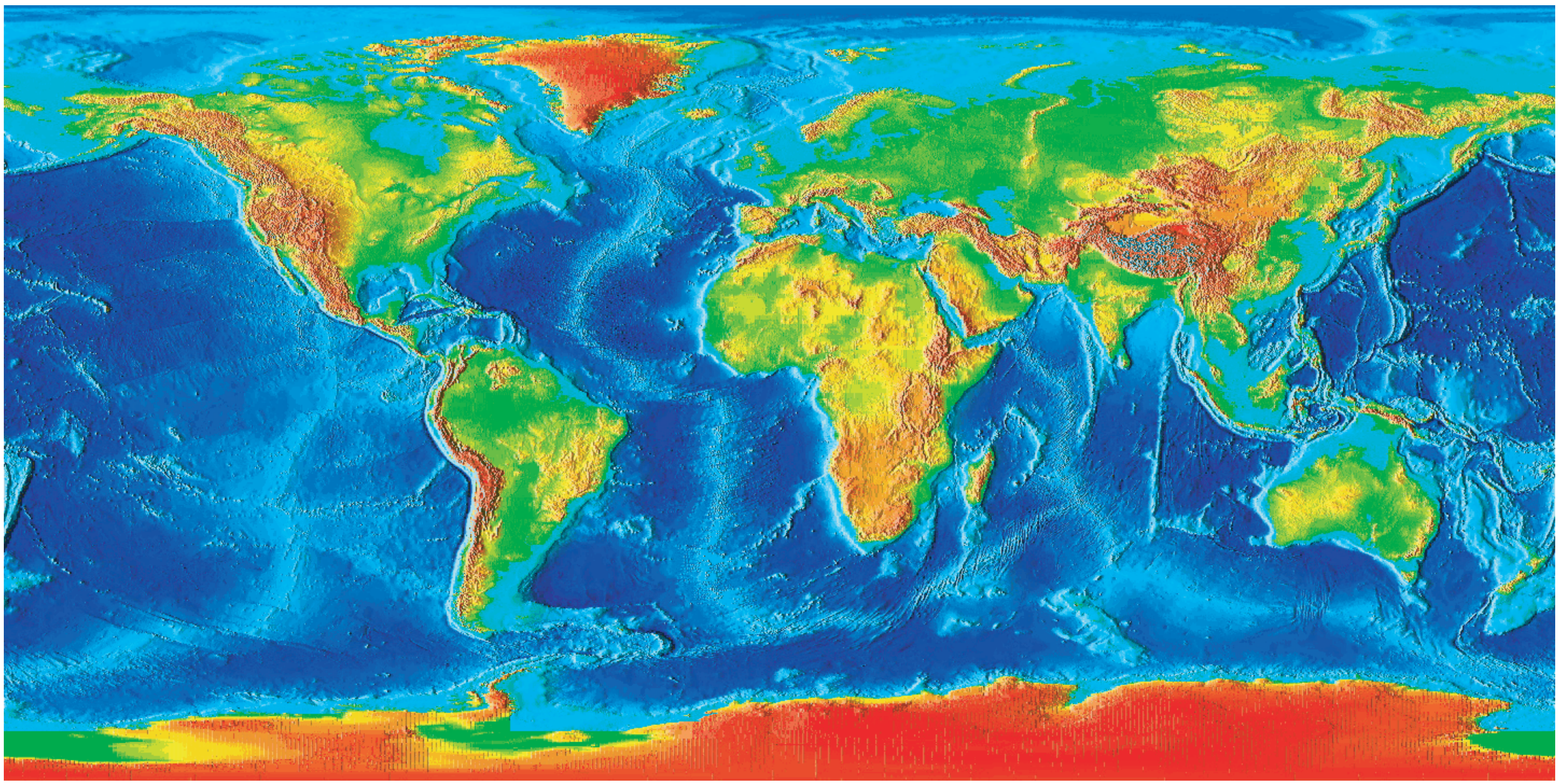
(a) Agent positions.

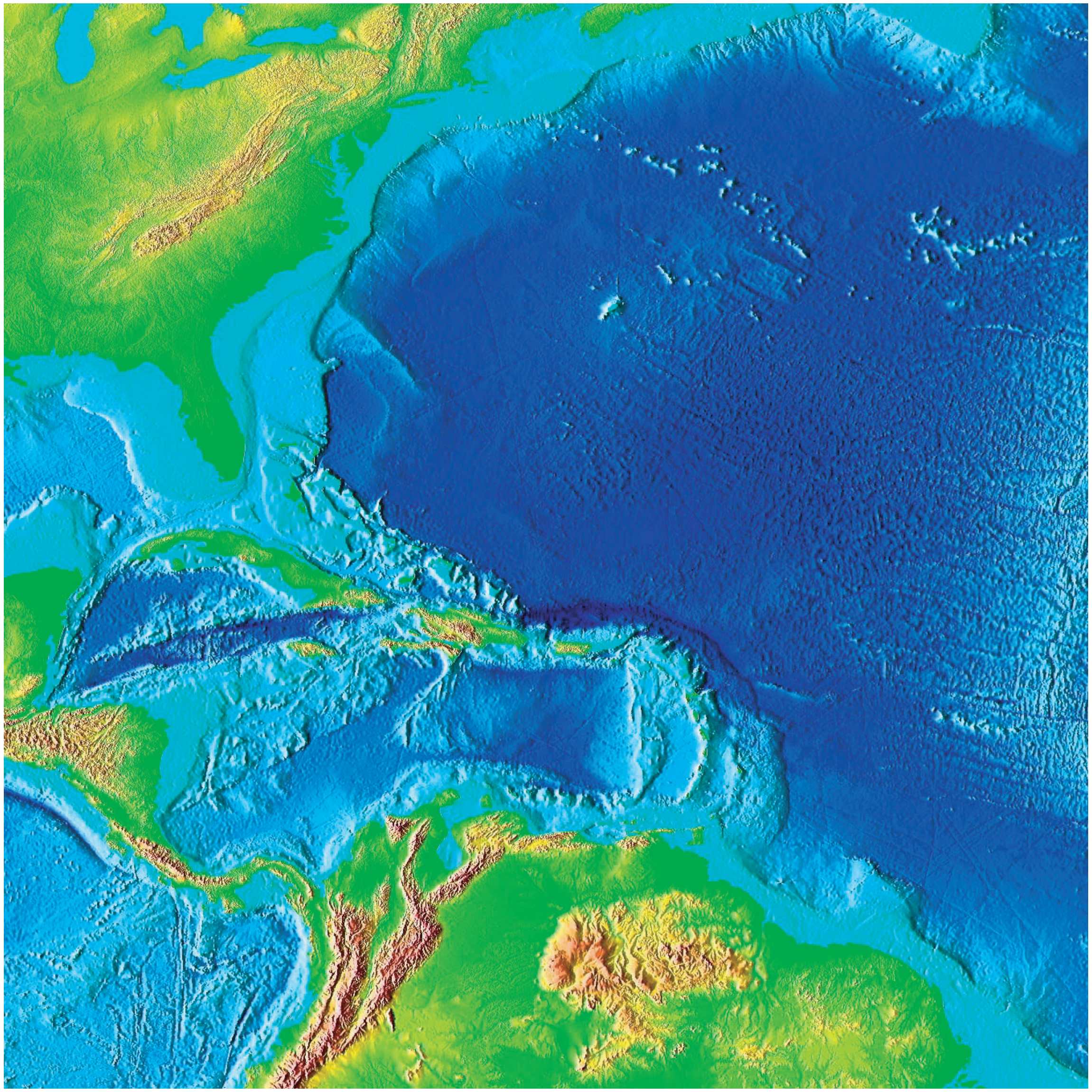


(b) Agent velocities.

Figure 4: Linear noise bounds case, plane profile ($N = 50$).

What about group climbing of more interesting surfaces? Mountains?





Social Coffee Foraging

- Arabica coffee bean grows best at elevations of about 1000 to 1800 meters
- Topographical data for Colombia:
  - National Geophysical Data Base, 5 minute data
  - Use linear interpolation for points in between available data

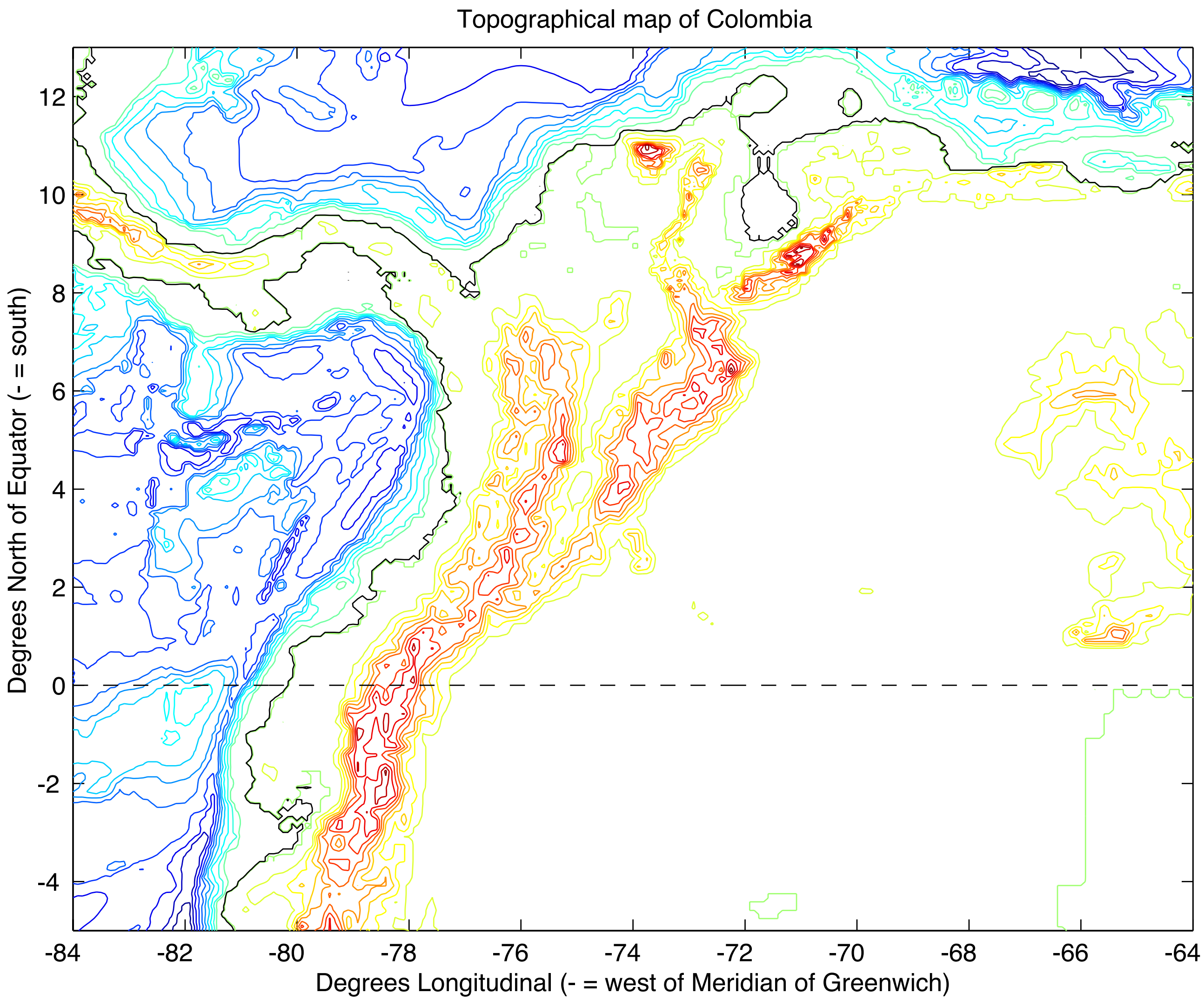
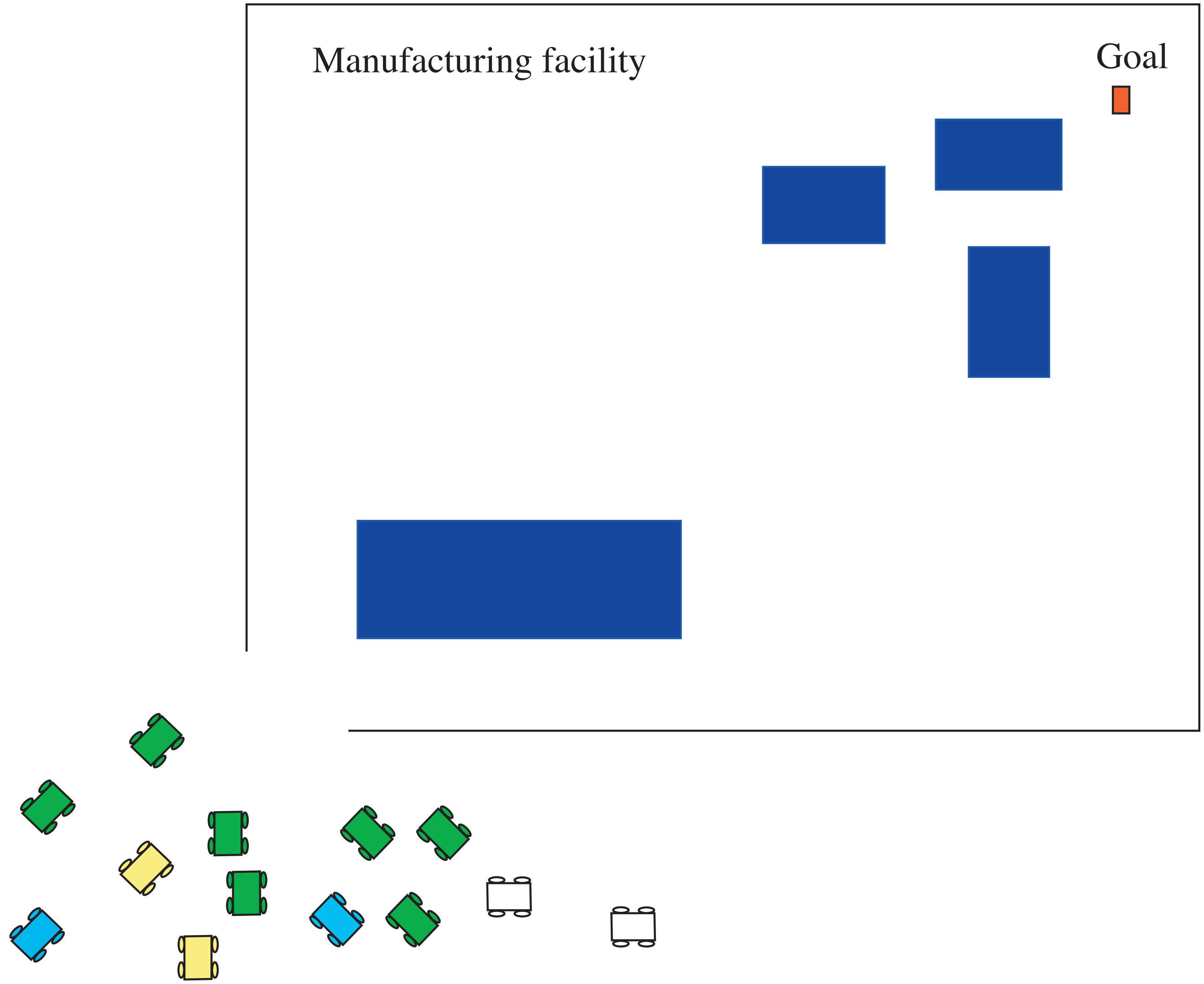


Figure 5: Topographical map of Colombia.

- Given an altimeter can agents socially climb mountains to find all coffee growing regions in Colombia?
  1. Avoid each other
  2. But try to stay together (helps each other)
  3. Use modified terrain map...
- Cost function: Gaussian function of elevation, centered at 1400 meters
- Movie: Due to Yanfei Liu...

Movie...

Application: Robotic Swarms



"Potential fields approach" to autonomous vechicle guidance, no noise...

With noise...

Intelligent Vehicle Swarms

Use ideas from intelligent social foraging?

- Planning, attention, learning, etc. How?
- What are network effects (delays, topology)?

Mathematical analysis possible? Important? Yes! (verification and validation)

- What can we achieve via cooperative robotic systems?
- Many challenges!

Concluding Remarks

✔ Foraging swarms:

1. Bio-inspiration, optimization models
2. Mathematical stability analysis of swarm cohesion
3. Application: Robotic swarms in manufacturing

★ Book: "Biomimicry for Optimization, Control, and Automation," to appear

★ http://eewww.eng.ohio-state.edu/~passino/ciiee03.html

References

[1] V. Gazi and K. M. Passino. Stability analysis of swarms. *To appear, IEEE Trans. on Automatic Control*, 2003.

[2] V. Gazi and K.M. Passino. Modeling and analysis of the aggregation and cohesiveness of honey bee clusters and in-transit swarms. *Submitted to J. of Theoretical Biology*, 2002.

[3] Y. Liu and K. Passino. Biomimicry of social foraging behavior for distributed optimization: Models, principles, and emergent behaviors. *J. of Optimization Theory and Applications*, 115(3), December 2002.

[4] Y. Liu, K. M. Passino, and M. M. Polycarpou. Stability analysis of m-dimensional asynchronous swarms with a fixed communication topology. *IEEE Transactions on Automatic Control*, 48(1):76–95, 2003.

[5] K.M. Passino. Biomimicry of bacterial foraging for distributed optimization and control. *IEEE Control Systems Magazine*, 22(3):52–67, June 2002.